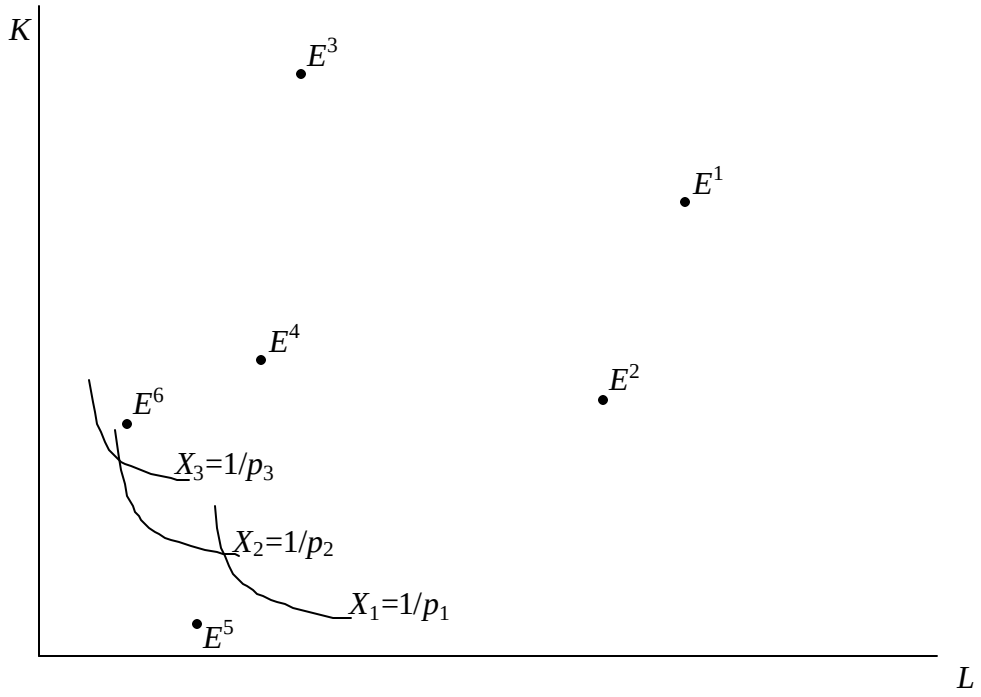
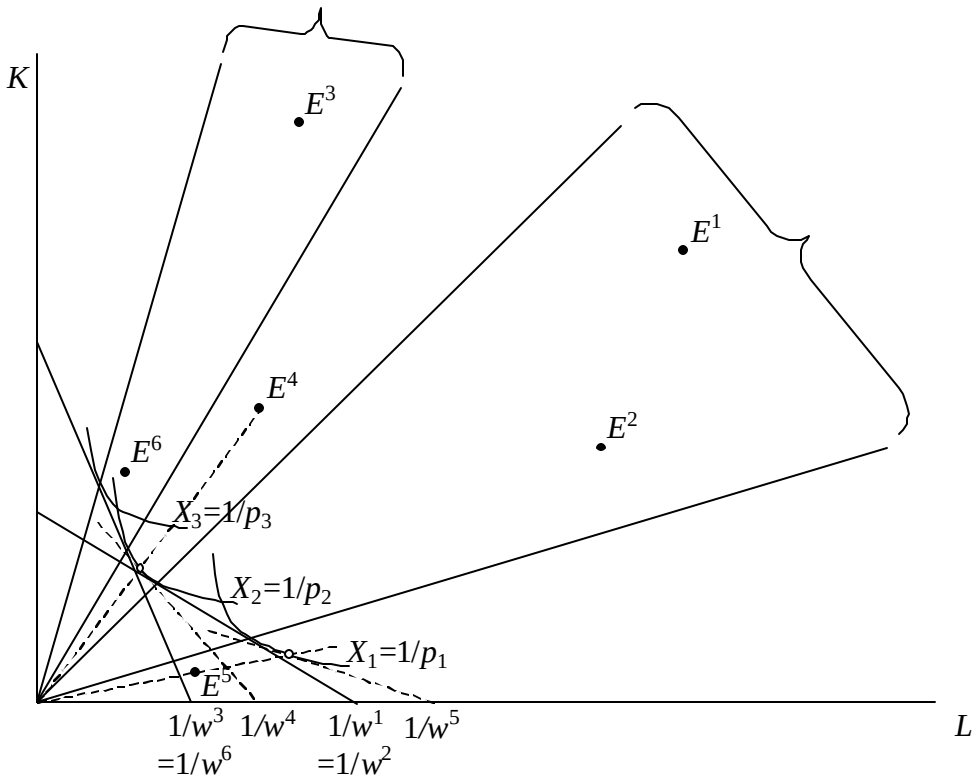
Problem Set 4 - *Answers*
Two-Cone Model and Gains from Trade

1. The graph below shows unit value isoquants for three goods, X_1, X_2 , and X_3, based on prices that are assumed to prevail throughout a world of many countries with free trade. Also shown are points representing the factor endowments of several countries, E^1, E^2, etc. Complete the two-cone Lerner diagram to identify the factor prices, factor ratios, and vectors of factors that will be employed in each sector by each country, and use these to answer the following questions:

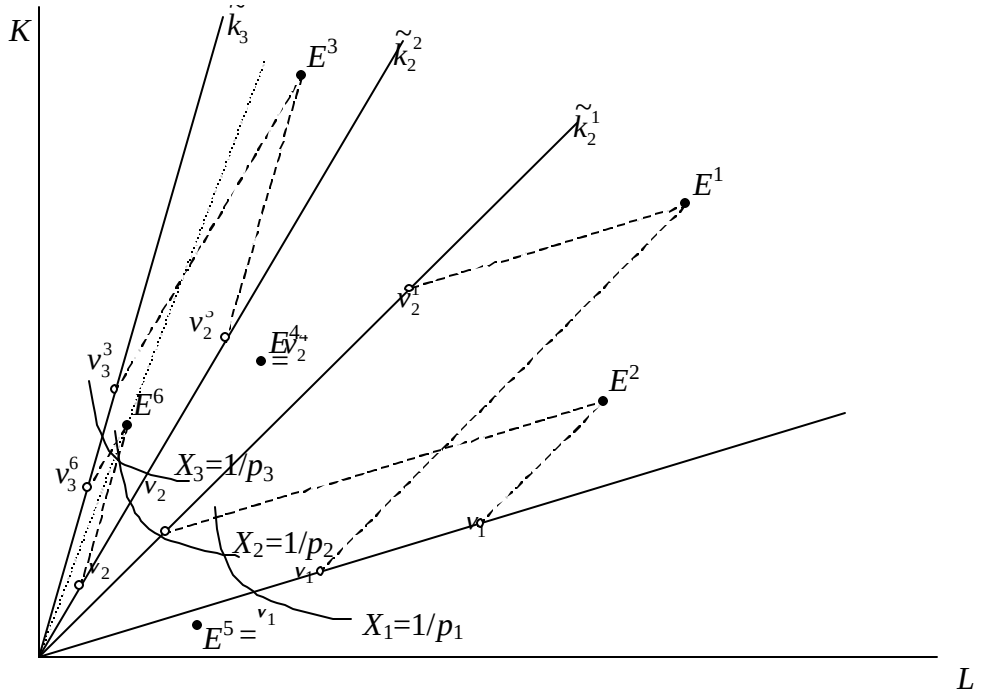


a. In which country or countries will the wage in units of good X_1, be highest, and in which will it be lowest? Would the answer be any different for the wage in units of goods X_2 or X_3?



Drawing the tangencies to pairs of adjacent isoquants we find the two cones of this equilibrium. Countries 1 and 2 are in the labor-intensive cone with identical nominal wages, and countries 3 and 6 are in the capital-intensive cone also with identical, but higher, wages. Countries 4 and 5 are not in either cone, and they therefore will specialize in goods 1 and 2 respectively. To find their factor prices, we draw rays from the origin through their endowment points and find the isocost lines tangent to their respective isoquants where they cross these rays. This gives us the wages shown as w^4 and w^5. Reading from these, we see that the lowest nominal wage is in country 5, while the highest is in both country 3 and country 6. Since prices of goods are the same for all countries, these are the lowest and highest respectively in terms of good 1 or any other good.

b. Which country will produce the largest quantity of each good?



For the four countries inside cones, the parallelogram construction identifies the vectors of factors they employ in each of the two goods that they produce, v_1^1 and v_2^1 for goods 1 and 2 in country 1, for example. For countries 4 and 5, which completely specialize, their endowment points are themselves the vectors of factors they employ in their only sector. Comparing these vectors, we can observe that country 2 employs more factors in producing good 1 than countries 1 and 5 (the only other countries that produce it), and therefore country 2 has the largest output of good 1. Similarly, country 1 produces the most of good 2 (which is produced by all countries except 5), and country 3 produces the most of good 3.

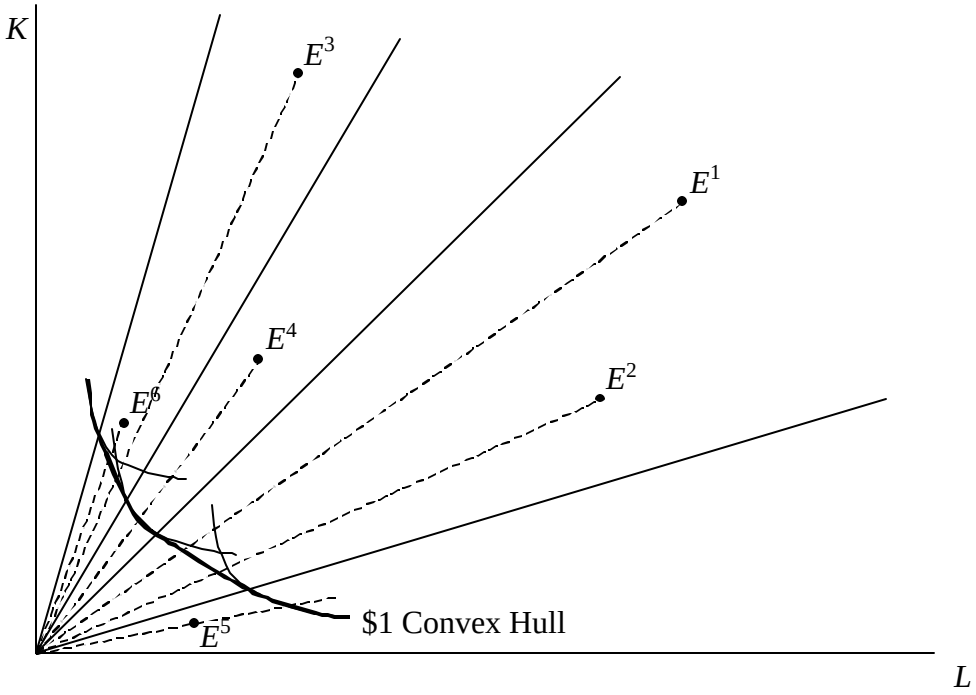
c. Which country will produce the largest *ratio* of good X_3 to good X_2?

The dotted line above through E^6 shows that country 6 has a higher ratio of capital to labor than does country 3 (or any other). Therefore, it produces a higher ratio of the more capital-intensive good 3 to good 2 than does country 3.

d. (Hard) Which country has the largest national income?

The value of each country's national income will be the value of its factors, measured in a numeraire that is common to all countries. From the convex hull of the unit value isoquants, we know a vector of factors in each country, proportional to its endowment, that is worth \$1. Comparing the length of the endowment vector itself to this gives national income. The figure from part (a) is

duplicated below, emphasizing just the convex hull, the endowment points, and the endowment rays for each country.

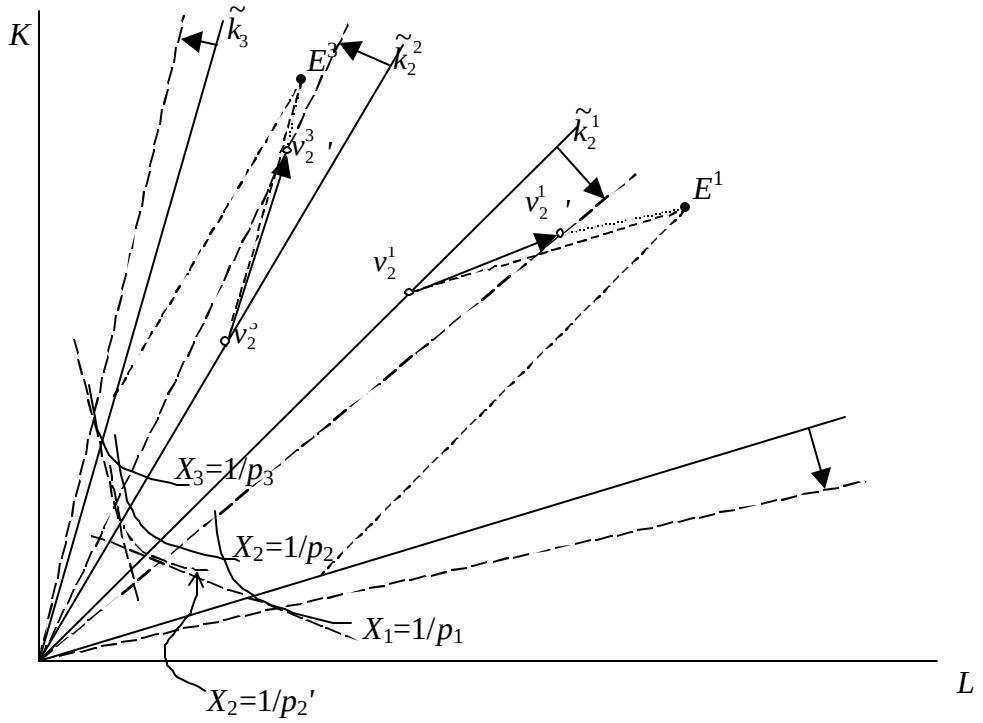


For example, E^5, is certainly worth less than \$1, since it is inside the convex hull, and it looks to be about \$0.60. E^6, on the other hand, is worth a little more than \$1, while E^4 appears to be a bit more than twice as far from the origin as the convex hull, and therefore worth a little more than \$2. Actually measuring these distances for all the countries it is clear that the country with the largest national income is country 1.

e. Suppose that consumer preferences in the world were to shift towards good 2 so as to cause a small increase in the price of good 2, the relative price of goods 1 and 3 remaining constant. Which country or countries would increase their output of good 2?

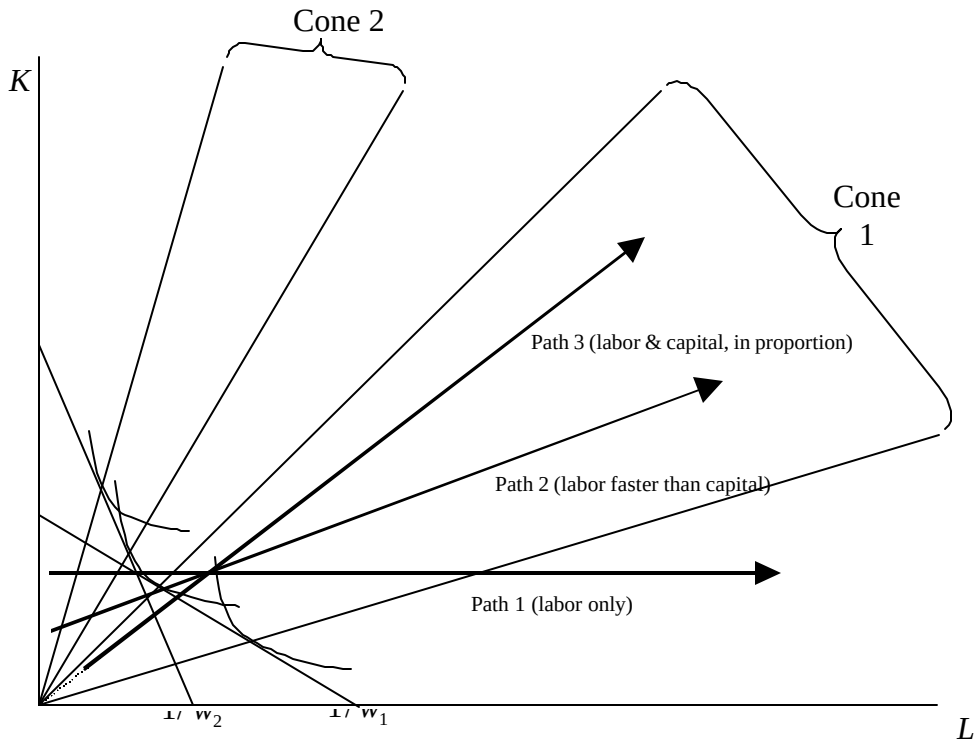
Holding p_1 and p_3 constant and increasing p_2, the unit value isoquant for X_2 shifts in towards the origin, as shown below. This gives rise to new tangencies with the isoquants. For those countries that were in one of the cones (all but 4 and 5), this causes new capital-labor rays in each sector, and a new parallelogram construction to determine vectors of inputs and thus outputs. The result is that all countries that were previously producing both good 2 and another good now produce more of it, which means that all countries except countries 4 and 5 produce more good 2. (If the price increase were large enough, country 5 would start to produce it too, since the labor-intensive cone would rotate clockwise to

encompass its endowment point. However, we are told that the price increase is small.) Rather than show all of this in one diagram, I show this construction below only for countries 1 and 3, and only for good 2.



2. Suppose that population grows in an open economy, in a world characterized by a two-cone equilibrium, and that the country is too small to affect world prices, even after this population growth. If the country's capital stock fails to grow as rapidly as its labor force, what will happen to the real wage of labor, and how will this depend on its pattern of specialization? Would your answer be any different if the country were able, instead, to expand its capital stock in proportion to its population?

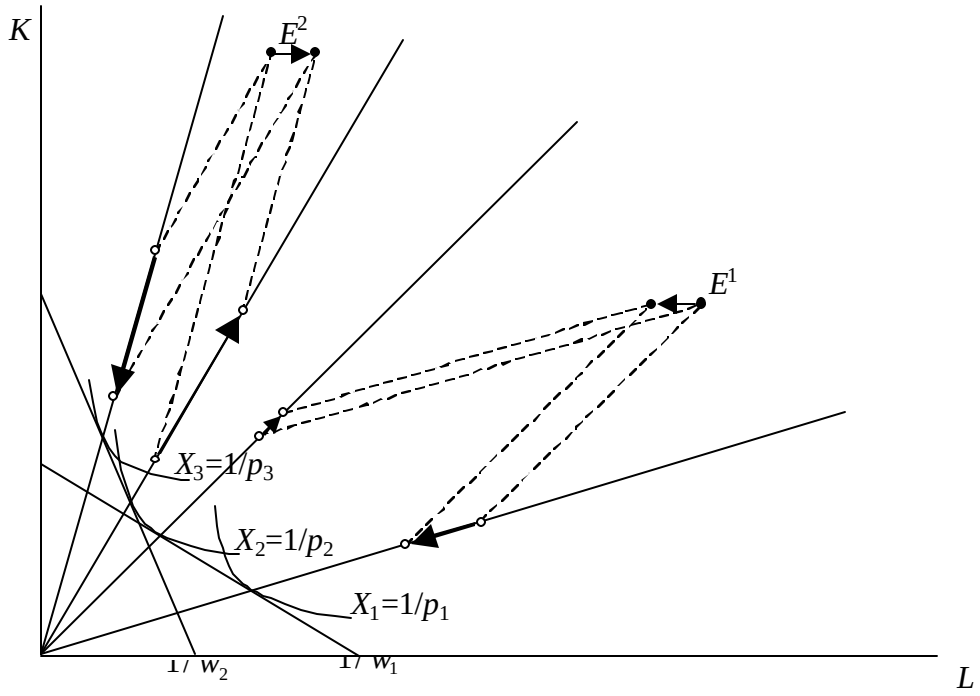
   Population growth, by itself, will cause the labor endowment to expand while the capital endowment does not, causing the endowment point in a Lerner diagram to move straight to the right as along Path 1 below. As it moves, depending on how far to the left it started (how little labor and how much capital), it will pass into and then out of first cone 2, with a higher wage, and then cone 1. Within each cone the wage will be constant, but outside of them the wage will decrease as L increases. Thus, whenever the country is producing two goods, its wage remains constant in spite of the population growth, but whenever it is producing only one, the wage falls.



The same story holds if capital is growing along with labor, as long as capital grows more slowly, as along Path 2 above. If capital is growing in proportion to labor, however, as along Path 3, then the capital-labor ratio of the endowment will remain fixed and the economy will stay, either within a cone or outside them, depending on where it started, and its wage will remain fixed.

3. Suppose the world consists of just two countries and three goods, initially in a two-cone equilibrium with country 1 producing the most labor intensive good, X_1, country 2 producing the most capital-intensive good, X_3, and both producing the good of intermediate capital intensity, X_2. Suppose now that a small part of the labor force in country 1 moves to country 2.

a. At initial prices, what happens to the real wage of the labor that moves? Does it



rise, fall, remain unchanged, or is the effect ambiguous?

Prices are fixed here, so a change in the real wage is the same as the change in the nominal wage. The labor moves from a wage of $\tilde{w}_1$ to the higher wage $\tilde{w}_2$, so the migrating labor gets a rise in its real wage.

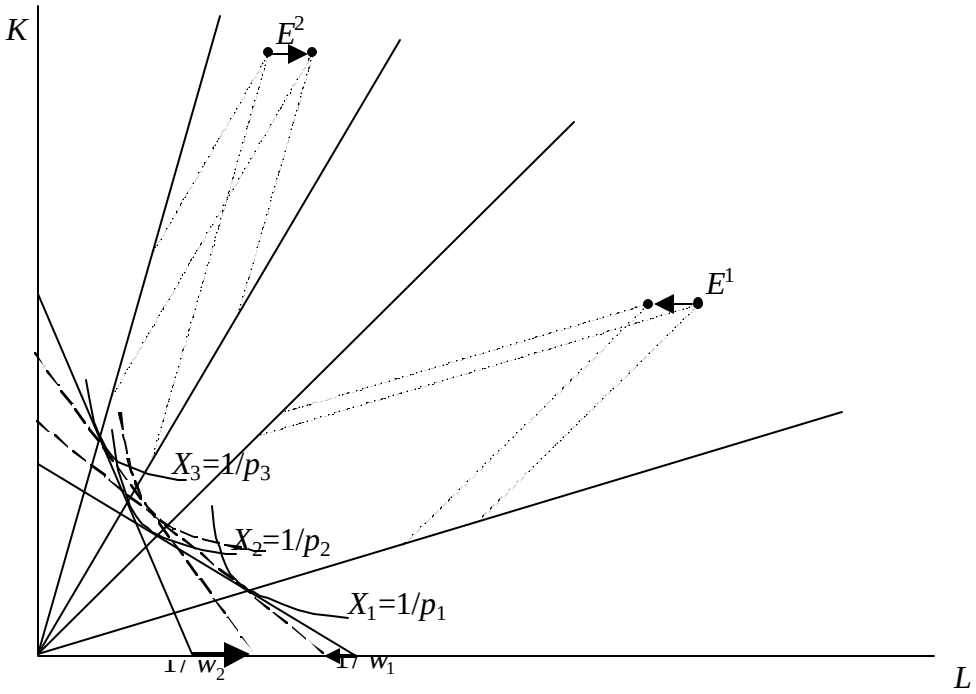
b. Also at initial prices, how, if at all, will this movement of labor affect the world's outputs of goods 1, 2, and 3?

From the parallelograms above, we see that both countries produce more of good 2, while the only producers of goods 1 and 3 both reduce their outputs of those goods.

c. Based on your answer to part (b), how would you expect world prices to change as a result, and how would this in turn affect real wages in the two countries? (Without details about preferences, you can't be sure of the answers to this, but you should be able to give answers that are plausible.)
With more output of good 2 and less of goods 1 and 3, we would expect the price of good 2 to fall, while the prices of 1 and 3 rise. Which of the latter rises by more depends on demand conditions, among other things, but there is no particular reason to expect one or the other, so it may be reasonable to assume, for a start, that p_1/p_3 remains unchanged. If so, then we can examine a fall in p_2 holding p_1 and p_3 constant, as below.

The price change shifts the unit-value isoquant for good 2 outward, causing the common tangent in the labor-intensive cone to rotate clockwise and the common tangent in the capital-intensive cone to rotate counter-clockwise. The result, as

shown by the labor intercepts of these tangents, is for the nominal wage to rise in country 1, which lost labor, and to fall in country 2, which gained labor. We know from the Stolper-Samuelson theorem that these changes are also changes in

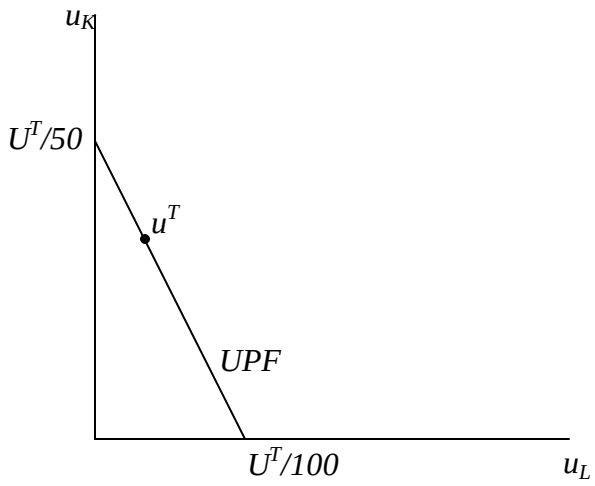


the same direction relative to the changed price of good 2, and therefore, since the other prices have been held constant, they are changes in real wages.

4. Consider a small open economy in a two-sector, Heckscher-Ohlin world where all the labor is owned equally by 100 people in one group, and all the capital is owned equally by a different 50 people in another group. As usual in HO models, both labor and capital are perfectly mobile between sectors, and initially the economy is producing both goods.

   a. Assuming that non-distorting transfers are possible, what can you say about the shape of the utility possibility frontier (UPF) for this economy?

   The main thing that we can say is that, whatever is the maximum utility available, call it U^T, the maximum utility per person in the labor-owning group will half that in the capital-owning group, since the former has twice as many people. With nondistorting transfers and a simple measurement of utility, the UPF will simply be a straight line with slope of minus two:

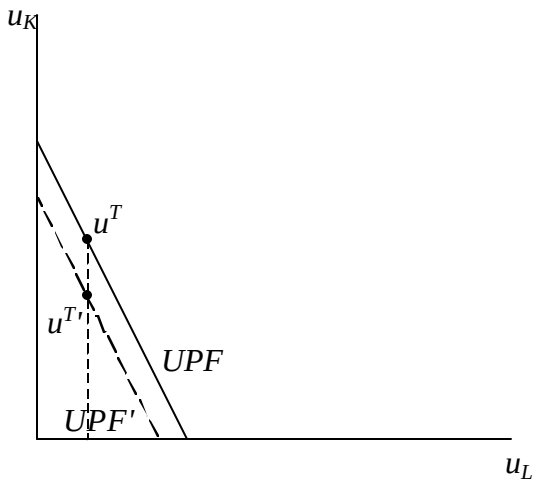


b. If transfers are not in fact used, what can you say about the actual utilities of the members of the two groups?

We can say very little, except that it will be a combination of utilities somewhere along this UPF. Exactly where depends on the equilibrium returns to labor and capital, which in turn depends on the prices of goods, which we don't know. There is no particular reason, at least in this abstract model, to expect the total return to capital to be, say, more than the total return to labor, so that capitalists would be assured of twice as high utility as workers. However, I have to put it somewhere, and that's what I've done above.

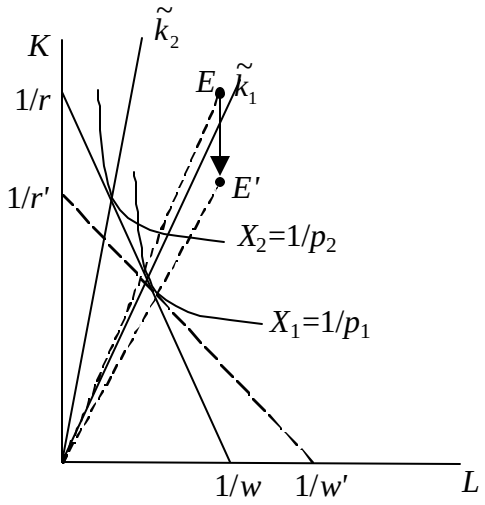
c. Suppose now that some sort of disaster destroys 20% of the capital of each capitalist, leaving population and labor unchanged. Assuming that the country continues to diversify, what will happen to its UPF, and what will happen to the actual utilities of the members of both groups?

The capital stock of the country, and of each capitalist, is cut by 20%. But because this is a small open economy facing fixed prices of goods, and because it is assumed to continue to diversify, we know from the factor price equalization theorem that factor prices will not change. Therefore each worker continues to earn the same wage as before, and the capitalists get the same rental on their remaining capital. Their actual incomes therefore fall by 20%. The actual income of labor is not reduced, but their potential income through redistribution is, since the economy can now produce less, with less capital. Since the numbers in the groups have not changed, the slope of the UPF will still be minus two, and it is parallel to the old one:

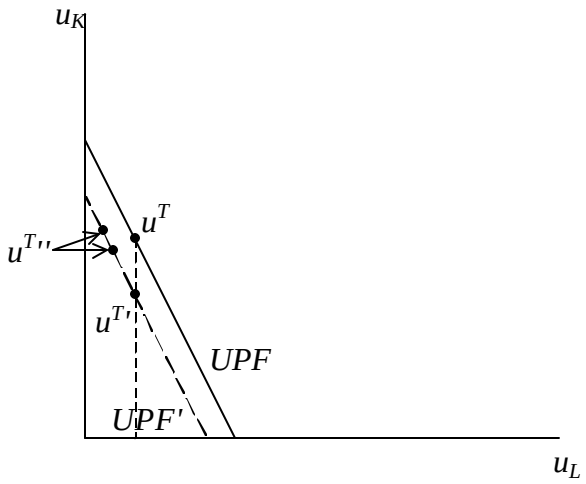


d. If instead the loss of capital causes the country to specialize in producing only one good, what then will happen to the UPF and to actual utilities?

Now we are leaving the cone of the HO model, and factor prices change, as can be seen in the following Lerner diagram:



The loss of capital moves the endowment point from E, inside the cone (between $\tilde{k}_1$ and $\tilde{k}_2$), down to E', below the cone. The wage falls from w to w' and the rental rises from r to r'. Compared to part (c), owners of labor do worse and owners of capital do better, possibly even raising their income and utility compared to before the disaster. The post-disaster utilities are now at points like those labeled u^{T}" below.



e. Using the same criteria that we usually use for judging gains from trade, how would we judge the welfare effects of this loss of capital in the two cases of parts (c) and (d)?

In both cases, the fact that utility possibilities have been reduced would cause us, using the usual gains-from-trade criteria, to conclude that the country has been made worse off by the disaster. This is true even though in case (c) there is a large groups of workers who have not been hurt, and in case (d) there may have been a large group of capitalists who have been helped.

5. Suppose that a small open economy that is exporting labor-intensive goods experiences an improvement in its terms of trade. In what sense does it "gain" from that improvement, in both the HO and the specific factors models?

In both cases, the "improvement in the terms of trade" means a rise in the relative price of the good it exports, and therefore a rise in the relative price of the labor-intensive good.

In the HO model, this means – according to the Stolper-Samuelson Theorem – that the real wage of labor rises and that the real rental on capital falls. Thus, if the two factors are owned by different groups, the group that owns capital is actually made worse off, unless the improvement of the terms of trade somehow stimulates a policy to redistribute income from workers to capitalists. We say nonetheless that the country gains from this change, because the income of the country as a whole goes up, shifting its utility possibility frontier outward, as below.

In the specific factors model, the story is similar, except that the winners and losers from the change in the terms of trade are different. Here it is owners of any specific factors in the labor-intensive industry that clearly gain, including the owners of capital there, while the owners of capital in the capital-intensive industry lose. Workers may gain or lose depending on the proportions they want to consume of the two goods. The diagrams below would be essentially unchanged, especially on the

left, while the utility possibility curves would now include owners of specific capital in the export sector on the horizontal axis. (Strictly speaking, there are three groups here, not two, and the UPF should therefore be extended to three dimensions.)