Midterm Exam #2 - Answers

March 27, 1997

Instructions: Answer all questions directly on these sheets. Points for each part of each question are indicated, and there are 100 points total. Budget your time.

1. (43 points) Last year, farmers in the country of Autonomia (which does not engage in international trade) harvested 300 tons of wheat, which they sold to consumers for \$200 per ton. This year, their output was only 250 tons and the price rose to \$210 per ton. Concerned about both the cost of wheat to consumers and the incomes of the farmers, the government of Autonomia is considering paying a subsidy on wheat production for next year.

   a. Assuming that the demand curve for wheat is linear and is the same each year, derive both the demand function and the inverse demand function for wheat.

   Ans: P=a–bQ; 200=a–b(300); 210=a–b(250); 10=b(50); b=10/50=0.2; a=200+0.2(300)=260; **P=260–0.2Q**; Q=(260–P)/0.2; **Q=1300–5P**

   b. If the cost of producing wheat each year is a constant per ton, and if that cost rises next year by the same percentage that it did this year, in the absence of any subsidy determine next year's

   i.) equilibrium price of wheat:

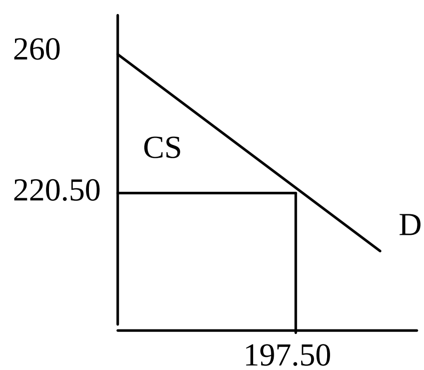
   ΔP/P=10/200=5%, P2=P1(1.05)=210(1.05)=**\$220.50**

   ii.) quantity of wheat consumed: Q=1300–5P=1300–5(220.50)=**197.50 tons**

   iii.) revenue of wheat farmers: Rev=PQ=220.50(197.50)=**\$43548.75**

   iv.) total consumer surplus in the wheat market

   CS=1/2(260–220.5)197.5=**\$3900.625**



   i.) total producer surplus in the wheat market

   Producer surplus is zero since producers just cover cost.

c. If the government instead pays a subsidy to wheat farmers of $40 per ton produced next year, again determine next year's

   i.) equilibrium price of wheat: 220.50–40=**$180.50**

   ii.) quantity of wheat consumed: Q=1300–5(180.5)=**397.5 tons**

   iii.) revenue of wheat farmers: Rev=397.5(220.5)=**$87,648.75**

   iv.) total consumer surplus in the wheat market:

   ½(260–180.5)397.5=**$15,800.625**

   v.) total producer surplus in the wheat market: **Still zero**

   vi.) amount paid out by the government on the subsidy: 40(397.5)=**$15,900**

d. Comparing parts b and c, calculate the net benefit or cost of the subsidy to the country as a whole

   ΔCS=15,800.625–3,900.625=$11,900; ΔPS=0; ΔSub= –15,900

   ΔNet = ΔCS+ΔPS+ΔSub = –**$4,000**

e. Which, if any, of your answers above would be altered if you also knew that wheat production yielded an external benefit to society worth $50 per ton? Indicate what you can about what the new answers would be.

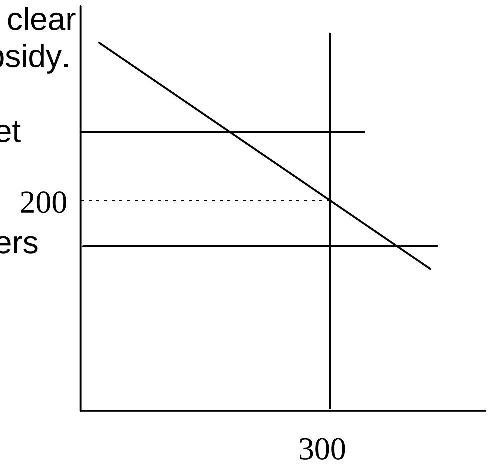
   Only part c-vii would be altered. Producers and consumers all do the same things as without the externality, but now the increase in quantity increases the externality by $50(ΔQ) = 50(200) = $10,000. ΔNet is now positive: –4,000 + 10,000 = **$6000**

f. Which, if any, of your answers above would be altered if you knew instead that wheat *consumption* yielded an external benefit to society worth $50 per ton? How does your answer here compare to part e, and why?

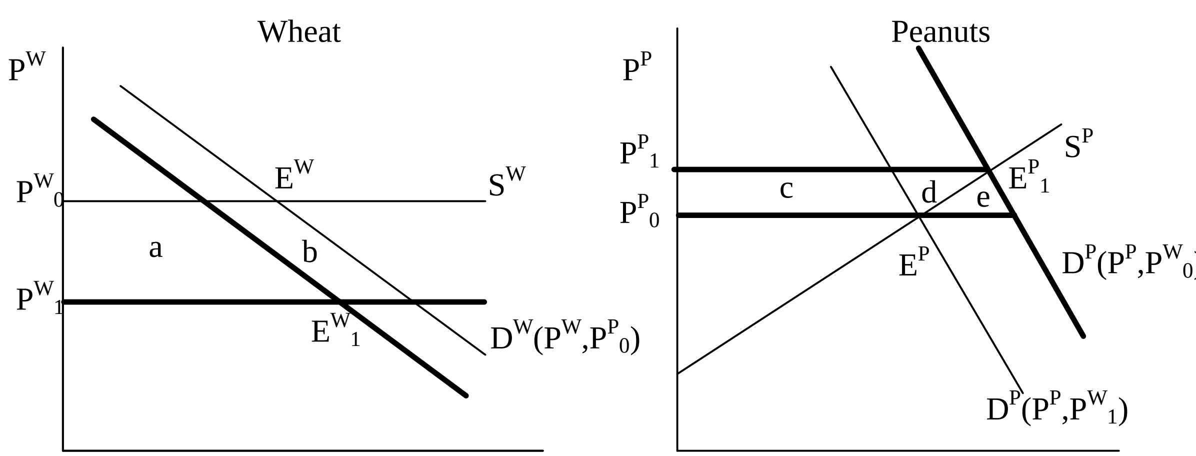
   All answers are the same as with a production externality. This is because without international trade, production and consumption are the same in equilibrium.

g. Suppose, now *without* the presence of any externalities, that the technology for producing wheat actually involves constant costs only up to a capacity constraint of 300 tons per year. Under the same assumptions as part c above (that is, including the $40/ton production subsidy), are wheat farmers better off, worse off, or unaffected by this capacity constraint, and by how much? Why?

At capacity output of 300, price must be 200 to clear the market, and producers get this plus the subsidy.
Profit=300(240-220.5)=**$5850**
Wheat farmers are better off, since they now get a positive profit instead of zero.
The reason is that the capacity constraint has caused wheat to become scarce, and the farmers are getting a rent from that.



h. I forgot to mention that demanders of wheat in Autonomia use it primarily to make bread for peanut-butter-and-jelly sandwiches, and as a result, bread is highly complementary with peanuts (and jelly too, but we'll ignore that). Peanuts are produced by an increasing-cost industry, the supply curve of which is shown in the figure on the right, below. The other curves in the two figures show the constant-cost supply of wheat (assumed now not to have a capacity constraint) and the initial demands for wheat and peanuts that would prevail next year without any subsidy. The initial equilibria are at E^W and E^P.



i. Show qualitatively in the figures (don't worry about numbers here) how a subsidy to wheat production will affect these markets. That is, shift the curves to the new positions that they will occupy in the presence of the subsidy, due to the subsidy itself and due to the price changes in the two markets. Mark clearly, as E^W_1 and E^P_1, the new equilibria that will obtain in both markets.

ii. Label the figures in some way so that you can indicate clearly, in the spaces below, each of the following effects of the subsidy:

The change in welfare of wheat producers: **zero**

The change in welfare of peanut producers: **+(c+d)**

The change in welfare of wheat and peanut demanders:

in wheat market: **+(a+b)**

in peanut market: –**(c+d+e)**

iii. Has allowing for the peanut market increased, decreased, or left unchanged your evaluation of the net benefits from the subsidy?

The net benefit from the subsidy has been **reduced** (because the effect on the peanut market has hurt peanut demanders more than it has benefited peanut suppliers).

2. (27 points) The town of West Stickendale, current population 2000, recently suffered a fire that destroyed its Civic Center Building, a facility that provided all sorts of services to the community. The question is whether, and on what terms, to rebuild it. The real interest rate is 3%, and it is known that the average citizen derives benefits from the civic center that are worth $150 a year to each of them. Architects have provided plans for two versions of a rebuilt Civic Center, each of which will yield these same benefits over its lifespan, but with different lifespans and different costs, which are shown in the table below:

	Version A	Version B
Time for construction	1 year	2 years
Cost of construction	$4,000,000	$8,000,000
Cost of operation	$1000/year	$2000/year
Expected lifespan	10 years	infinite

As you can see, Version A is cheaper to build and to operate, but it falls apart 10 years after construction is completed. Version B takes longer to build, costs more, and lasts forever.

a. Write the formulas for, and then calculate, the present values of the benefits and operating costs for both versions:

Benefits per year are $150 per person times 2000 persons = $300,000.

i. Version A - benefits

$$= \sum_{t=1}^{10} \frac{B}{(1+r)^t} = \frac{B}{r}\left(1 - \frac{1}{(1+r)^{10}}\right) = \frac{\$300{,}000}{0.03}\left(1 - \frac{1}{1.03^{10}}\right) = \$2{,}559{,}061$$

ii. Version A - operating costs $= \dfrac{\$1000}{0.03}\left(1-\dfrac{1}{1.03^{10}}\right) = \$8{,}530$

iii. Version B - benefits

$$\sum_{t=2}^{\infty}\frac{B}{(1+r)^t} = \frac{B}{r} - \frac{B}{(1+r)} = \$300{,}000\left(\frac{1}{0.03}-\frac{1}{1.03}\right) = \$9{,}708{,}738$$

iv. Version B - operating costs $= \$2000\left(\dfrac{1}{0.03}-\dfrac{1}{1.03}\right) = \$64{,}725$

b. Assuming that, for both versions, costs of construction are borne all at once as construction begins, what are the net present values of the two versions?

Version A: = –4,000,000 – 8,530 + 2,559,061 = – **$1,449,469**

Version B: = –8,000,000 – 64,725 + 9,708,738 = **+ $1,644,013**

Which version of the Civic Center, if any, should be built? **Version B**

c. Now suppose that the population of West Stickendale is expected to fall at a rate of 2% a year indefinitely, starting immediately. How would this change your calculations, and what effect, if any, do you think it would have on your conclusions? (You do *not* need to redo your calculations. Just indicate the direction of any effect, and what changes you would expect in the answers.)

Ans: Since the benefits accrue per person, a declining population means that the benefits shrink over time at a 2% rate. This will make all future benefits smaller than calculated above, the more so the further away they are in time. Version A will become even less desirable, and it seems likely that Version B will come to yield a negative net benefit as well.

3. (30 points) Everyone in the city of Amberton currently buys their electricity from an outside regional supplier at a price of $.14 per Mw (I don't know what that stands for, but it is not important). The demand for electricity in the city is given by the linear inverse demand curve, P=1.00–0.0002Q, where Q is quantity demanded in Mw per day. A salesperson from the U-Peddle Corp. has come to town offering to sell the city a system with which it will be able to generate its own electricity using a collection of stationary bicycles installed in a building to be constructed on the edge of town. By employing 10 workers each working 2000 hours per year at the Amberton wage of $5 per hour (all Ambertonians are the same), the company has estimated that the town will be able to generate all the electricity it demands at a marginal cost (and price) of only $.06 per Mw. The system itself can be purchased and installed for the entire

town at a cost of $1,100,000, and it will be ready to operate in one year. With routine maintenance provided by the company at $10,000 per year, the facility should last forever.

a. Calculate the quantity of electricity currently demanded by Amberton at the $.14 price, the quantity that will be demanded at a price of $.06, and the change in consumer surplus per day that will result from this peddle-power plant.

Ans: $P=1-0.0002Q \Rightarrow Q=5{,}000(1-P)$

$Q(.14) = 5{,}000\ (.86) =$ **4300 Mw**

$Q(.06) = 5{,}000\ (.94) =$ **4700 Mw**

$\Delta CS = (0.14 - 0.06)(4300 + 4700)/2 =$ **$360 /day**

b. Assume first that the Amberton labor market is in equilibrium, and that it is large enough for the equilibrium wage to remain essentially unchanged when 10 more workers are hired. What is then the net present value of this project, using a real interest rate of 4%? (Assume that every year has 365 days.)

Ans: $\Delta CS = \$360\ (365) = \$131{,}400$ /year

$B = \Delta CS$ – maintenance cost = $131,400 – 10,000 = $121,400

$$PV = -1{,}100{,}000 + \sum_{t=1}^{\infty} \frac{B}{(1-r)^t} = \frac{121{,}400}{0.04} - 1{,}100{,}000 = \mathbf{\$1{,}935{,}500}$$

c. Assume instead that the equilibrium wage is only $4.80, but that the project would still have to pay $5 per hour to compensate workers for the known probability of 0.000004 (that's four in a million) that a given worker will have a stroke and die during any given ~~day~~ hour of peddling. [Original "day" corrected to "hour".]

i. How much do these workers' lives appear to be worth to them, based on this information? [With "day", of 8 hours, answer is 8x50,000=400,000.]

Ans: Let V=value of life. By accepting a $.20 wage premium to compensate for an increased probability of 4/1,000,000 of losing life, the worker is equating

$$0.20 = \frac{4}{1{,}000{,}000} V \quad \text{or} \quad V = \frac{0.20 * 1{,}000{,}000}{4} = \mathbf{\$50{,}000}$$

ii. Assuming that workers correctly perceive this danger on the job, is the net present value of the project to the city and its residents now more than, less than, or the same as it was in part b? (Direction is enough – no need to quantify.)

Ans: Net present value is the **same**, since the danger has been built into the wage costs.

d. Assume finally (and instead) that the wage of $5 is paid to all workers (there is now no risk of death), but that it is a legal minimum wage, at which there is excess supply in the labor market. The labor supply curve is given in inverse form by the equation w=1+0.0002(L) where w is the wage and L is the number of workers, and labor demand without the project is 15,000 workers at the $5 wage. Jobs are allocated randomly among those willing to work at the $5 wage.

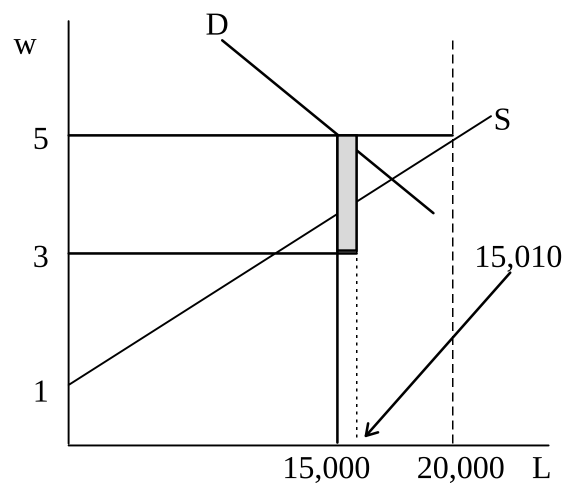
i. Draw a graph of labor supply and demand, showing and also calculating the amount of unemployment (without the project).

5 = 1 + 0.0002 L

L = 20,000

U = 20,000 – 15,000

= **5,000 workers**



i. Show in the diagram, and also calculate, the net benefit (surplus) to workers of employing 10 more of them for this project?

Ans: Workers are willing to work for wages from $1 to $5, so that with random allocation, the average worker requires (1+5)/2=$3 / hr. The average surplus per worker is therefore $5–3=$2 /hr. With 10 workers each working 2,000 hours per year, the surplus is therefore $2*10*2,000 = **$40,000 / year.**

ii. What is now the present value of the project to the citizens of Amberton?

Ans: The present value of this worker surplus is $40,000/0.04 = $1,000,000. Added to the present value of the rest of the project calculated in part b, the new present value is $1,935,000 + 1,000,000 = **$2,935,000.**