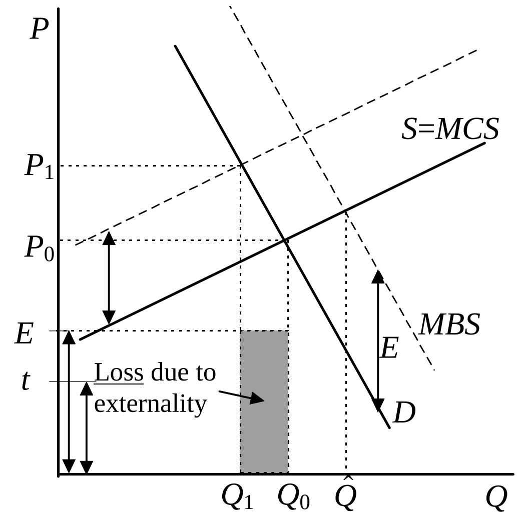
Name: ________________________

Midterm Exam #2 - Answers
November 24, 1997

Instructions: Answer all questions directly on these sheets. Points for the parts of each question are indicated, and there are 84 points total. Budget your time.

1. [14 Points]The figure below shows the supply and demand curve in the market for oranges. Also measured, on the vertical axis, are an externality, *E*, and a tax, *t*. The externality is a positive external benefit arising from consumption of oranges, a result of their ability to fend off the infectious common cold and therefore improve the health of others who come in contact with those who eat oranges. The tax, *t*, has been proposed in order to raise revenue for an unrelated purpose.



a) [4 points]Find in the figure, drawing them if necessary, the curves that indicate the marginal benefit to society and the marginal cost to society of oranges, as functions of the quantity produced. Label them MBS and MCS. If you need to add anything to the diagram in order to do this, be sure to indicate what you have done.

b) [2 points]Identify in the figure the *optimal* level of output of oranges, and label it $\hat{Q}$.

c) [4 points]Starting from the initial equilibrium at Q_0 and P_0, find the new equilibrium that will arise if the tax, *t*, is levied on production. Label it Q_1 and P_1.

d) [4 points]Identify and label in the figure the gain or loss to society due to the tax as a result *only* of the externality. That is, ignoring the effects of the tax on producers and consumers of oranges, by how much does the tax raise or lower the welfare of society through its effects on those who come in contact with consumers of oranges? Indicate clearly whether it is a gain or a loss.

2. [45 points]The town of Gateway, South Dakota, current population 3,500, is considering the construction of a new water treatment plant. The plant will take two years to build and would cost, at today's (Jan 1, 1998) prices, \$300,000 in 1998 plus \$400,000 in 1999. The plant will begin operating Jan 1, 2000, and it will provide a benefit to the city's population, in today's prices, of \$20 per year per person. Cost of operation, at today's prices, will be the same regardless of the population: \$7,000 a year. The plant is expected to operate for 20 years, starting Jan 1, 2000.

   Under each of the following sets of assumptions, calculate the present value of this project. In each case, first show the formulas, with appropriate numbers plugged in but not evaluated, for

   i) present value of construction cost
   ii) present value of operating cost
   iii) present value of benefits to population

   Then, evaluate the formulas for each, and combine them to get the total net benefit or cost, using your calculator if you like. (Most of the credit will be given for using correct formulas with correct values in them, so be sure to make that clear.)

   a) [15 points]The nominal interest rate is 8%, inflation is zero, and population growth is zero.

      Ans: Real interest rate = r = 8% – 0% = 8%

$$PV(CC) = C_0 + \frac{C_1}{1+r} = 300{,}000 + \frac{400{,}000}{1+0.08} = 670{,}370$$

(Costs for years 0 and 1, discounted by the real interest rate.)

$$PV(OC) = \frac{C_{op}}{r}\left(1 - \frac{1}{(1+r)^T}\right) - \frac{C_{op}}{1+r} = \frac{7{,}000}{0.08}\left(1 - \frac{1}{1.08^{21}}\right) - \frac{7{,}000}{1.08} = 63{,}636$$

(Costs incurred in the 20 years, numbered 2, …, 21. First term is formula for constant cost in years 1,…,T. Second term removes the cost for year 1.)

$$PV(B) = \frac{B_1}{r}\left(1 - \frac{1}{(1+r)^T}\right) - \frac{B_1}{1+r} = \frac{70{,}000}{0.08}\left(1 - \frac{1}{1.08^{21}}\right) - \frac{70{,}000}{1.08} = 636{,}361$$

(Benefit is \$20×3,500 -- \$20 per person times 3,500 persons – in years 2,…,21. Formula is same as for operating costs.)

$$PV(NB) = PV(CC) + PV(OC) + PV(B) = -97{,}645$$

b) [15 points]The nominal interest rate is 9%, inflation is 3%, and population growth is zero.

Ans: Now r = 9% – 3% = 6%. Formulas are otherwise the same as before:

$$PV(CC) = 300{,}000 + \frac{400{,}000}{1+0.06} = 677{,}359$$

$$PV(OC) = \frac{7{,}000}{0.06}\left(1 - \frac{1}{1.06^{21}}\right) - \frac{7{,}000}{1.06} = 75{,}745$$

$$PV(B) = \frac{70{,}000}{0.06}\left(1 - \frac{1}{1.06^{21}}\right) - \frac{70{,}000}{1.06} = 757{,}448$$

$$PV(NB) = 4{,}344$$

c) [15 points]The nominal interest rate is 10%, inflation is zero, and population of the town is expected to grow at the rate of 4% a year, starting immediately.

Ans: The benefit now grows at 4% a year and the real interest rate is 10%. We have a formula for a growing benefit, starting in year 1 and ending in year T. It applies to a benefit that starts at a value B_1 in year 1 and grows at a constant rate. Here the benefit is \$20×3,500 in year 0 and will grow 4% already by year 1, so B_1=\$70,000×(1.04)=\$72,800. The formulas for costs are the same as before:

$$PV(CC) = 300{,}000 + \frac{400{,}000}{1+0.10} = 663{,}636$$

$$PV(OC) = \frac{7{,}000}{0.10}\left(1 - \frac{1}{1.10^{21}}\right) - \frac{7{,}000}{1.10} = 54{,}177$$

$$PV(B) = \frac{B_1}{r-a}\left(1 - \frac{(1+a)^T}{(1+r)^T}\right) - \frac{B_1}{1+r} = \frac{72{,}800}{0.10-0.04}\left(1 - \frac{1.04^{21}}{1.10^{21}}\right) - \frac{72{,}800}{1.10} = 773{,}528$$

$$PV(NB) = 55{,}715$$

3. [25 points]For each of the policies in parts (a) and (b) below, by which a government could help the poor, calculate

   i) the cost to the government,

   ii) the benefit to the poor,

   iii) the cost or benefit to others, and

   iv) the weight, $\hat{w}_P$, that would have to apply to the poor (if the weight is 1 for everyone else) in order for the policy to break even (in the sense of its weighted net benefit being zero).

   a) [8 points]The government gives checks for $100 to each of 1000 poor people, incurring administrative costs of $20,000 plus $5 per check.

   Cost to Government = CG = $100×1000 + 20,000 + 5×1000 = $125,000

   Benefit to Poor = BP = $100×1000 = $100,000

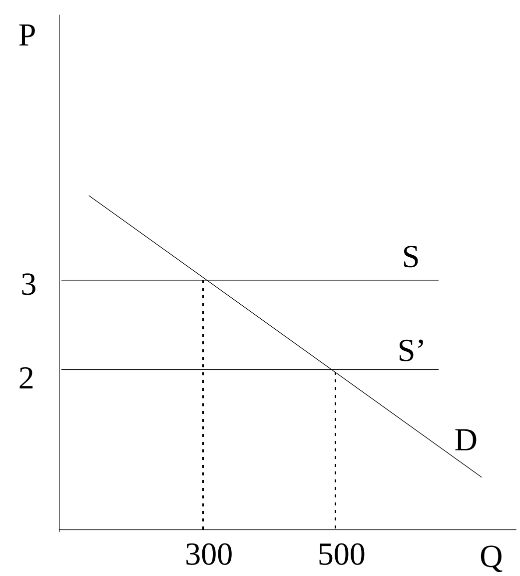
   Benefit to Others = BO = 0

$$\hat{w}_P = \frac{\text{Net Cost to Nonpoor}}{\text{Net Benefit to Poor}} = \frac{125{,}000}{100{,}000} = 1.25$$

   b) [12 points]The government pays a subsidy of $1.00 per can for purchases of the luncheon meat, Spam, noting that prior to the policy, 300,000 cans of Spam are sold each year at a price of $3.00 each and two thirds of the buyers of Spam are poor. Spam is known to be produced at constant cost and sold perfectly competitively, and the elasticity of demand for Spam is 2.

Ans: Spam market appears at right. Subsidy can be viewed as shifting the (horizontal, since constant cost) supply curve down by $1, lowering the price from $3 to $2, a 1/3 reduction. With demand elasticity of 2, the % change in quantity is twice the % change in price, so a 1/3 reduction in price causes quantity to increase by 2/3, to 500,000.

Government pays the subsidy of $1 on all cans, costing it $500,000. Demanders gain consumer surplus of the area left of the demand curve between the prices. Two thirds of this is a gain to the poor, while one third is a gain to



others:

CG = \$500,000

ΔCS = (3–2)(300,000 + 500,000)/2 = \$400,000

BP = (2/3) ΔCS = \$266,667

BO = (1/3) ΔCS = \$133,333

$$\hat{w}_P = \frac{\text{Net Cost to Nonpoor}}{\text{Net Benefit to Poor}} = \frac{500{,}000 - 133{,}333}{266{,}667} = 1.375$$

c) [5 points]Which of these policies transfers resources to the poor more efficiently? Why?

In this case, the direct payment (policy (a)) transfers resources more efficiently. We can see this from the fact that one needs a smaller extra weight on the poor, 1.25, in order to justify that policy, as compared on the weight on the poor needed to justify the Spam subsidy, 1.38. Alternatively, this can be seen from the fact that a larger percentage of the net cost to the non-poor gets to the poor under policy (a) (100/125= 80%) than under policy (b) (266,667/366,667 = 73%), or that the leaky bucket coefficient for (a) (20%) is less than (b) (27%).