$$V = \sum_{t=0}^{\infty} \frac{X_t}{(1+r)^t} \rightarrow\rightarrow\rightarrow V = \frac{X}{r}$$

where

$t = 1, 2, ..., \infty$

$X_t = X$

$$V = \frac{X}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

where

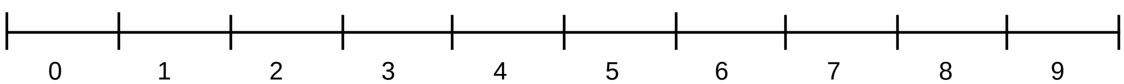
$t = 1, 2, ..., T$

$X_t = X$

<u>Time in discounting</u>

Having selected years as our time period, how do you deal with events within those years?

Time extends along a line, divided into periods:



The initial period is the present, so the first period is labeled 0. Anything that happens during that period needs no discounting because it's in the present already.

If the event in question is a day at the beginning of the year, you might be tempted to re-label the first period as 1 and discount it, but you shouldn't do that. If it's important to differentiate between days, you should be discounting using days as your time unit measurement. The formulas we use here are all annually-based.

If you have an exchange rate or price that varies moment to moment, the most correct one to use for purposes of discounting is to try to get an average over the discounting period.

<u>Special cases</u>

Suppose this thing isn't constant over time, but changes in a systematic way over time. How do we handle that?

<u>Decaying value</u>

Suppose that you have something that decays at a constant rate.

$$X_1 = X \qquad X_2 = \frac{X}{(1+d)} \qquad X_3 = \frac{X}{(1+d)^2} \rightarrow\rightarrow\rightarrow X_t = \frac{X}{(1+d)^{t-1}} \rightarrow\rightarrow\rightarrow \infty$$

plug these values for x into the discounting formula.

For derivation of finite formula with depreciation, see discounting handouts

Example: build bridge, charge toll, assumptions re: time change the formula you use. (come up with a bunch of different tweaks on the same facts).

<u>appreciation</u>

for derivations see handouts

Think of this as negative depreciation.

What if d is both negative and larger than r? Your discounting term is negative, which means that your result will be negative. This seems strange because your benefit is appreciating at a huge rate! How to explain this?

In our derivation you have a final term. One of these terms is left at the end (one series is longer than the other), but the final term is extremely small. But if you have a D that is negative, that final term is huge because you're growing faster than you're discounting. In this case there's no justification for canceling all the terms. Your answer is wrong.

What is the <u>right</u> answer?

If the benefit grows at a rate faster than the discount rate and the stream is infinite, the benefit is infinite!

<u>inflation</u>

Suppose that all prices increase at the same rate, g. Here, we're dealing with real prices that are constant, but nominal prices that increase at rate g.

$P_t = (1+g)^t$ where P = 1

$$\sum_{t=0}^{\infty} \frac{P_t X_t}{(1+r)^t (1+g)^t}$$

Suppose that the real interest rate is coupled with real numbers. (real oranges, real orange growth rate). Then we can use our original formulas.

Suppose that we're using prices instead? Must use nominal values.

$N_t = P_t X_t$ = nominal value

$(1+m) = (1+r)+(1+g)$ = nominal interest rate

$$\sum_{t=0}^{\infty} \frac{P_t X_t}{(1+r)^t (1+g)^t} \rightarrow\rightarrow\rightarrow \sum_{t=0}^{\infty} \frac{N_t}{(1+r)^t (1+g)^t} \rightarrow\rightarrow\rightarrow \sum_{t=0}^{\infty} \frac{N_t}{(1+m)^t}$$

Note that the form is the same as our original equation. you just use the real interest rate plus the rate of inflation. Note that you observe nominal dollars and nominal interest rates in the real world, so as long as all your dollar values and interest rates are in the same terms your answers will be fine.

The only problem is that we don't know what future inflation will be; the real interest rate is more stable because it's not influenced by monetary policy. Today's dollars are in real terms (?), so you can match the nominal interest rate minus the inflation rate with today's dollars in your discounting. Of course, the real interest rate is difficult to determine because how you measure it changes the answer.

Bottom line: you should justify how you settled on a real interest rate when you do your analysis. You could also handle the problem by doing a sensitivity analysis: try different rates and see how much the answer changes.

<u>We'll argue tomorrow that riskless measures are the ones to use.</u>