Introduction to the Lerner Diagram

Alan V. Deardorff
University of Michigan
July 1, 2002

The Lerner Diagram originated with Lerner (1952) as a tool for relating goods prices and factor prices in a two-factor, two-good, Heckscher-Ohlin model. Its essential feature, in my view, is that it uses unit-value isoquants, not unit isoquants, permitting it to identify from the common tangent between them the factor prices that are consistent with producing two goods. Although sometimes called the Lerner-Pearce Diagram because Pearce (1952), in his debate with Lerner, used unit isoquants to make his point, the more important innovation of using unit-value isoquants seems clearly to have been due to Lerner alone. How very useful it could be, however, was shown somewhat later by Findlay and Grubert (1959), who attributed it to Lerner and who used it for the quite different purpose of sorting out the effects of growth. Since then, trade theorists have used it extensively in a variety of contexts.

Intro to the Intro

An easy entrée to the Lerner Diagram can be provided by first considering the simpler case of a one-sector economy shown in Figure 1. The only good, X, is produced from two factors, K and L, with constant returns to scale, so that the entire production function for the good can be

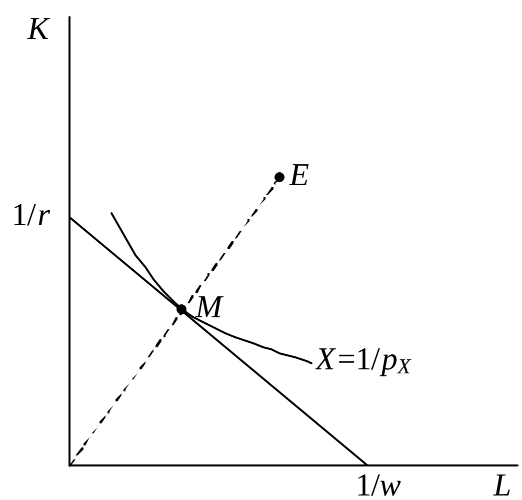

Figure 1

represented by any single isoquant. With only one sector, it doesn't mean much to speak of its price, but do so anyway, and let this be the isoquant for producing one dollar's worth of *X*, $X=1/p_X$, or its unit-value isoquant.

Suppose next that the country's supplies of the two factors, its endowments, are those shown at point *E*. What can we say about the market-clearing prices of factors under perfect competition? Whatever the factor prices may be, all firms will use the technology available to produce *X*, selecting factor proportions that will minimize their costs. In order to induce them to demand the factors in the same proportions as point *E*, this cost minimization must lead them to the point labeled *M*, on the same ray from the origin occupied by *E*. In addition, to avoid a profit or loss that would lead to entry or exit, this cost-minimizing bundle of factors must also be worth exactly one dollar, just like the amount of *X* that it produces. Therefore, the isocost line drawn through *M* must represent one dollar's worth of factors. Hence its horizontal intercept is one dollar's worth of labor, or one over the wage *w*, while its vertical intercept is one dollar's worth of capital, one over the rental *r*, as labeled. If one wished, one could also measure the country's national income using a parallel isocost line through *E* and comparing it to this one. In this case, national income appears to be about $2.

From such a diagram we can see immediately how factor prices depend on factor endowments in a one-sector economy. In Figure 2, starting from the same factor endowment *E*, I consider both a proportional expansion of both endowments to *E¢*, and an expansion of only the labor endowment to *E¢¢*. Clearly, the former leaves factor prices unchanged, while the latter requires a fall in the wage and a rise in the rental.

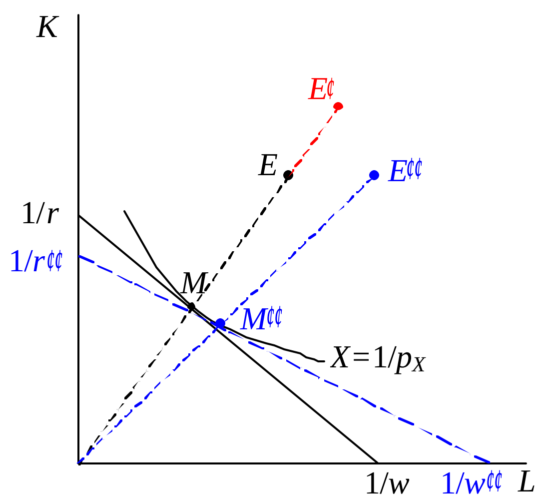

Figure 2

Two Sectors, and Lerner Emerges

Now suppose that there are two sectors, producing goods *X* and *Y*, and that the prices of these goods are given as p_X and p_Y. The unit-value isoquants for the two sectors in Figure 3 now give two different sets of techniques for producing a dollar's worth of output. And under the standard assumptions of constant returns to scale and perfect divisibility of both factors and goods, these are not all. Any convex linear combination (that is, any weighted average) of these techniques will serve the purpose as well. Thus, all straight lines connecting points on the *X*-

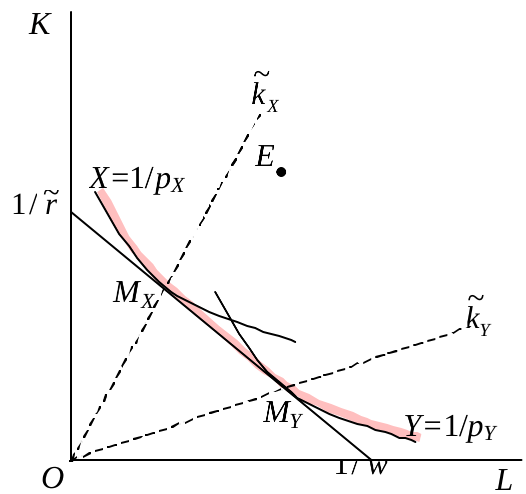

Figure 3

and *Y*-unit-value isoquants will also produce a combination of the two goods that will be worth a dollar. Of these, the least cost combinations are those along the common tangent to the two unit-value isoquants, shown in Figure 3 as the line segment $M_X M_Y$. Thus the isoquant for producing a single dollar, can be thought of as this line segment, together with the upper-left portion of the *X* isoquant and the lower-right portion of the *Y* isoquant, or the "convex hull" of the isoquants shown as the shaded curve, XM_XM_YY.

This convex hull can be used in exactly the same way as the single unit-value isoquant of the one-sector economy in Figure 1. That is, for any factor endowment such as *E*, a ray from the origin to it will identify the point on this convex hull where production must, in a sense, take place. If this point is on one of its curved portions, this will require producing only one of the goods, with the technique indicated and with factor prices given by the tangent to that curve (not shown in Figure 3). But if this point is on the straight portion of the hull, as it would be for the endowment *E* shown in Figure 3, then least-cost production of a dollar's worth of output requires that both *X* and *Y* be produced, using the techniques at M_X and M_Y respectively, in proportions so that their combined use of factors matches the proportions at *E*. That is, endowments between the rays $O\tilde{k}_Y$ and $O\tilde{k}_X$ require that the economy produce both goods. Hence the name of this region in trade theory, the "diversification cone." Endowments outside that cone, on the other hand, will require that the economy completely specialize in either *X* or *Y*.

Of course, the most important message is not this, but what the Figure tells us about factor prices. The unit-value hull can be used in exactly the same way as the unit-value isoquant of the one-sector economy to determine factor prices, but the message is quite different because of its straight segment. For any factor endowments in the diversification cone, the tangent to the hull is that straight segment itself, and the factor prices must therefore be the same: those shown in Figure 3 as $\tilde{w}$ and $\tilde{r}$. This is the familiar message of the Factor Price Equalization (FPE)

Theorem: that two countries trading freely and thus facing the same prices of goods, if their technologies are the same and their factor endowments are sufficiently similar so that they are in the same diversification cone, will have the same factor prices.

Lerner Meets Edgeworth

Lerner did not do this, but it is a small step to integrate the Edgeworth (production) Box into the Lerner diagram, now that it includes both factor endowments and the factor proportions in the two industries. One can imagine an Edgeworth Box with corners at O and E, and a contract curve connecting them, but all of that is hardly needed. Just complete the parallelogram between these points by drawing lines parallel to $O\tilde{k}_Y$ and $O\tilde{k}_X$ down and to the left from E, and the new intersections must then be the factors allocated to the two industries. This is done in Figure 4 for endowment E, which will be allocated to the two industries at points (L_X,K_X) and (L_Y,K_Y), which sum to E. The isoquants through these points denote the country's outputs of the two goods, the values of which can be inferred by comparison with the unit isocost line. As drawn, it looks like the country produces about a third of a dollar's worth of Y and a dollar and two-thirds of X.

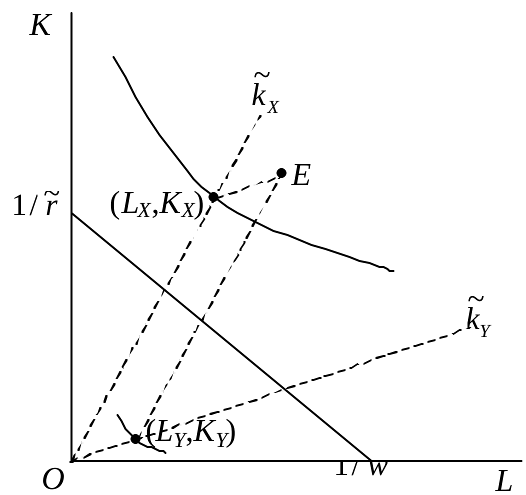

Figure 4

It is then quite easy to use the diagram to find the effects, at constant prices, of a change in endowments on outputs. In Figure 5, for example, is shown the effect of an increase in the endowment of capital holding the labor endowment constant. The advantage of doing this in the Lerner diagram is that it readily shows the effects both inside and outside of the diversification cone. As shown, a not-too-large increase in capital endowment from E to E' increases the factor allocation in X from v_X to v_X' and reduces the allocation in Y from v_Y to v_Y', showing the usual Rybczynski result that the capital-intensive output rises more than in proportion to the capital increase and the other output falls. A further increase in endowment on the other hand, to E'', moves the endowment point outside the diversification cone, reducing the factor allocation in the Y industry to zero and no further, while extending but also tilting the vector of

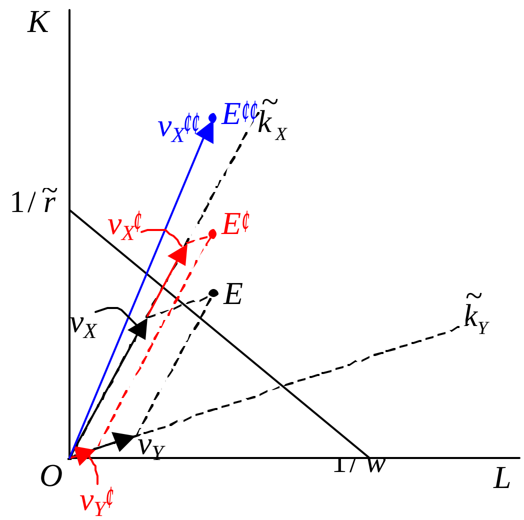

Figure 5

factors in X to v_X¢¢. Beyond the border of the cone, output of X rises but less than in proportion to the increase in capital.

Stolper-Samuelson

The original purpose of the Lerner Diagram in Lerner (1952) was to examine price changes, so it is fitting to close with that here. An increase in the price of, say, good Y, holding the price of X constant, means that a dollar can be earned from production of a smaller quantity of Y. Thus the unit value isoquant of good Y shifts radially inwards toward the origin by the percentage of the price increase. This is shown in Figure 6. This inward shift causes the common tangent to the X and Y unit-value isoquants to rotate clockwise, as shown, showing immediately that the

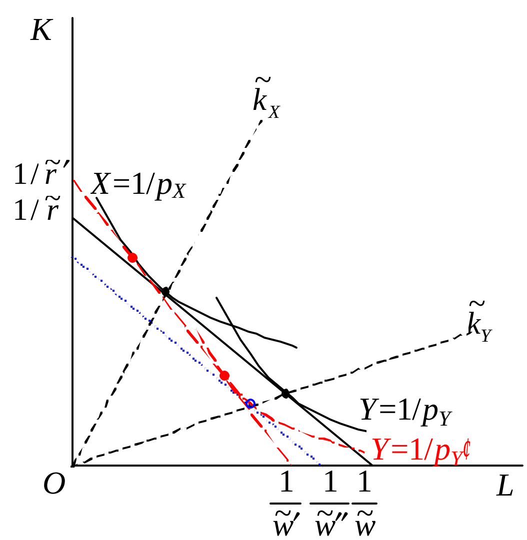

Figure 6

wage-rental ratio corresponding to diversification, $\tilde{w}/\tilde{r}$, must rise, since it is the slope of this common tangent. Its intercepts also show immediately that the nominal wage of labor rises, while the nominal rental on capital falls, both with good *X* as numeraire. The latter is sufficient for the Stolper-Samuelson result that the real rental on capital (the factor *not* used intensively in the sector whose price has risen) falls. To finish the Stolper-Samuelson Theorem we need the effect on the real wage, which requires comparing the nominal wage to the increase in price of *Y*. The construction of the (blue, dotted) line parallel to the old isocost line but tangent to the new isoquant serves this purpose, since it identifies a wage, $\tilde{w}''$, that has increased by the same percentage as p_Y. Since $\tilde{w}'$ is larger than this, the real wage rises.

The diagram can also be used to find the effects of a price change on factor allocations and thus outputs. This is done in Figure 7 for the same increase in p_Y as Figure 6. Using the same construction of factor allocation vectors, v_X and v_Y, as in Figure 4, we find that more factors are

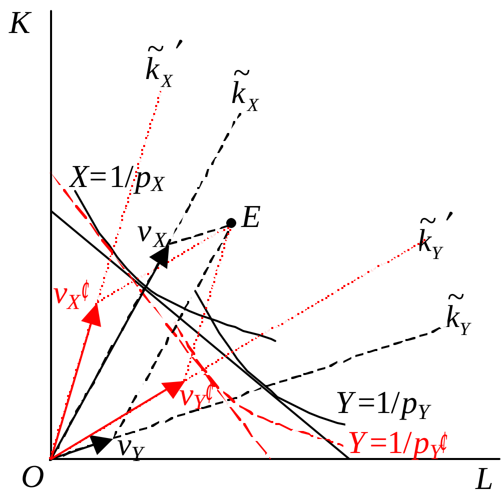

Figure 7

allocated to good *Y*, and less to good *X*, than before the price change. Hence output of *Y* rises and output of *X* falls. These vectors also, if one is interested, trace out the contract curve in the implicit Edgeworth production box between the origin and the endowment point *E*.

References

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