Last time the series should have looked like

t	Xt	
0	0	
1	x	results in $x_t = x(1-d)^{t-1}$ where $t=1,\ldots,\infty$
2	x(1-d)	
3	x(1-d)^2	PV=Σt=0to∞

See discounting handouts.
One additional point: when benefits appreciate at a rate larger than the discount rate <u>and</u> the stream of payments is infinite, the benefit is infinite. This is because you can't borrow enough now to make you indifferent between investing in your project and lending elsewhere. r won't return enough to make up your opportunity cost!

Risk

Risk is about uncertainty. This is not necessarily bad: chances for a big gain, chances for a big loss are both forms of "risk." Some projects have risk as a central feature.

How do we quantify benefits and costs when returns are uncertain? Credit markets are sources of information about risk management: lenders build in an interest premium to the market interest rate to adjust for the fact that some loans default. Government doesn't pay the same interest rate that we do because the government is viewed as a riskless investment.

Example: bridge that has a probability of falling down/becoming useless in 10 years.

- tempting to increase the discount rate, but that won't work because credit markets (where interest rates come from) insure against a particular kind of risk, which is not the kind of risk that we're talking about in the case of a derelict bridge.
- another option would be to use the expected value of the outcome. This would also be wrong in many circumstances because attitudes toward risk are not yet accounted for.
  - dollar on the flip of a coin example: e=50¢ but even that's too high.
  - dollar on the flip of a coin when wanted Coke: willing to pay more than expected value. This is how the lottery works.

dollar on the flip of a coin at a price of 20¢ when you only have 20¢ and you need it for bus fare; willing to pay much less than the expected value.

So what should we use? Certainty equivalents of risky outcomes. Certainty equivalent is another version of willingness to pay, but with a twist: it's the certain benefit/cost that the recipient/payer would regard as equally desirable, in terms of utility, as the gamble s/he is offered.

Outcome X
E(x)
CE(x)
CE≠E (note that CE isn't always smaller, though often is).

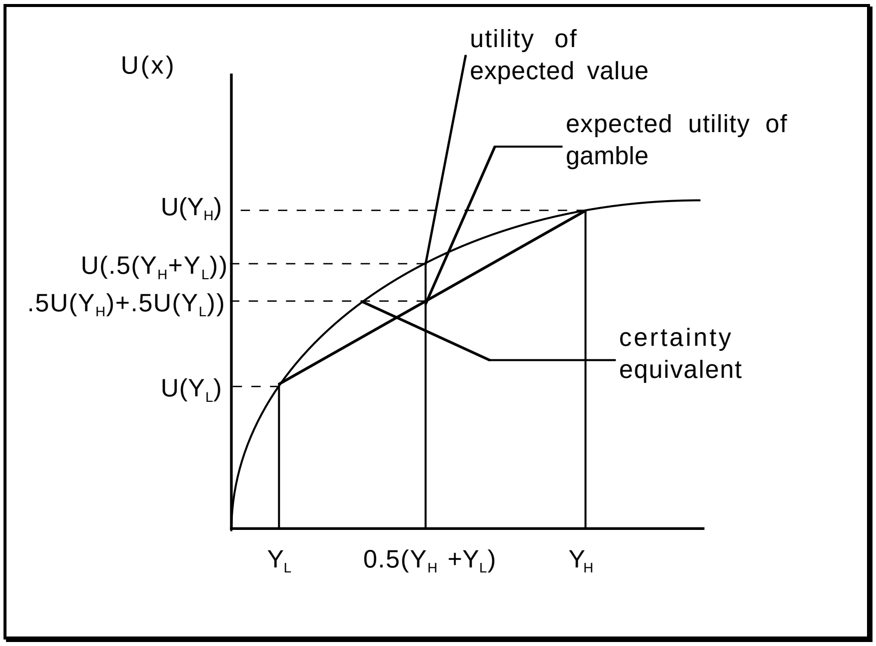
If x is repeated many times, then <u>maybe</u> E(x) equals the CE(x). Each time you flip the coin, there's a 50-50 chance that you'll win; but if you flip the coin ∞ times, you're certain to win half the time and the result converges to 50¢. why only <u>maybe</u>? You're paying many times when you do the slot machine or the lottery, but your expected value is still negative.

If in general, CE(x)<E(x), the individual is "risk averse." We believe that people generally fall into this category.
If in general, CE(x)>E(x), the individual is "risk loving."
If in general, CE(x)=E(x), the individual is "risk neutral."

Note that this whole concept is very subjective: it's all about perceptions and preferences; in other words, personal utility determinations.

How do we deal with this stuff? Don't look at the expected value of the outcome, but look at the utility of the expected value!

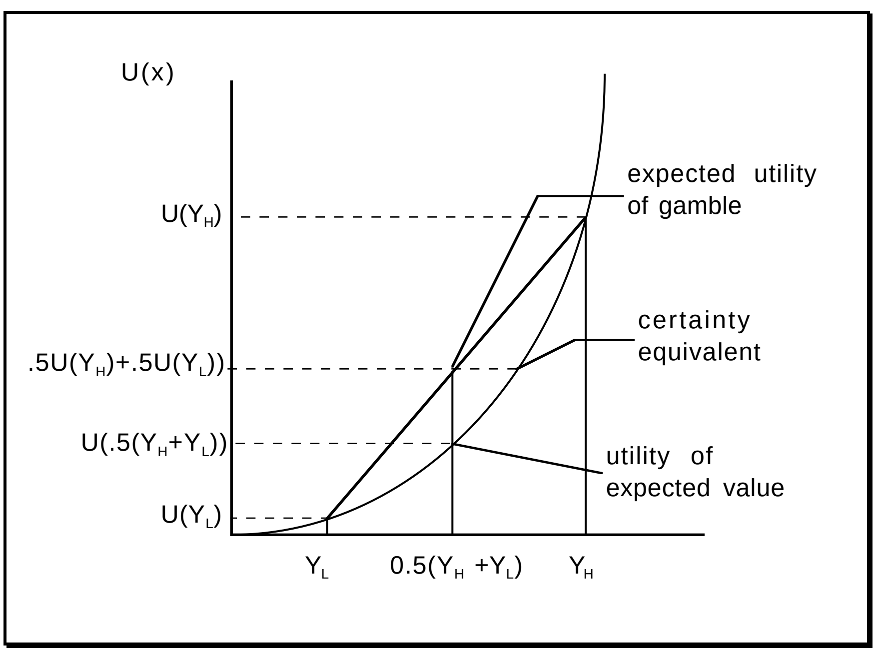
Expected utility framework (income example)



This graph shows increasing total utility as income increases, but decreasing marginal utility. This person is facing a gamble between two outcomes: low and high, with equal probabilities.

Certainty equivalent is the amount of certain money that gives the same amount of utility as the gamble. In this case, $CE(x) < E(x)$.
Expected utility of a gamble is not the same as the utility of the expected value of the gamble.
$E(u(x)) \neq U(E(x))$

why did this work? Because of the shape of the utility function—this person is risk averse. See what happens with a risk lover:



Here, $CE(x) > E(x)$.

What explains people that both have insurance and gamble? different situations, different perceptions of utility. Funny curve.

What good is this? You get a nice answer, but unless you know the shape of the utility function this isn't particularly useful. Insurance markets are a source of some utility function information, but as we've shown is that curves are different for different people for different situations.

What Ned concludes from this: if the gambles are small compared to overall income, we can approximate your utility function as a straight line and risk doesn't need to be adjusted for (use expected value). Problem is that many of the problems we use benecost for are big things where risk is important. What do we do instead? Look at all the different outcomes, see what happens to who in different situations, and determine how we feel. Sensitivity analysis of a different type.

Case : Ice Cream Social

You're the president of the PTO and you're trying to decide whether to hold one of these things. You'll raise some money by selling tickets on the day of the event, and people will enjoy themselves, but you have to set the date ahead of time, it might rain, and you have to spend some money to set up the event.

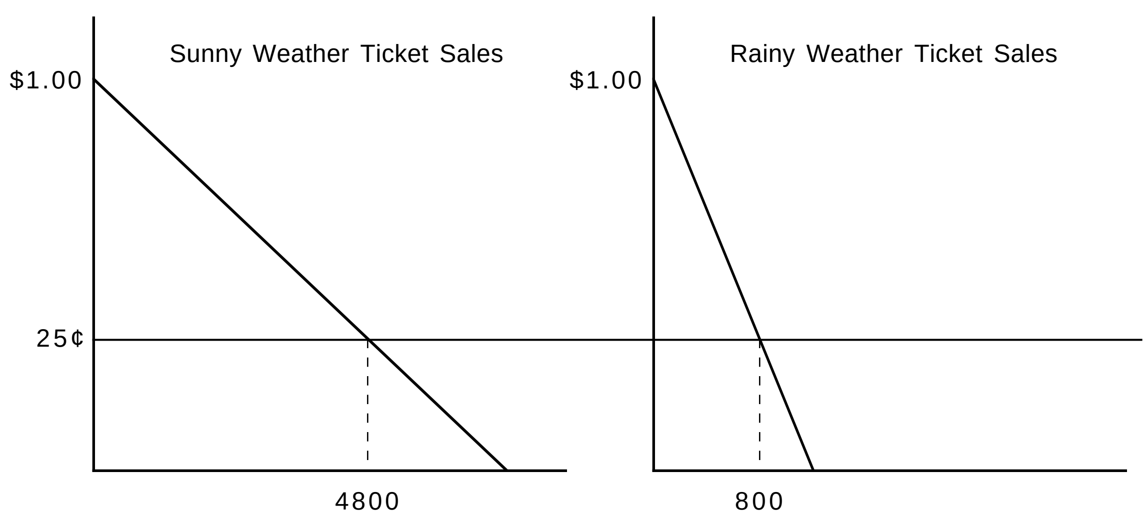
Setup costs=FC=$500 from PTO treasury

Custom or regulation tells you that ticket can only be 25¢

Half of the ticket price comes back as profit (before deducting FC); in other word, variable costs are half of the ticket price.

Probability of sunny weather is 90%, tickets sold are 4800

Probability of rain is 10%, tickets sold are 800



What decision would you make if you were risk averse, risk loving, profit maximizer? tweaks: variable ticket price, diehard members sure to come plus less definites.