

MATH 217: MID-TERM EXAM
FALL 2004

NAME: _____

Professor/Section: _____

For each problem show **ALL** your steps and clearly **BOX** your final answer.

Problem	Points Possible	Points Earned
1	12	
2	10	
3	12	
4	14	
5	18	
6	14	
7	10	
8	10	
Total	100	

1. **a)** Let $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -3 \end{bmatrix}$. Find all the eigenvalues and eigenvectors for A .

b) Is A diagonalizable? If so find an invertible matrix P such that $AP = PD$ (or equivalently $A = PDP^{-1}$).

2. Let $A = \begin{bmatrix} -2 & 12 \\ -1 & 5 \end{bmatrix}$. A has eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$ with corresponding eigenvectors $v_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.
- a) Show that A is diagonalizable by writing $A = PDP^{-1}$.

- b) Use the factorization computed in a) to compute A^5 .

3. Suppose A is a 3×3 matrix where $v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ are in the range of A and $v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is in the kernel of A . Find a matrix that satisfies these conditions.

4. In each part below give a simple example or explain why no example exists.

a) A 3×5 matrix A_1 with the dimension of $Nul(A_1) = 4$.

b) A 3×5 matrix A_2 with the dimension of $Col(A_2) = 4$.

c) A 3×5 matrix A_3 with the dimension of $Row(A_3) = 4$.

d) A 3×5 matrix A_4 with $rank(A_4) = 1$.

e) A 3×5 matrix A_5 with dimension of $Col(A_5) = 2$ and the dimension of $Nul(A_5) = 2$.

5. Let H be the subspace of \mathbb{R}^4 given by $H = \text{Span}\{S\}$ where

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 0 \end{bmatrix} \right\}.$$

a) Find a basis, B , for H .

b) Find an implicit description of H . That is, complete the following:

$$H = \left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \text{ in } \mathbb{R}^4 \text{ such that } \dots \right\}$$

c) Can you find a basis B' for \mathbb{R}^4 that includes the vectors in B ? If so write one down. If not, explain why not.

d) Find a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^m$ for some m such that T is onto and $\ker(T) = H$.

6. Let $W = \{p \in \mathbb{P}_4 \text{ such that } p'(0) = p''(0)\}$.

a) Show that W is a subspace of \mathbb{P}_4 .

b) Find a basis for W .

c) What is the dimension of W ?

7. Let V be a vector space and B be an ordered basis for V . Use the definition of basis to prove that if $[v]_B = [w]_B$ then $v = w$.

8. Let T be a linear transformation and let v_1, v_2 , and v_3 be eigenvectors for T associated to the eigenvalues $\lambda_1 = 3$, $\lambda_2 = 5$, and $\lambda_3 = -2$ respectively. Find $T(v_1 - v_2 + 2v_3)$.