

IOE 510 HW 4 Dylon 6 Feb 2002.
 Three formulation Problems. Formulate only.

1. **Toy store problem:** A toystore chain has several stores in the midwest. For the coming X-mas season, they need to put orders with their overseas suppliers before the end of May for delivery in time for the X-mas sales period.

Since unsold toys at the end of the X-mas season do not contribute much to the profit of the company, they base their order quantities quite close to the expected sales volume. From experience over the years they observed that the X-mas sales volume has a high positive correlation with the DJA = Dow Jones average (a measure of the economic status of the region prior to the sales period), and a high negative correlation with the % unemployment rate in the region. Following table gives data on the DJA during the months of Feb, Mar, Apr (these are independent variables x_1, x_2, x_3), the % unemployment in the region during this period (independent variable x_4), and the toy sales volume in the region during the X-mas sales season (dependent variable y) between 1990-2001.

From above discussion it is reasonable to assume that the expected value of y can be approximated by a function $a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4$, where the parameters satisfy $0 \leq a_1 \leq a_2 \leq a_3$, and $a_4 \leq 0$. Write the LP formulation of the problem of finding parameter values that give the closest fit to data by the L_1 measure of deviation.

Year	x_1	x_2	x_3	x_4	y
2001	10690	10185	10306	4.3	64
2000	10533	10525	10798	4.0	65
1999	9356	9550	10307	4.3	59
1998	8307	8664	8940	4.6	53
1997	6828	6727	6800	5.2	42
1996	5435	5527	5579	5.5	35
1995	3927	4080	4239	5.5	26
1994	3898	3723	3634	6.5	25
1993	3344	3405	3434	7.1	23
1992	3247	3253	3294	7.4	22
1991	2798	2903	2895	6.6	19
1990	2607	2665	2673	5.2	16

1.18 In constructing a hydrological model using the data in the table given at the bottom, it is required to obtain the expected runoff, denoted by R_i , during the i th period, as a linear function of the observed precipitation. From hydrological considerations the expected runoff depends on the precipitation during that period and the previous two periods. So the model for expected runoff is $R_i = b_0 p_i + b_1 p_{i-1} + b_2 p_{i-2}$, where p_i equals precipitation during the i th

period; and b_0, b_1, b_2 are the coefficients that are required to be estimated. These coefficients have to satisfy the following constraints from hydrological considerations: $b_0 + b_1 + b_2 = 1$, where $b_0 \geq b_1 \geq b_2 \geq 0$. Obtain the best estimates for b_0, b_1, b_2 , if the objectives are

- (i) to minimize the sum of absolute deviations $\sum |R_i - b_0 p_i - b_1 p_{i-1} - b_2 p_{i-2}|$, and
- (ii) to minimize the maximum absolute deviation $\max |R_i - b_0 p_i - b_1 p_{i-1} - b_2 p_{i-2}|$.

Period	1	2	3	4	5	6	7	8	9	10	11	12
Precipitation (inch hours)	3.8	4.4	5.7	5.2	7.7	6.0	5.4	5.7	5.5	2.5	0.8	0.4
Runoff (acre feet)	0.05	0.35	1.0	2.1	3.7	4.2	4.3	4.4	4.3	4.2	3.6	2.7

—(R. Deininger)

1.29 A farmer can lease land up to maximum of 1000 a. He has to pay \$5 per acre per year if he leases up to 600 a. Beyond 600 a, he can lease at \$8 per acre per year. He grows corn on the land. He can grow corn at the *normal* level or at an *intense* level (more fertilizer, frequent irrigation, etc.) Normal level yields 70 bushels per acre. Intense level yields 100 bushels per acre. The requirements are given in the table at the bottom. Harvesting requires 0.5 man-hours of labor per bushel

harvested. The farmer can sell corn at the rate of \$2.50 per bushel in the wholesale market. He can also raise poultry. Poultry is measured in poultry units. To raise one poultry unit requires 25 bushels of corn, 20 man-hours of labor, and 25 ft² of shed floor-space. He can either use the corn that he has grown himself or buy corn from the retail market. He gets corn at the rate of \$3.50 per bushel from the retail market. He can sell at the price of \$175 per poultry unit in the wholesale market up to 200 units. Any amount of poultry over 200 units sells for \$160 per unit. He has only one shed for raising poultry with 15,000 ft² of floor space. He and his family can contribute 4000 man-hours of labor per year at no cost. If he needs more labor, he can hire it at \$3 per man-hour up to 3000 man-hours. For any amount of labor hired over 3000 man-hours, he has to pay \$6 per man-hour. Maximize his net profit.

Requirements per Acre per Year	Normal Level	Intense Level
Labor (man-hours)	6	9
Materials (seed, fertilizer, water etc.)	\$20	\$35