Answer all questions, in blue book. Plan and budget your time. The questions are worth a total of 80 points, as indicated, and you will have 80 minutes to complete the exam.

1. [20 points] In the (standard) Specific Factors Model, analyze the effects on factor prices of an increase in the relative price of good X. That is,
   a. [14 points] Show how a rise in the price of good X, holding the nominal price of good Y constant, would change the market equilibrium values of the
      i. nominal wage
      ii. nominal rental on capital employed in X
      iii. nominal rental on capital employed in Y
      iv. real wage
      v. real rental on capital employed in X
      vi. real rental on capital employed in Y
   Use the usual Specific Factors diagram to help you get your results, together with other relationships that you know must hold, but also be sure to say explicitly what you have found and why.

   [It has been pointed out to me that 14+4≠20, and I have confirmed this with my calculator. I have therefore given everyone a free 2 points on this question.]

   Holding the price of good Y constant, the rise in price of good X shifts the $VMP_{\text{LX}} = p_X MP_{LX}$ curve upward, as shown below:

   ![Diagram showing the shift in the VMP curve](image)

   Thus, the rise in $p_X$ causes more labor to be employed in the X sector ($L_X$ rises) and less in the Y sector ($L_Y$ falls). These changes in turn, since capital stocks in the sectors are fixed in the specific factors model, mean that the ratios of capital to labor, on which marginal products depend, change as follows:
   $K_X/L_X$ falls
   $K_Y/L_Y$ rises.
It follows, since in each sector $MP_L$ rises with $K/L$ and $MP_K$ falls with $K/L$, that $MP_{LX}$ falls, $MP_{LY}$ rises, $MP_{KX}$ rises, and $MP_{KY}$ falls.

From the figure we see immediately that the nominal wage rises, from $w_1$ to $w_2$, as does the nominal rental price of capital in the $X$ sector (since the triangle above $w_2$ and below $VMP_{LX}'$ is larger, meaning that payments to $X$-sector capital increase, while the quantity of capital there has not changed), while the nominal rental price of capital in the $Y$ sector falls (the triangle for it gets smaller).

As for real factor prices, these can be inferred from their ratios to goods prices, which are equal to marginal products, as shown in the list below.

\[ w = p_X MP_{LX} = p_Y MP_{LY} \text{ rises, from figure} \]
\[ r_X = p_X MP_{KX} \text{ rises, from figure, or because } p_X \text{ and } MP_{KX} \text{ both rise} \]
\[ r_Y = p_Y MP_{KY} \text{ falls, from figure, or because } MP_{KY} \text{ falls} \]

\[ w/p_X = MP_{LX} \text{ falls} \]
\[ w/p_Y = MP_{LY} \text{ rises} \]
\[ \text{hence effect on real wage is ambiguous} \]

\[ r_X/p_X = MP_{KX} \text{ rises} \]
\[ r_X/p_Y = (r_X/p_X)(p_X/p_Y) \text{ rises, since both terms rise} \]
\[ \text{hence real rental on } X\text{-sector capital rises} \]

\[ r_Y/p_Y = MP_{KY} \text{ falls} \]
\[ r_Y/p_X = (r_Y/p_Y)/(p_X/p_Y) \text{ falls, since numerator falls and denominator rises} \]
\[ \text{hence real rental on } Y\text{-sector capital falls} \]

b. [4 points] Which of your answers would be changed if it were the nominal wage that were fixed, instead of the nominal price of $Y$?

The figure would be harder to draw, since now in addition to what is shown above, we would need to lower both goods prices in the same proportion to get $w$ back where it started. But you don’t need to do this. Rather, just note that real variables are unaffected by nominal ones, so the answers for real factor prices must be the same as above. (Each ratio of a factor price to a goods price depends just on a real marginal product and, in some cases, the relative prices of the goods, never on any nominal prices alone.)

Since $w = p_X MP_{LX} = p_Y MP_{LY}$, and since the shift of labor has caused $MP_{LX}$ to fall and $MP_{LY}$ to rise, the constant wage requires that $p_X$ rise while $p_Y$ falls. Thus $r_X = p_X MP_{KX}$ rises, since both $p_X$ and $MP_{KX}$ rise. And $r_Y = p_Y MP_{KY}$ falls, since both $p_Y$ and $MP_{KY}$ fall.
2. [22 points] Consider a small open economy that, in the absence of any policy, would export good Y.
   a. [4 points] Of the following four policies,
      i. A production tax on X
      ii. A consumption tax on X
      iii. A production tax on Y
      iv. A consumption tax on Y
   which policy, if large enough, could cause the country to export good X instead of exporting Y? Note: You do not necessarily need to draw any diagrams for this part, unless you find that it helps you to find the answer.

   A production tax on X causes less of it to be produced, and thus more of it to be imported, not less.
   A consumption tax on Y would cause less of it to be consumed, and thus more of it to be exported, not less.
   But a consumption tax on X would cause less of it to be consumed, thus less imported, and if large enough could reduce consumption below production and cause X to be exported.
   Likewise, a production tax on Y would reduce production of Y, and thus reduce exports of Y, and if the tax were large enough could also reverse the pattern of trade.
   So it is policies ii and iii that have the potential to reverse the pattern of trade: a consumption tax on X and a production tax on Y.

   b. [6 points] Let the relative price of X on the world market be $p^*$, and the country’s relative price of X in autarky be $p^A$. For just one of the policies you’ve identified in part (a) (if there were more than one), suppose it were set equal to the percentage difference between $p^*$ and $p^A$. Show that this would not be enough to reverse the pattern of trade.

   $Y$-Production Tax
   
   $X$-Consumption Tax

   A production tax on Y of this size will lower the price received by Y-producers to its autarky level, and thus raise the relative price of X to producers to $p^A$. A consumption tax on X will raise the price of X to consumer to its autarky level,
and thus raise the relative price to consumers to $p^A$. We show these changes in
the figures above, while leaving the untaxed group facing the world price $p^*$. 

That is, a production tax on Y of this size would move production to the autarky
point, but since consumers would still face the lower world price of X, they would
still consume more than is being produced, and thus the country would still
import X, as shown above.

A consumption tax of this size on X would confront consumers with the autarky
prices, but since producers now will still face world prices, they would continue
to produce at free-trade levels. Thus, again, production of X would be less than
consumption of X, and the country would import X, as also shown above.

c. [6 points] For the same policy you chose in part (b), draw and explain an
equilibrium in which the policy does reverse the pattern of trade.

A sufficiently large production tax on Y will lower the price received by Y-
producers sufficiently below the world price (and thus raise producers’ relative
price of X) to reduce its output of Y below consumption, perhaps even to zero. It
produces at $P'$ in the figure, and consumes at C', exporting X.

A sufficiently high consumption tax on X will raise the price paid by consumers
sufficiently above the world price to reduce its consumption below production,
also perhaps to zero if preferences permit that. With consumption at C' and
production at $P'$, it too now exports X.

d. [6 points] Without doing any formal analysis for this part of the question, but
drawing on what you know from our various models, who gains and who loses
from the policy that you’ve illustrated in part (c)?

Production tax on Y: Because this changes prices received by producers, it also
changes factor prices. The rise in relative price of X to producers will raise the
real return either to the factor used intensively in the X industry (in the HO
model) or the specific factor(s) in the X industry (in the Specific Factors Model),
while lowering the real return to the other (or other specific) factor. Thus the
owners of these factors, if that is their only source of income, gain and lose correspondingly. However, the tax revenue must also be used, and if as usual we assume that it is redistributed to somebody, that gives them an additional gain. Nonetheless, we know from the move to a lower indifference curve above that the country as a whole is worse off, so the losers just mentioned must surely lose.

Consumption tax on X: As shown above, this leaves prices faced by producers unchanged at their world levels, and therefore does not change factor prices relative to world prices. However, all residents of the country now pay a higher-than-world price for X as consumers, by the amount of the tax, while perhaps getting a share of the redistributed tax revenue. If all persons are alike as consumers and share in this redistribution in proportion to their income, then we know from the lower indifference curve reached by the country as a whole that all of them must lose. If they are not alike, and/or if they share unequally in the redistribution, some may gain who don’t want to consume much X anyway, and/or who share disproportionately in the redistribution.

3. [18 points] We have argued that when an industry in a country is imperfectly competitive, then trade can provide an additional, “pro-competitive” benefit, over and above the gains from comparative advantage, to the extent that it forces producers to behave more like perfect competitors. But what if trade drives them out of business? Are there still gains from trade? And if there are, how do these compare to what would have happened with perfect competition? To answer this question, first

a. [4 points] show and explain an autarky equilibrium in which one sector is a monopolist, and then

The figure below depicts autarky equilibrium in an economy in which the X sector is a monopoly, and therefore charges consumers a price higher than marginal cost. This is reflected in a price line to consumers, $p^C$, tangent to an indifference curve that is steeper (higher relative price of X) than the slope of the PPF.
b. [4 points] open to free trade at a world price such that all production in that sector ceases.

This will happen if the world relative price of X is smaller than the slope of the PPF at it Y-axis intercept. The result, shown below, is zero output of X at Pf and consumers consuming at, and trading to Cf, the tangency shown of the world price line, p*, to an indifference curve.

c. [4 points] Show whether there are gains from trade, and

Clearly, from the figure above, there are gains from trade, since we move from the indifference curve um to uf.

d. [6 points] show how these gains or losses compare to what would have happened with trade if the sector had been perfectly competitive.

With perfect competition we would have started from the undistorted autarky equilibrium, which is found at point A below as the tangency between the PPF and an indifference curve. Adding this to the figure above, and labeling it as ua, we see that the gains from trade are larger when we start with the monopoly, and trade moves us from um to uf, than they would have been if we had started from A, moving from ua to uf. This is true even though the monopoly has been driven completely out of business by the low priced competition from free trade.
4. [20 points] Use the External Increasing Returns to Scale (EIRS) Model, from lecture and the handout, to answers the questions below. Use, for each, both the tools of the model and an explanation in words. As in the handout and in lecture, let the increasing returns good be $M$=machines and the constant returns good be $F$=food, and let the Home country have a larger labor force than the Foreign country.

a. [6 points] Why, if two countries are otherwise identical, does the one with the larger labor force have a lower relative autarky price of machines?

The too simple, but correct, answer is that the larger country is better able to take advantage of the increasing returns to scale and it thus enjoys a lower relative price in autarky for the IRS good. But to show this you must show not just that it can produce more $M$, but that it will, using demand conditions.

Formally (and easy for me, since I can copy the diagram from the handout), the picture is as appears below. The two countries share the same AC curve out to the limit of their output permitted by their labor forces. This limit in this case is higher for the home country, $\bar{M}$, than for the foreign country, $\bar{M}^*$. They have different demand curves, however, because of their different sizes. At any $M<\bar{M}^*$, the Home country has more labor left over to produce the other good, Food, and so produces a lower ratio of $M/F$ and requires a higher relative price, $p=p_M/p_F$, to get consumers to demand this low ratio. This places its demand curve above (and to the right) of Foreign’s, as shown. But with $D$ to the right of $D^*$, the intersection with AC (and AC*) occurs to the left for Foreign and thus at a higher price. This proves that the relative price of machines (the IRS good) is lower in the country with more labor.

b. [6 points] Why, if these two countries open to free trade, will the larger country expand production of machines, and the other contract production of machines?

When they open to free trade, if we assume that initially that outputs do not change, then the market will have to clear with the outputs they are currently producing. Initially, as indicated by its low relative price, Home is producing a higher ratio of $M$ to $F$ than Foreign. When their outputs combine, the ratio available in the world lies between these two, higher than in $M$ but lower than in $F$, so the world price settles between the two autarky prices, rising in Home and
falling in Foreign. But this means that producers in Home find themselves facing a price that is higher than their average cost, and they now make a profit. At the same time, Foreign producers face a world price lower than their average cost and make a loss. We therefore expect Home producers to expand, and/or new firms to enter the market in Home, so that output of \( M \) in the Home economy expands. And we expect the opposite to happen in Foreign, since they are making a loss, and Foreign output of \( M \) contracts.

c. [8 points] Assume a new free trade equilibrium in which the larger country does indeed produce more \( M \) than does the smaller country, as they started to do in part (b). In fact, assume that Home produces only \( M \) and Foreign produces only \( F \), and further that the equilibrium relative price of \( M \) is above the autarky price in Foreign. Which of these countries gains more from trade, and why?

The free trade equilibrium is found by constructing an \( AC^W \) curve for the world

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\begin{align*}
\text{I have placed the world demand curve so as to create an equilibrium with the properties listed in the question: Both countries completely specialize, and the price is above the autarky price of Foreign (see the point marked } A^*, \text{ below the } p^W \text{ line on } AC^*. \\
\text{To see what happens to welfare, however, we need to look at the PPFs and}
\end{align*}
\]

\[
\begin{align*}
\text{You can see the graphical representation of the welfare analysis here.}
\end{align*}
\]
indifference curves. These are drawn below for the two countries. These show first the two autarky equilibria, at points $A$ and $A^*$, prices $p^A$ and $p^{*A}$ with corresponding utility $u^A$ and $u^{*A}$. Because Home specializes completely in $M$, it produces at the corner of its PPF on the $M$ axis, point $P$, and consumers are able to trade from there at the price $p^W$ that is well above its average cost. Foreign, on the other hand, produces only $F$ at $P^*$, at the other corner of its PPF, and trades at a price that is higher (steeper) than its autarky price. As shown, Home gains quite a lot, while Foreign actually loses from this trade. Given the configuration of production and price stated in the question, this is the way that it has to turn out, since Foreign must import $M$ at a price that is higher than it produced it itself in autarky.