Assessing moderation from intensive longitudinal data: Application to mHealth interventions

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Intensive longitudinal intervention data
cf. Walls and Schafer (2006)

$t$ treatment occasion
$X_t$ individual and contextual characteristics at $t$
$A_t$ binary treatment at $t$
$Y_{t+1}$ continuous response following $t$ and before $t + 1$
$H_t$ history through $t$: $(\bar{X}_t, \bar{Y}_t, \bar{A}_{t-1})$

An observation in chronological order is

$$X_1, A_1, Y_2, X_2, A_2, Y_3, \ldots, X_T, A_T, Y_{T+1}.$$
Example

BASICS-Mobile (Witkiewitz et al. 2014)
Proximal moderated effect

$A_t$ on $Y_{t+1}$

$Y_{t+1}(\tilde{a}_t)$ response, had the treatments $\tilde{a}_t$ been provided

$S_{1t}(\tilde{a}_{t-1})$ vector of candidate moderators from the history through $t$, had the treatments $\tilde{a}_{t-1}$ been provided

The proximal treatment effect is

$$E\left[ Y_{t+1}(\tilde{A}_{t-1}, 1) - Y_{t+1}(\tilde{A}_{t-1}, 0) \mid S_{1t}(\tilde{A}_{t-1}) \right].$$

This is averaged over any variables in $H_t$ not represented in $S_{1t}$. 
Delayed moderated effect

\( A_t \) on \( Y_{t+2} \)

The delayed treatment effect is

\[
E \left[ Y_{t+2}(\tilde{A}_{t-1}, 1, A_{t+1}) - Y_{t+2}(\tilde{A}_{t-1}, 0, A_{t+1}) \mid S_{2t}(\tilde{A}_{t-1}) \right].
\]

These are averaged over any variables in \( H_t \) not represented in \( S_{kt} \), as well as future treatment \( A_{t+1} \).
Identification

Under sequential ignorability, consistency and positivity

The proximal treatment effect in terms of the observed data is

\[
E \left[ E \left[ Y_{t+1} \mid A_t = 1, H_t \right] - E \left[ Y_{t+1} \mid A_t = 0, H_t \right] \mid S_{1t} \right]
= E \left[ \frac{1(A_t = 1)}{\rho_t(H_t)} Y_{t+1} - \frac{1(A_t = 0)}{1 - \rho_t(H_t)} Y_{t+1} \mid S_{1t} \right],
\]

where \( \rho_t(H_t) = \Pr(A_t = 1 \mid H_t) \).

The delayed treatment effect can be identified similarly.
Model

For parsimony we consider the linear model

$$E \left[ E \left[ Y_{t+1} \mid A_t = 1, H_t \right] - E \left[ Y_{t+1} \mid A_t = 0, H_t \right] \mid S_{1t} \right] = S_{kt}^T \beta_k^*,$$

for $k = 1, 2$. 
Estimation

Treatment probability known

The weighted least squares estimator solving

\[ \mathbb{P}_n \sum_t \left( Y_{t+k} - \tilde{S}_{kt}^\top \alpha_k - A_t S_{kt}^\top \beta_k \right) \]

\[ \times \left\{ \frac{\rho}{\rho_t(H_t)} \right\}^{A_t} \left\{ \frac{1 - \rho}{1 - \rho_t(H_t)} \right\}^{(1-A_t)} \left( \begin{array}{c} \tilde{S}_{kt} \\ A_t S_{kt} \end{array} \right) = 0, \]

for some \( \rho \in (0, 1) \), is consistent for \( \beta^*_k \) provided that \( \tilde{S}_{kt} \) contains \( S_{kt} \) as a sub-vector.

Here \( \tilde{S}_{kt}^\top \alpha_k \) is a working model for \( \mathbb{E}[Y_{t+k} \mid A_t = 0, H_t] \).
Observational data
Treatment probability unknown

Use $\rho_t(H_t; \hat{\eta})$ in place of $\rho_t(H_t)$.

Correct SEs for sampling error in $\hat{\eta}$.

If the true treatment probability can be expressed as $\rho_t(S_{kt}; \eta)$ or $S_{kt}$ omits no underlying moderators from $H_t$, weighting is doubly robust.
Implementation

Estimation can be implemented with standard GEE software.

Only the independence working correlation structure may be employed. Alternative structures induce bias.

Only baseline candidate moderators can be considered in weight stabilization.

Extra code is needed for SEs with observational data.
Example

\( A_t \) indicator that the user received a mindfulness-based EMI
\( Y_{t+1} \) smoking rate reported at the EMA following \( A_t \)
\( S_{1t} \) indicator of increased self-regulation from \( t - 1 \) to \( t \)
\( S_{2t} \) equal to 1

<table>
<thead>
<tr>
<th>Effect</th>
<th>Estimate</th>
<th>95% CI</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proximal, ↑ self-regulation</td>
<td>−0.5</td>
<td>(−2.0, 1.1)</td>
<td>0.553</td>
</tr>
<tr>
<td>Proximal, ↓ self-regulation</td>
<td>−2.5</td>
<td>(−5.0, −0.1)</td>
<td>0.045</td>
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<tr>
<td>Delayed</td>
<td>−1.0</td>
<td>(−2.2, 0.2)</td>
<td>0.100</td>
</tr>
</tbody>
</table>
Thanks

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