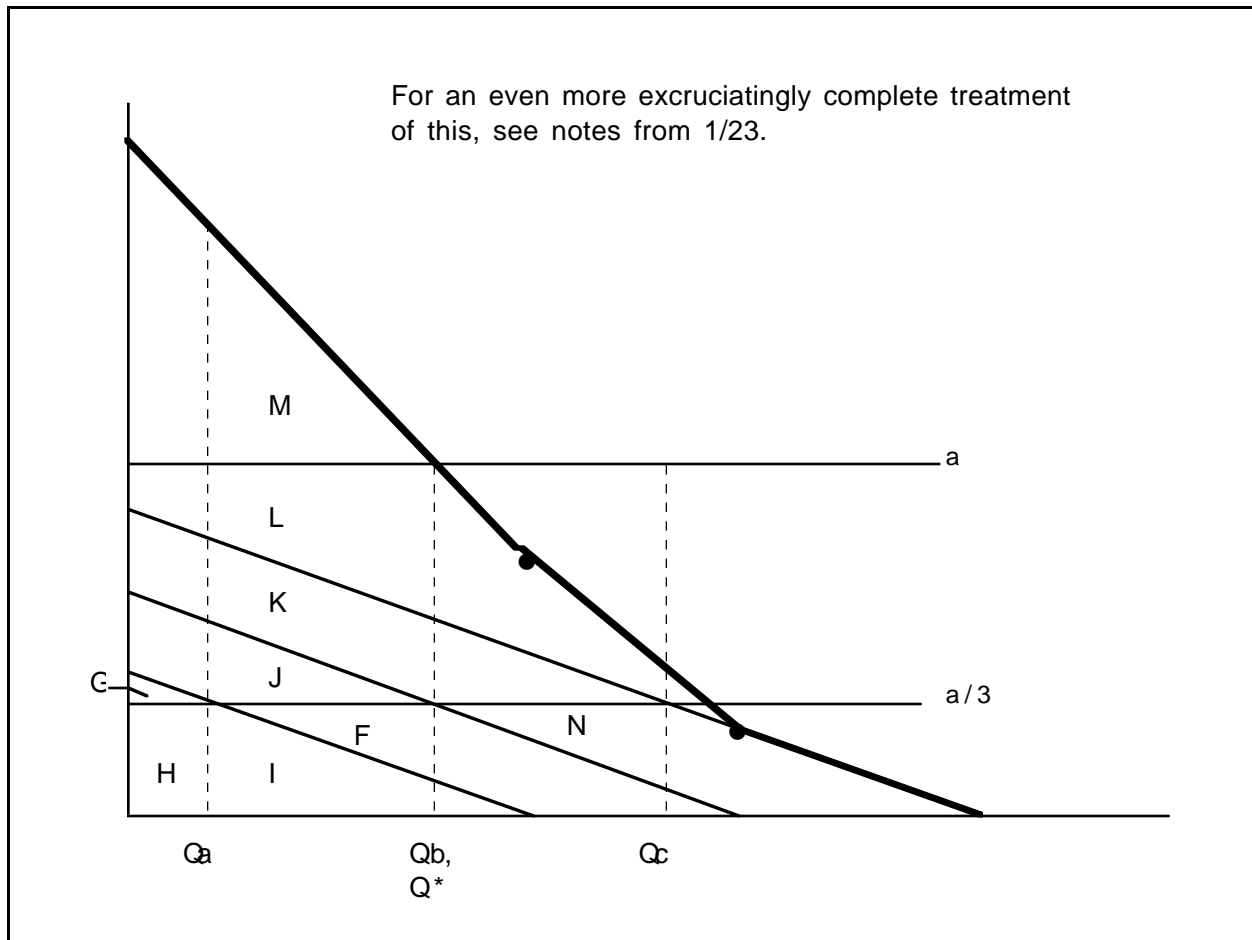


Back to Kaldor-Hicks: If the losers *could* be compensated for their losses by the winners *and* at least one winner would still be better off, the K-H test is passed. If that compensation is *actually* made, the move is also Pareto efficient.

Example:



Here, we have three consumers with parallel demands, but different levels of demand.

First, find the vertical sum of the demands across all relevant ranges of Q . To find the optimal level of Q for society (Q^*), find the point where $MC = MBS$.

If there is no collective action in this economy for this good and $MC = a$, none will be produced because the marginal cost (price) is higher even than the highest level of demand. If the group gets together, forms a collective (government or less formal joint group), and decides to

purchase some of this good, the collective should purchase a level of Q that maximizes the net gain to society. That level is Q^* , the quantity indicated at the intersection of MC and MBS.

Why is this so?

The marginal cost and MBS curves are found by adding individual demands together. Because of this, the area under the MBS curve and above the MC curve is the sum of all the individual gains and losses.

Here, the net gains and losses of A, B, and C in the move from Q_a to Q_b (which in this case happens to be Q^*) are:

$$\begin{aligned} \text{A: } & -F \\ \text{B: } & J \\ \text{+ C: } & J + K \\ & 2J + K - F. \end{aligned}$$

But how do we know that this is a net positive?

In the move from Q_a to Q_b , $MBS > MC$; that is, benefits are greater than costs and the net sum is positive. Therefore, $2J + K - F$ is certainly positive if it equals area M. Let's find out whether it does.

We assert that the sum of gains (benefits) and losses (costs) of individuals equal the sum of gains and losses of society.

$$\begin{aligned} \underline{\Sigma \text{ costs \& benefits of society}} &= \underline{\Sigma \text{ costs \& benefits of A, B, \& C}} \\ (\text{benes})-(\text{costs}) &= (\text{A benes}-\text{costs})+(\text{B benes}-\text{costs})+(\text{C benes}-\text{costs}) \\ (I+F+J+K+L+M)-(I+F+J+K+L) &= [I-(I+F)]+[(I+F+J)-(I+F)]+[(I+F+J+K)-(I+F)] \\ \mathbf{M} &= \mathbf{2J+K-F} \end{aligned}$$

We reach the same conclusion if we simply compare the areas under the curves all the way down to the x axis. The first step is realizing that, by definition,

$$3(MC/3) \equiv MC \quad \rightarrow \quad 3(I+F) \equiv (I+F+J+K+L) \quad \rightarrow \quad 3I+3F \equiv I+F+J+K+L$$

Area under MBS = Σ Areas under A, B, and C

$$(I+F+J+K+L+M) = I+(I+F+J)+(I+F+J+K)$$

$$3(I+F)+M = I + (I+F+J)+(I+F+J+K)$$

$$3I+3F+M = 3I+2F+2J+K$$

$$F+M = 2J+K$$

$$\mathbf{M=2J+K-F}$$

\therefore we've established that the move from Q_a to Q_b (Q^*) satisfies the K-H test, because the sum of the gains and losses is positive and compensation

is theoretically possible. If payments are made, the move is Pareto efficient.

Generally speaking, all points where $MBS \geq MC$ are solutions that pass the K-H test and, if compensating side payments are made, are also Pareto efficient. All points between zero and Q^* meet this definition, so a move rightward to any point up to Q^* passes the K-H test. If compensating side payments are actually made, such a move is also Pareto efficient.

If side payments are not necessary, the move is *Pareto preferred* (in our example everyone wins and no-one loses in a move from zero to Q_a , so across that range side payments are not required).

Note that compensating merely changes who bears the loss; it doesn't change the total social benefit of the move.

Q: If the majority desires a change, is it socially optimal by definition?
In the case we've described above, B and C would vote for Q^* ; A would not, but 2/3 majority wins and we'd get Q^* .

Generally speaking, in a series of votes on various levels of Q , society will move toward the median level of Q . To show this, let's look at individual preferences.

Preferences		
A	B	C
Q_a	Q_b	Q_c
Q_b	$Q_a = Q_c$	Q_b
Q_c	$Q_a = Q_c$	Q_a

Note that B is indifferent between Q_a and Q_c because all the demands in this system are parallel and equidistant, so triangles J (benefit foregone if we remain at Q_a) and K (cost imposed if we move to Q_c) are the same size and are equally negative outcomes for B.

Given the above preferences, what happens in a series of elections?

Q_a v. $Q_b \rightarrow Q_b$ (A votes for Q_a , B & C vote for Q_b)

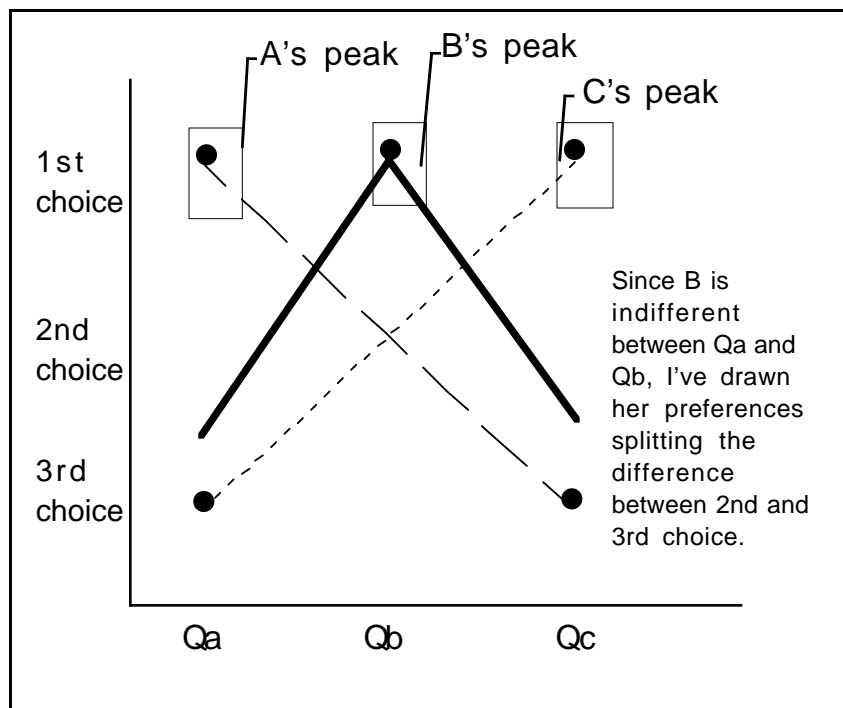
Q_a v. $Q_c \rightarrow$ indeterminate (depends on what B chooses)

Q_b v. $Q_c \rightarrow Q_b$

\therefore majority rule leads us to the median outcome Q_b . This is a general rule: the **median voter** (in this case, B) controls the outcome of the series of elections. In this case, $Q_b = Q^*$, so we arrive at Q^* .

This rule depends on a few things:

1. single-peaked preferences (where utility consistently falls away from the preferred outcome)
2. one-dimensional variable (a yes or no answer to a single question: no confounding factors)



If we graph the preferences, we can see the peaks.

In terms of preference,

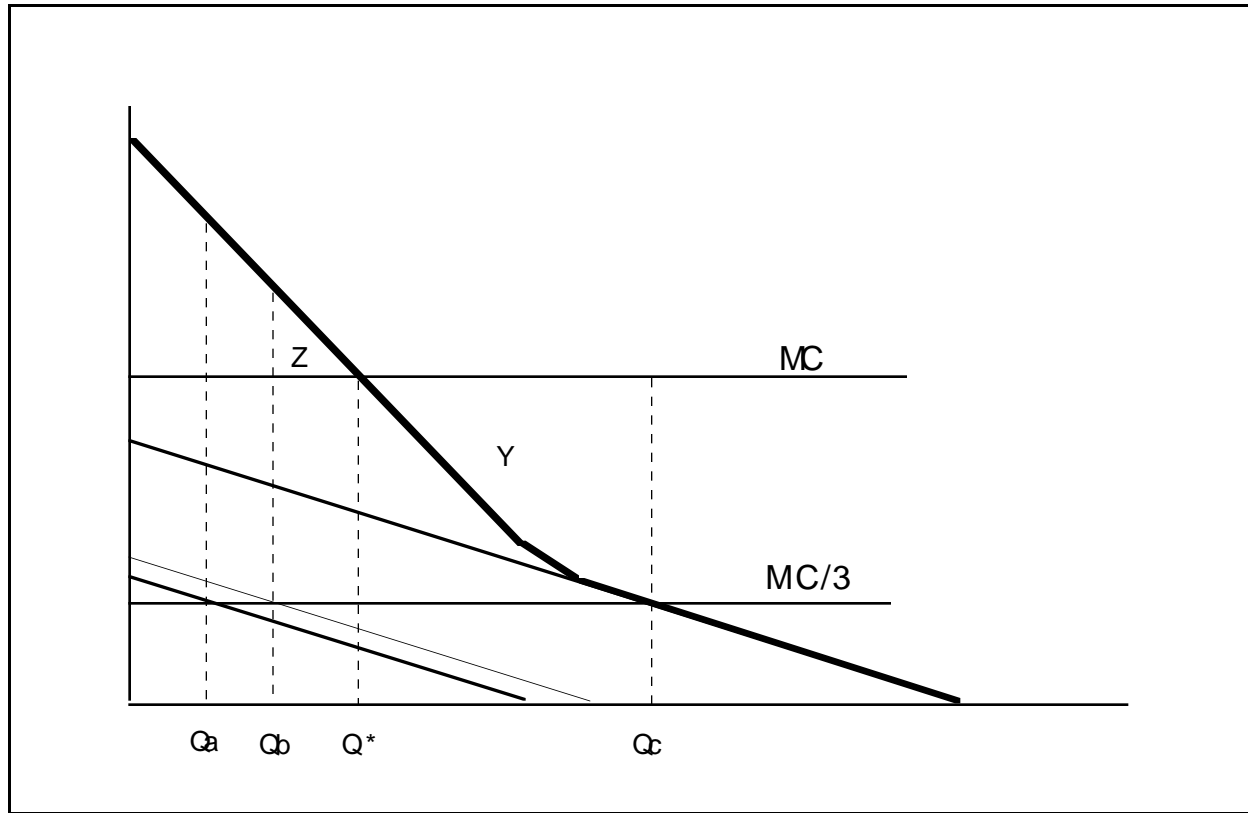
for A, $Q_a > Q_b > Q_c$

for B, $Q_a < Q_b > Q_c$

for C, $Q_a < Q_b < Q_c$.

"Single-peaked" means that utility consistently when moving away from the

preference point.



We do not reach Q^* by majority vote unless $Q^* = \text{median voter's preference}$.

In this graph, demands are not symmetrical. Q^* no longer meshes with Q_b .
 \therefore the median voter outcome and the voting mechanism for making public choice will not necessarily arrive at the socially optimal solution.

Again assuming single peaks, even if all 4 options were voted for, Q_b would win:

VOTES	Q_a	Q_b	Q^*	Q_c
Q_a v. Q_b	A	B, C	XXX	XXX
Q_a v. Q^*	A, B	XXX	C	XXX
Q_a v. Q_c	A, B	XXX	XXX	C
Q_b v. Q^*	XXX	A, B	C	XXX
Q_b v. Q_c	XXX	A, B	XXX	C
Q_c v. Q^*	XXX	XXX	A, B	C

Note that B is not indifferent between Q^* and Q_a ; in this example, Q_a is closer than Q_b and so B's loss is less at Q_a . Note also that even if B were indifferent, it would have no effect on the ultimate outcome.

Moving from Q^* to Q_b cannot be a K-H winner, because C loses more (triangle Z = benefits foregone) than the others gain.

The general rule is that wherever $MBS > MC$, a move to the right is a K-H winner; a move to the left is a K-H loser.

Note that the size of triangle Z equals the amount that C has left over after bribing A and B to vote for Q^* as opposed to Q_b . Of course, the problem is that in many cases bribes and collective action are illegal.

Triangle Y is the loss that society would bear if it produced at Q_c .

Problems of truthfulness: If people know they're going to get side payments/pay side payments, they have an incentive to lie about their preferences. How, then, can we ever get to Q^* ?

Ways of getting Q^* :

Clarke taxes: ask people to quantify the extent of their preferences without knowing they'll be paying taxes. However, the taxes don't go to the individuals. You have winners pay an amount equal to the loss of the losers as they reported it. Losers have no incentive to inflate because they don't get the money. This is not a real-world solution.

Logrolling: trading votes. A wants C to vote with him on issue a, so offers to do so in return for C voting with A on an issue in the future. This allows those with strong preferences to express them by convincing people without strong preferences to vote their way and thus increasing the power of their vote.

Filibuster: C might filibuster and make things really painful for A and B until they give in. This is an inferior method as compared to logrolling because it expends a lot more resources than the others.