SPP/ECON 573 Alan Deardorff
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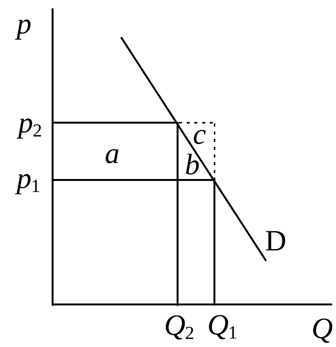
Consumer Surplus

Calculating the Change in Consumer Surplus in the Case of Linear Demand:



Let

$$\Delta p = p_2 - p_1 > 0$$

$$\Delta Q = Q_2 - Q_1 < 0$$

From the figure:

$$\begin{aligned}\Delta CS &= -[a+b] = -\Delta p Q_2 - \tfrac{1}{2}(\Delta p)(-\Delta Q)\\ &= -[(a+b+c)-c] = -\Delta p Q_1 + \tfrac{1}{2}(\Delta p)(-\Delta Q)\end{aligned}$$

If you are lucky enough to know both the initial and final quantity and price, then you can calculate the various areas directly, as in these formulas. If however you do not know, say, the new quantity, because the policy that will raise price to p_2 has not happened yet (or it has happened but amidst other changes that may have shifted the demand curve), then you will need to estimate Q_2 somehow. One way of doing this is to first obtain an estimate of the elasticity of demand, and then use that to estimate ΔQ from Δp. For example, let E be the (positive) arc elasticity of the demand curve between the two points, defined as

$$E = (-)\frac{\frac{\Delta Q}{\overline{Q}}}{\frac{\Delta p}{\overline{p}}}, \text{ where } \overline{Q} = \frac{Q_1 + Q_2}{2} = Q_1 + \tfrac{1}{2}\Delta Q \text{ and } \overline{p} = \frac{p_1 + p_2}{2} = p_1 + \tfrac{1}{2}\Delta p.$$

Then the equation defining the elasticity can be solved for ΔQ in terms of Δp as follows:

$$\boxed{E = -\frac{\frac{\Delta Q}{Q_1 + \frac{1}{2}\Delta Q}}{\frac{\Delta p}{p_1 + \frac{1}{2}\Delta p}}}$$, inserting the definitions of $\overline{Q}$ and $\overline{p}$;

$$E\frac{\Delta p}{p_1 + \frac{1}{2}\Delta p} = -\frac{\Delta Q}{Q_1 + \frac{1}{2}\Delta Q}$$, multiplying both sides by $\frac{\Delta p}{p_1 + \frac{1}{2}\Delta p}$;

$$E\Delta p(Q_1 + \tfrac{1}{2}\Delta Q) = -\Delta Q(p_1 + \tfrac{1}{2}\Delta p)$$, multiplying both sides by the two denominators;

$$\Delta Q(p_1 + \tfrac{1}{2}\Delta p) + E\Delta p\,\tfrac{1}{2}\Delta Q = -E\Delta p Q_1$$, moving terms with ΔQ to the left and others to the right;

$\Delta Q[p_1 + ½\Delta p + ½E\Delta p] = -E\Delta pQ_1$, factoring out ΔQ;

$\Delta Q[1 + ½(1+E)\frac{\Delta p}{p_1}] = -E\frac{\Delta p}{p_1}Q_1$, dividing through by p_1 and grouping terms;

$$\boxed{\Delta Q = -EQ_1 \frac{\frac{\Delta p}{p_1}}{\left(1 + ½(1+E)\frac{\Delta p}{p_1}\right)}}$$, dividing by the bracketed expression.

Now substitute this expression for ΔQ into the last expression above for ΔCS:

$$\begin{aligned}\Delta CS &= -\Delta pQ_1 + ½(\Delta p)(-\Delta Q)\\ &= -\Delta pQ_1 + ½(\Delta p)\left[EQ_1 \frac{\frac{\Delta p}{p_1}}{\left(1 + ½(1+E)\frac{\Delta p}{p_1}\right)}\right]\\ &= -\frac{\Delta p}{p_1}p_1Q_1 + ½(\frac{\Delta p}{p_1})p_1Q_1 \frac{E\frac{\Delta p}{p_1}}{\left(1 + ½(1+E)\frac{\Delta p}{p_1}\right)}\end{aligned}$$

Thus, collecting terms,

$$\boxed{\Delta CS = -p_1Q_1\frac{\Delta p}{p_1}\left[1 - \frac{½E\frac{\Delta p}{p_1}}{\left(1 + ½(1+E)\frac{\Delta p}{p_1}\right)}\right]}$$

Notice that this expression only requires data on:

- The percent change in price (relative to initial price): $\frac{\Delta p}{p_1}$
- The <u>value</u> of initial demand, p_1Q_1; and
- The arc elasticity of demand, E.