Econ 102
Savings, Investment, and the Financial System

1. Savings-Investment Identity
   a) Derive the identity between national savings (i.e. sum of private savings and
government savings) and investment for a closed economy. Show your steps.
   Start with \( Y = C + I + G \)
   Subtract \( C \) and \( G \) from both sides, yields:
   \( Y - C - G = I \)
   Now add and subtract \( T \) on the left hand side, to get:
   \( (Y - T - C) - (T - G) = I \),
   where \( (Y - T - C) \) is private saving and \( (T - G) \) is public saving.

   b) For comparison, derive the corresponding identity i.e. \( (I + NX = NS) \) for the open
economy. Again, show your steps.
   Start now with \( Y = C + I + G + NX \)
   Subtract \( C \) and \( G \) from both sides, yields:
   \( Y - C - G = I + NX \)
   Now add and subtract \( T \) on the left hand side, to get:
   \( (Y - T - C) + (T - G) = I + NX \)
   This identity states that private savings plus the budget surplus (national savings)
equals investment plus the trade balance. That is, \( S = I + NX \)
   (Thus in an open economy, it appears that savings does not equal investment. In fact, we will learn
later in the course that \( NX \) is very much like investment, since it implies that the country is
accumulating foreign assets. You don’t need to worry about that for now, however.)

c) Based on the identity for the closed economy, under what circumstances will an
increase in government purchases lead to lower growth in output per worker? Do
you think it is reasonable to believe that such a trade-off exists?
An increase in government purchases reduces the government’s surplus (assuming
that the taxes it collects are unchanged). If there is no change in private savings,
then national savings declines. Since \( S = I \), investment must decline as well. But
investment is the increase in the country’s capital stock, \( K \). Since \( Y/L = A \times F(1,K/L,H/L,N/L) \), slower accumulation of \( K \) will lead to slower growth in \( Y/L \).

3. Suppose you are given the following information about a closed economy:

\[
\begin{align*}
Y &= \$40,000 \\
T &= \$6,000 \\
S_{pr} &= \$1,000 + 0.15(Y-T) + 1,000r \\
I &= \$5,600 - 2,000r
\end{align*}
\]

Real GDP = Income
Net tax collections
Private saving function; \( r \) = real interest rate
Total investment function
Look carefully at the equations for private saving and total investment. Note that investment is a decreasing function of the real interest rate. That is, as the rate of interest increases, the level of desired investment (i.e. the demand for loanable funds) decreases. Also, private saving is an increasing function of both the real rate of interest and disposable income \((Y-T)\).

The parameter 0.15 in the saving equation, which is multiplied by disposable income, is called the marginal propensity to save (MPS) and tells us how much extra saving is generated by an increase in disposable income.

a) Give some intuition on why private saving might be increasing in the real interest rate.

Recall question 4 on homework 1: the real interest rate is the compensation an impatient consumer receives for delaying her consumption for an extra period. The higher this compensation, the more are consumers willing to wait to consume. Waiting to consume implies that the consumer is not spending all her income, but is saving it for next period. The less the consumer spends out of her income today, the more she has available to save. Therefore, a higher real interest rate rewards saving more than a lower real interest rate.

b) Does this government run a deficit or a surplus?

In this case, tax revenues of the government \((T)\) are lower than its purchases \((G)\). Therefore, the government is running a deficit. The amount of government deficit is:

\[
G - T = 6,800 - 6,000 = $800
\]

c) What is the amount of national saving? That is, write an equation for national saving \((S)\), showing how it depends on the interest rate.

Since national savings is the sum of private and public savings, we have:

\[
S = S_{pr} + (T-G)
\]

\[
= $1,000 + 0.15(Y-T) + 1,000r + (T - G)
\]

\[
= $1,000 + 0.15(Y-T) + 1,000r - 800
\]

\[
= $200 + 0.15 (40,000 - 6,000) + 1,000r
\]

\[
= $200 + $5,100 + 1,000r
\]

\[
= $5,300 + 1,000r
\]

d) Without any calculations, what are the implications of changes in the amount of government deficit/surplus for the equilibrium interest rate in the economy?

Part of the national savings equation is public savings. Therefore, a change in the amount of budget deficit or surplus will affect the supply side of the loanable funds market. An increase in the budget deficit or a decrease in the surplus will decrease the supply of loanable funds, increasing the interest rate. Similarly, a decrease in the budget deficit or an increase in the surplus will increase the supply of loanable funds, decreasing the interest rate.
Note that in this model, a reduction in investment in the economy does not change long run GDP. Here the level of GDP was given exogenously; that is, the model cannot explain changes in GDP. However, our simple model here makes predictions about changes in the composition of GDP.

e) Suppose that the government increases its purchases without increasing its tax collection. What prediction would you make about changes in the shares of consumption, investment, and government purchases in GDP?

As discussed above, a decrease in the amount of government surplus will lead to a higher real interest rate. In order to determine what happens to G/Y, I/Y and C/Y, we only need to look at the implied changes in the components of GDP, since Y is fixed in this example. An increase in G will lead to an increase in G/Y. Since I is a decreasing function of r, I/Y will decrease. Since C depends negatively on r, C/Y will fall as well. This exercise illustrates that in a closed economy an increase in government spending comes at the cost of less investment and less consumption in the long run.

f) Calculate the equilibrium real interest rate in this economy. Also, if you are told that the rate of inflation in this economy is 2.5%, what is the nominal rate of interest?

Setting National Savings equal to Investment we have:

\[
I = 5,600 - 2,000r = 5,300 + 1,000r \Rightarrow 300 = 3000r \Rightarrow r = \frac{300}{3000} = 0.10 = 10\%
\]

If \( r = 10\% \), and the inflation rate = 2.5\%, then \( i \) (the nominal rate of interest) is 12.5\% (because we know that \( r = i - \pi \) or \( i = r + \pi \), where \( \pi \) is the rate of inflation.)

g) What are the total levels of saving and investment at this rate of interest? How much of the total saving is comprised of government saving and how much is private saving?

Total Savings = \$5,300 + 1,000r = 5,300 + 1,000 \times (0.10) = \$5,400

Public Savings = - \$800
Private Savings = \$6,100 + \$100 = \$6200

4. Using a market for loanable funds diagram, analyze the effects of the following events on investment and on the real interest rate.

a. An increase in government purchases.

The increase in government purchases causes a decrease in \( S_{LF} \), for any \( r \), since it enters negatively in the net budget surplus part of the supply of loanable funds equation. The leftward shift in the supply of loanable funds causes an increase in
the real interest rate, which then causes investment to decrease along the given $D_{LF}$ curve.

\begin{figure} 
\centering 
\includegraphics[width=0.5\textwidth]{interest_rate_graph.png} 
\caption{Diagram showing the real interest rate and the shifts in the supply of loanable funds (SLF) curves.} 
\end{figure}

b. A tax credit for personal savings (kept revenue neutral by increasing another tax)

The tax credit for savings (a reduction in tax liability for every $1 saved) makes personal savings more attractive for consumers ($Y – C – T$ increases at any real interest rate) and thus shifts the supply of loanable funds to the right. This causes a decrease in the real interest rate and an increase in investment.

\begin{figure} 
\centering 
\includegraphics[width=0.5\textwidth]{tax_credit_graph.png} 
\caption{Diagram showing the effects of a tax credit on the real interest rate and the supply of loanable funds (SLF) curves.} 
\end{figure}

c. A decrease in the income tax rate (careful here).

The effects of this policy appear at first to be ambiguous. At a lower income tax, consumers have more disposable income and thus personal savings increases, but at the same time, the government collects less in taxes and thus saves less. As in question 3 above, government savings falls dollar-for-dollar with the fall in taxes, while consumers use their increased disposable income partly to save and partly to consume (in 3, they saved only 15% of the increase in disposable income). Thus private savings rises but by less than government savings falls, and the level of national savings for any interest rate is reduced. Thus the SLF curve shifts to the
left, and the picture is the same as in part (a) above. The real interest rate rises and the level of investment falls.

5. By how much does a high saving rate enhance a family’s future living standard? Find out by working out the following example:

The Spends and the Thrifts are similar families, except that the Spends save 7% of their total income each year and the Thrifts save 17%. The two families began saving in 1980 and plan to continue to save until their respective breadwinners retire in the year 2015. Both families earn $35,000 a year in real terms in the labor market, and both put their savings in a mutual fund that has yielded a real return of 6% per year, a return they expect to continue into the future. Compare the amounts that the two families consume in each year from 1980 to 2015, and compare the families’ wealth at retirement. Note that in any given year both families’ total income consists of wages (i.e. income earned in the labor market) and interest (i.e. income earned in the financial market).

You may find it convenient to do this exercise in a spreadsheet, such as MS Excel.

See the table on the next page, which has been constructed in the Excel spreadsheet HW3-Spends&Thrifts.xls. You can look at the formulas in the spreadsheet to see how it was done.

These dramatic differences depend in part on the assumption that the real rate of return is 6 percent – lower than the actual return to mutual funds since 1980 but still a relatively high rate of return from a historical perspective. On the other hand, the Spend family actually saves more than the typical U.S. household!
### Comparison

<table>
<thead>
<tr>
<th>Year</th>
<th>Consumption Spends</th>
<th>Thrifts</th>
<th>Wealth Spends</th>
<th>Thrifts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>$32,550</td>
<td>$29,050</td>
<td>$2,450</td>
<td>$5,950</td>
</tr>
<tr>
<td>1981</td>
<td>$32,687</td>
<td>$29,346</td>
<td>$4,910</td>
<td>$11,961</td>
</tr>
<tr>
<td>1982</td>
<td>$32,824</td>
<td>$29,646</td>
<td>$7,381</td>
<td>$18,033</td>
</tr>
<tr>
<td>1983</td>
<td>$32,962</td>
<td>$29,948</td>
<td>$9,862</td>
<td>$24,167</td>
</tr>
<tr>
<td>1984</td>
<td>$33,100</td>
<td>$30,253</td>
<td>$12,353</td>
<td>$30,363</td>
</tr>
<tr>
<td>1985</td>
<td>$33,239</td>
<td>$30,562</td>
<td>$14,855</td>
<td>$36,623</td>
</tr>
<tr>
<td>1986</td>
<td>$33,379</td>
<td>$30,874</td>
<td>$17,368</td>
<td>$42,946</td>
</tr>
<tr>
<td>1987</td>
<td>$33,519</td>
<td>$31,189</td>
<td>$19,891</td>
<td>$49,334</td>
</tr>
<tr>
<td>1988</td>
<td>$33,660</td>
<td>$31,507</td>
<td>$22,424</td>
<td>$55,788</td>
</tr>
<tr>
<td>1989</td>
<td>$33,801</td>
<td>$31,828</td>
<td>$24,968</td>
<td>$62,307</td>
</tr>
<tr>
<td>1990</td>
<td>$33,943</td>
<td>$32,153</td>
<td>$27,523</td>
<td>$68,892</td>
</tr>
<tr>
<td>1991</td>
<td>$34,086</td>
<td>$32,481</td>
<td>$30,089</td>
<td>$75,545</td>
</tr>
<tr>
<td>1992</td>
<td>$34,229</td>
<td>$32,812</td>
<td>$32,665</td>
<td>$82,265</td>
</tr>
<tr>
<td>1993</td>
<td>$34,373</td>
<td>$33,147</td>
<td>$35,252</td>
<td>$89,055</td>
</tr>
<tr>
<td>1994</td>
<td>$34,517</td>
<td>$33,485</td>
<td>$37,850</td>
<td>$95,913</td>
</tr>
<tr>
<td>1995</td>
<td>$34,662</td>
<td>$33,826</td>
<td>$40,459</td>
<td>$102,841</td>
</tr>
<tr>
<td>1996</td>
<td>$34,808</td>
<td>$34,171</td>
<td>$43,079</td>
<td>$109,840</td>
</tr>
<tr>
<td>1997</td>
<td>$34,954</td>
<td>$34,520</td>
<td>$45,710</td>
<td>$116,911</td>
</tr>
<tr>
<td>1998</td>
<td>$35,101</td>
<td>$34,872</td>
<td>$48,352</td>
<td>$124,053</td>
</tr>
<tr>
<td>1999</td>
<td>$35,248</td>
<td>$35,228</td>
<td>$51,005</td>
<td>$131,268</td>
</tr>
<tr>
<td>2001</td>
<td>$35,545</td>
<td>$35,950</td>
<td>$56,345</td>
<td>$145,921</td>
</tr>
<tr>
<td>2002</td>
<td>$35,694</td>
<td>$36,317</td>
<td>$59,032</td>
<td>$153,359</td>
</tr>
<tr>
<td>2003</td>
<td>$35,844</td>
<td>$36,687</td>
<td>$61,729</td>
<td>$160,873</td>
</tr>
<tr>
<td>2004</td>
<td>$35,995</td>
<td>$37,061</td>
<td>$64,439</td>
<td>$168,464</td>
</tr>
<tr>
<td>2005</td>
<td>$36,146</td>
<td>$37,440</td>
<td>$67,159</td>
<td>$176,133</td>
</tr>
<tr>
<td>2006</td>
<td>$36,297</td>
<td>$37,821</td>
<td>$69,891</td>
<td>$183,879</td>
</tr>
<tr>
<td>2007</td>
<td>$36,450</td>
<td>$38,207</td>
<td>$72,635</td>
<td>$191,705</td>
</tr>
<tr>
<td>2008</td>
<td>$36,603</td>
<td>$38,597</td>
<td>$75,390</td>
<td>$199,610</td>
</tr>
<tr>
<td>2009</td>
<td>$36,757</td>
<td>$38,991</td>
<td>$78,157</td>
<td>$207,596</td>
</tr>
<tr>
<td>2010</td>
<td>$36,911</td>
<td>$39,388</td>
<td>$80,935</td>
<td>$215,664</td>
</tr>
<tr>
<td>2011</td>
<td>$37,066</td>
<td>$39,790</td>
<td>$83,725</td>
<td>$223,813</td>
</tr>
<tr>
<td>2012</td>
<td>$37,222</td>
<td>$40,196</td>
<td>$86,527</td>
<td>$232,046</td>
</tr>
<tr>
<td>2013</td>
<td>$37,378</td>
<td>$40,606</td>
<td>$89,340</td>
<td>$240,363</td>
</tr>
<tr>
<td>2014</td>
<td>$37,535</td>
<td>$41,020</td>
<td>$92,165</td>
<td>$248,765</td>
</tr>
<tr>
<td>2015</td>
<td>$37,693</td>
<td>$41,438</td>
<td>$95,002</td>
<td>$257,252</td>
</tr>
</tbody>
</table>