Using Vote Counts' Digits to Diagnose Strategies and Frauds: Russia*

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Abstract

Tests of the digits of vote counts have been proposed to diagnose election fraud. Both the second-digit Benford's-like Law (2BL) and the idea that the last digits should be uniformly distributed have been proposed as standards for clean elections. Many claim that election fraud is rampant in recent Russian federal elections (since 2004), so Russia should be a good setting in which to see whether the digit tests add any diagnostic power. Using precinct-level data from Russia, I first use a randomization test to identify sets of precincts (called UIKs in Russia) in which vote counts for candidates are augmented compared to vote counts in a comparison sets of UIKs. These are a subset of UIKs in which turnout percentages or the percentage of votes for Putin (or United Russia) are divisible by five. Then I run tests of the second and last digits of the UIK vote counts both for the entire set of UIKs in an election year and separately for various sets of UIKs. The digit tests produce surprising and on balance implausible results. For example, they suggest that none of the votes for Putin in 2004 and 2012 or for United Russia in 2011 were fraudulent, while votes for Medvedev in 2008 were fraudulent. The usefulness of simple and direct application of either kind of digit tests for fraud detection seems questionable, although in connection with more nuanced interpretations they may be useful.

1 Introduction

In authoritarian regimes in recent decades election fraud and its detection have been a concern (Lehoucq 2003; Bjornlund 2004; Schedler 2006; Alvarez, Hall and Hyde 2008; Myagkov, Ordeshook and Shaikin 2009; Cantu and Saiegh 2011; Fukumoto and Horiuchi 2011; Simpser 2013). Over the most recent election cycles Russian elections have become increasingly unfree and unfair, characterized by suppression of electoral competition, rising levels of administrative interference and drastic growth of electoral frauds (Freedom House 2010). The credibility of the elections has fallen to such an extent that one observer described them as "election type events" rather than genuine elections (Gel'man 2009). The growth of electoral manipulations and crude falsifications, and their widespread systematic pattern has been described as "mass administrational electoral technology" (Buzin and Lubarev 2008; Lubarev, Buzin and Kynev 2007). Evidence of Russian election frauds is found by inspecting vote distributions, which are distinctly bimodal and spiky (Shpilkin 2011; Klimek, Yegorov, Hanel and Thurner 2012), or by conducting field experiments (Enikolopov, Korovkin, Petrova, Sonin and Zakharov 2013). Fraudulent voter turnout is readily detected (Myagkov and Ordeshook 2008; Myagkov, Ordeshook and Shaikin 2008, 2009; Mebane and Kalinin 2009*b,a*). Ballot box stuffing and simply writing down false numbers are likely mechanisms (e.g. Boldyrev 2012), but phony voter registrations (Arbatskaya 2004) also occur.

Kalinin and Mebane (2011) argue that recent Russian electoral frauds may be partly explained by institutional incentives connected to recentralization. To address deficits in the Russian party system (Golosov 2004) and "the problem of dysfunctional bargaining" (Filippov, Ordeshook and Shvetsova 2004, 174), United Russia becomes "an integrated party that facilitates cooperation across levels of government" (Filippov, Ordeshook and Shvetsova 2004, 195) at the price of some "perversion of democracy" (Filippov, Ordeshook and Shvetsova 2004, 174). Kalinin and Mebane (2011) argue that the recentralization involves coordination among officials using particular values in election returns to

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implement a scheme of "signaling" between regional actors and the center. In particular Kalinin and Mebane (2011) claim that turnout values that are divisible by five, which as Mebane and Kalinin (2009b,a) show occur especially often, are used by regional and more local actors to signal loyalty to the Kremlin. The signaling is consequential for more than election outcomes: Kalinin and Mebane (2011) show that these "signals" are connected to systematic movements in transfer payments from the center to regional governments.

Evidence of election frauds in Russia is ample, so these elections provide a convenient setting for examining whether tests of the digits in precinct vote counts have anything to contribute to efforts to diagnose the manipulations the frauds involve. Notwithstanding the skeptical arguments (Carter Center 2005; Shikano and Mack 2009; López 2009; Deckert, Myagkov and Ordeshook 2011; Mebane 2011, 2013), the 2BL tests based on the second significant digits originally introduced by Pericchi and Torres (2004, 2011) are of interest.[1] Also there is a suggestion by Beber and Scacco (2012) that vote counts' last digits can be diagnostic. Beber and Scacco (2012) argue that if there are no frauds then the last digits of vote counts should be uniformly distributed. If fraud is as rampant in Russian elections as many observers claim, then they should be easy marks for the digit tests, if the tests actually work to detect fraud.

A caveat is that the motivation for the digit tests presumes that those who commit election fraud are unsophisticated or careless, or that the mechanisms used to commit the frauds do not allow precise control of what the fraudulent outcomes are. Beber and Scacco (2012) argue that humans who fake vote counts simply by writing down numbers they happen to think of are subject to psychological limitations that produce nonuniform patterns in the results. Such human limitations would be easily overcome, say by using a random number generator to create the fake numbers: using well-known algorithms, the fake numbers can have any desired distribution. Vote counts that are 2BL-distributed are easy to simulate as well, which would tend to undercut the test advocated by Pericchi and

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[1]Mebane (2006) introduces the location 2BL (for second-digit Benford's Law) tests, even though it is more precise (Mebane 2010) to use 2BL to refer to second-digit Benford-like tests.

Torres (2011). Indeed, Beber and Scacco (2012) point out that the simulated counts produced by one of the mechanisms in Mebane (2006) that was designed to produce counts that satisfy 2BL also have uniformly distributed last digits. Given that officials committing election fraud in Russia would know that precinct data would be released and that many would be scrutinizing the data, it is reasonable to believe that any such official might anticipate tests that might be performed.

I check whether digit tests are triggered by the votes cast in places that by other evidence feature many artificially manipulated and likely fraudulent votes. First I isolate UIKs[2] in which unusually large numbers of votes are counted for candidates or parties contesting the election. Then I compare digit test statistics for data from those UIKs to statistics for data from the other UIKs. Perhaps fraud is so pervasive in Russian elections that a UIK having an unusually large ballot count does not actually distinguish how much the votes counted there have been manipulated. But the effort to isolate places where fraud is more prevalent is at least plausible.

2 Randomization, etc.

For each UIK, turnout[3] or the proportion of votes for Putin (or, depending on the year, for Medvedev, for United Russia or for a candidate affiliated with United Russia) is multiplied by 100. Let T_i denote this value of turnout in UIK i, and let S_i denote the value of the vote percentage. Let V_{ik} denote the number of ballots counted for alternative k in UIK i. Using the indicator function $I(\cdot)$,[4] for each of the divisible-by-five values $j = 0, 5, \ldots, 100$ let

$$\bar{V}_{Tjk} = \frac{\sum_i I(|T_i - j| < .5)V_{ik}}{\sum_i I(|T_i - j| < .5)} \tag{1}$$

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[2]A UIK, *uchastkovaya izbiratelnaya komissiya*, is a precinct.

[3]Turnout is the sum of the number of ballots issued to voters who voted early, the number of ballots issued to the polling station on election day and the number of ballots issued outside the polling station on election day divided by the number of voters included in the voters list.

[4]$I(\cdot) = 1$ if the argument condition is true, $I(\cdot) = 0$ otherwise.

denote the mean number of ballots counted for alternative k in UIKs whose rounded value of T_i would be j. For each $j = 0, 5, \ldots, 100$ let $\mathcal{T}_{jk} = \{\bar{V}_{Tjk} - V_{ik} : .5 \leq |T_i - j| < 2.5\}$ denote the set of differences between $\bar{V}_{Tjk}$ and V_{ik} for UIKs that have T_i within 2.5 percent of the divisible-by-five value j but for which T_i would not round to j. Likewise based on

$$\bar{V}_{Sjk} = \frac{\sum_i I(|S_i - j| < .5)V_{ik}}{\sum_i I(|S_i - j| < .5)}, \tag{2}$$

define $\mathcal{S}_{jk} = \{\bar{V}_{Sjk} - V_{ik} : .5 \leq |S_i - j| < 2.5\}$ relative to vote percentages.

The values in $\mathcal{T}_{jk}$ and $\mathcal{S}_{jk}$ can be used to perform tests for ballot box stuffing (or other fakery that has the same effect on vote totals) that is tied to signaling among officials as described by Kalinin and Mebane (2011). The values in $\mathcal{T}_{jk}$ and $\mathcal{S}_{jk}$ represent comparisons of UIKs that have a divisible-by-five turnout figure or United Russia vote percentage to other UIKs that are similar in that they have similar turnout or vote percentage values. While in general these UIKs are quite diverse, notwithstanding their heterogeneity they ended up having similar turnout or vote percentages. They are in this sense "neighbors." So we might expect the ballot counts for each alternative k to be likewise similar among these "neighboring" UIKs. If UIKs with one of the divisible-by-five values for rounded T_i or rounded S_i do not have artificially elevated counts of ballots for alternative k, then the typical value of the elements of $\mathcal{T}_{jk}$ and $\mathcal{S}_{jk}$ should be zero. On the other hand, if some of the divisible-by-five turnout values are used to implement signaling among officials, as described by Kalinin and Mebane (2011), then we might expect the elements of $\mathcal{T}_{jk}$ to be typically positive for some values of j. While Kalinin and Mebane (2011) focus on signaling via turnout, analogous considerations suggest there may be signaling using vote percentages directly, hence such signaling may imply that the elements of $\mathcal{S}_{jk}$ are typically positive for some values of j.

One way to implement such tests with minimal assumptions is to use a randomization test procedure. To do this for turnout and a divisible-by-five number j, I consider each set

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of UIKs that have rounded T_i equal to j together with the "neighbors" of these UIKs: let $\mathcal{I}_j = (i : I(|T_i - j| < 2.5))$ denote an ordered set of the indices of these UIKs. For each permutation $\sigma(\mathcal{I}_j)$ of $\mathcal{I}_j$, compute

$$\bar{V}^\sigma_{Tjk} = \frac{\sum_i I(|T_i - j| < .5)V_{\sigma(i)k}}{\sum_i I(|T_i - j| < .5)} \tag{3}$$

and $\mathcal{T}^\sigma_{jk} = \{\bar{V}^\sigma_{Tjk} - V_{\sigma(i)k} : .5 \leq |T_i - j| < 2.5\}$. To test whether the mean $\bar{\mathcal{T}}_{jk}$ of the elements of $\mathcal{T}_{jk}$ is significantly large given the hypothesis that there is nothing special about the UIKs that have rounded T_i equal to j compared to the "neighbors" of these UIKs, find a p-value by computing the proportion of values $\bar{\mathcal{T}}^\sigma_{jk}$ among the permuted means $\bar{\mathcal{T}}^\sigma_{jk}$ that are greater than $\bar{\mathcal{T}}_{jk}$. That proportion is the p-value.

The procedure for vote percentages, based on $\bar{\mathcal{S}}_{jk}$, $\bar{V}^\sigma_{Sjk}$ and $\bar{\mathcal{S}}^\sigma_{jk}$, is exactly analogous.

I use $\bar{\mathcal{T}}_{jk}$ and $\bar{\mathcal{S}}_{jk}$ with data for each Russian presidential election during 2004, 2008 and 2012 and for each Duma election during 2003, 2007 and 2011 to run randomization tests of whether UIKs with rounded turnout or United Russia vote percentages that are divisible by five have excess ballots.[5] In 2007 and 2011 the elections used proportional representation electoral rules, but in 2003 half the seats were allocated based on proportional representation while half were determined by plurality rule elections in single-member districts. In 2003 I consider the proportional and plurality results separately. Because the number of UIKs is large (greater than 95,000), the number of elements in each set $\mathcal{T}_{jk}$ or $\mathcal{S}_{jk}$ is too large to allow all permutations to be enumerated. I use a Monte Carlo method based on using 10,000 random permutations to determine each p-value. Because the test for each year's votes involves multiple comparisons, I use methods based on the FDR (Benjamini and Hochberg 1995) to adjust the test level in each year for the set of divisible-by-five values j and choice alternatives k that occur in each year.[6]

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[5]UIK-level data are downloaded from sites connected to Central Election Commission of the Russian Federation (2013).

[6]To define FDR adjustment for J tests, sort the p-values with $\hat{p}_{(j)}$ being the j-th ordered value ($\hat{p}_{(1)}$ is smallest). For test level α, let d be the smallest value such that $\hat{p}_{(d+1)} > (d+1)\alpha/J$. The d smallest p-values match tests rejected after FDR adjustment (Benjamini and Hochberg 1995) .

The test results suggest that in the presidential election of 2012 there is significant artificial vote count augmentation that focuses on turnout values that are divisible by five. Figure 1 shows test results for all five candidates that relate to turnout ($\bar{\mathcal{T}}_{jk}$). Tests are attempted for the values $j = 20, 25, \ldots, 100$.[7] Several p-values in Figure 1 are less than the conventional test level $\alpha = .05$, and several are significant at level $\alpha = .05$ even given FDR adjustment for multiple testing. For Zhirinovskij, Mironov and Prohorov, the significant p-values occur for $j = 100$, while for Putin there is a significant p-value for $j = 85$. Other p-values that are not significant given multiple testing but are nonetheless small (less than .1) occur for Putin at $j \in \{45, 90\}$, for Zyuganov and Mironov at $j = 50$, for Prohorov at $j = 60$ and for Zyuganov, Mironov, Prohorov and Putin at $j = 80$.

*** Figure 1 about here ***

The test results suggest even more that in 2012 vote counts are artificially augmented when Putin has a vote percentage value that is divisible by five. In Figure 2, which shows results based on $\bar{\mathcal{S}}_{jk}$, significant (FDR adjusted) p-values occur for Zhirinovskij, Zyuganov, Mironov, and Putin at $j \in \{70, 75, 80, 85\}$, for Zyuganov at $j = 90$ and for Putin at $j = 100$. Small p-values also occur for Zyuganov at $j \in \{30, 65\}$, for Mironov at $j = 30$, for Prohorov at $j \in \{30, 35, 70, 75\}$ and for Putin at $j \in \{30, 65, 90\}$. In UIKs at many divisible-by-five values of Putin's vote percentage greater than or equal to 65 percent and at a couple of lower percentages, the counts of ballots for most or all of the candidates are typically greater than in "neighboring" UIKs.

*** Figure 2 about here ***

How often are UIKs that have turnout divisible by five the same as the UIKs that have Putin's vote percentage divisible by five? Figure 3, which displays how often each combination of rounded turnout and rounded Putin vote percentage values occur in the

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[7]The absence of p-values for $j = 20$ in Figure 1 results from there being no UIKs matching that value. p-values are missing for some values of j in Figures 7, 15 and 9 for the same reason.

2012 UIK data, provides some of the answer. The peak occuring with a frequency of 533 where both percentages are at 100 is the most notable feature of the graph. Somewhat harder to see are the local peaks that occur at combinations of values that are divisible by five. For instance there are local peaks at turnout equal 100 percent and Putin percentage equal to 50, 55, 60, 65 or 70. Other local peaks at divisible-by-five values are harder to pin down in Figure 3 but are clear upon inspecting the counts in each cell. For turnout greater than 50 and Putin percentage greater than 55, counts in twenty divisible-by-five cells are greater than in any of the "neighboring" cells (i.e., one or two units away). The list of these cells appears in Table 1.[8]

*** Figures 3 and Table 1 about here ***

Figure 4, which shows the smoothed joint density of the UIK-level turnout and Putin vote proportion variables, reinforces the message that divisible-by-five values of turnout and of Putin vote percentage occur occur relatively frequently for the higher values of each variable.[9] The density plot does not resolve the location of the local maxima as precisely as does inspecting the counts in the frequency distribution of the rounded variables, but it does clearly show the regularity of the pattern in which many of the local peaks occur. Peaks are apparent near turnout of .5, .7, .85 or 1.0 and Putin vote proportion of .55, .75, .85 or 1.0.

*** Figure 4 about here ***

If not for the evidence of the randomization tests reported in Figures 1 and 2, we might explain the greater frequency of turnout and Putin vote percentage figures that are divisible by five as merely due to officials at some level rounding off reported figures. Regarding this

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[8]Other divisible-by-five cells occur more frequently than cells one unit away but not for all cells that are two units away: the referent (turnout, vote percentage) pairs are $(80, 75)$, $(85, 65)$, $(90, 65)$, $(90, 95)$, $(95, 75)$ and $(95, 95)$.

[9]The density is computed using the `sm` package of **R** (R Development Core Team 2005). I call function `sm.density()` with argument `h=c(.002,.002)`, which produces a window for the normal kernel function that is narrower than optimal. With optimal smoothing parameters the density resembles Figure 3.

potential explanation, however, note that such rounding off would be occurring in computed percentages and not in the original counts of who voted and for whom. The randomization test results show that the UIK counts of votes received by the candidates in divisible-by-five UIKs are augmented in comparison to the "neighbors" of those UIKs. Rather than rounding off after the fact, one suspects that the divisible-by-five values came first and the vote counts were then concocted to go with the specified percentages.

The relationship between turnout and the leading candidate's vote proportion in the presidential elections of 2008 and 2004 is similar to the relationship in 2012. The plots of the smoothed densities for 2008 and 2004, shown in Figures 5 and 6, resemble the 2012 density in that the local peaks are in the same locations. The peaks when turnout and the vote proportion (for Medvedev in 2008 and for Putin in 2004) both equal 1.0 are higher in 2008 and in 2004 than the peak for the same values in 2012. The peaks near .75 and .85 for the candidates' vote proportion seem to be roughly the same size in all three years, while the peaks near .55 are higher in 2012 than in the other two years.

*** Figures 5 and 6 about here ***

Randomization tests using the earlier presidential election data produce results similar to those seen for 2012, but also there is evidence that the amount of artificial vote count augmentation tied to the divisible-by-five values increases over time. In 2008 the frequency of focused vote augmentation across distinct divisible-by-five values seems roughly comparable to the frequency observed in 2012. In 2008 all the significant p-values for divisible-by-five turnout values occur for vote counts for Medvedev (Figure 7(a)): at rounded turnout $j \in \{80, 90, 95\}$. Small p-values occur for all candidates at $j = 65$, for Medvedev and Zhirinovskij at $j \in \{75, 85\}$, and for Zhirinovskij at $j \in \{80, 90\}$. Tests focused on instances when Medvedev has a divisible-by-five vote percentage also show significant results about as frequently as in 2012. There are significant p-values for Medvedev at $j \in \{65, 70, 75, 80, 85\}$, for Zhirinovskij at $j \in \{70, 75, 80, 85, 90\}$, for

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Zyuganov at $j \in \{65, 70, 85\}$ and for Bogdanov at $j \in \{80, 85\}$ (Figure 8). Several small p-values also occur for all candidates for $j \geq 60$.

*** Figures 7 and 8 about here ***

In 2004 the number of distinct divisible-by-five values that test positive for vote augmentation is slightly smaller than in the subsequent two presidential election years. Only one p-value is significant among the p-values for turnout, for the Against All alternative at $j = 100$ (Figure 9(g)). Many p-values other than that one are small at $j \geq 55$, however, and vote counts for all candidates (and for Against All) are involved at various values of j. In this respect, the number of j values at which votes are augmented may be somewhat greater in 2004 than in 2012. Augmentation appears less prevalent in 2004 among the divisible-by-five values of Putin's vote percentage. A single p-value is significant for each of Putin, Haritonv and Malyshkin, at $j = 95$ (Figure 10). Several p-values are small for many candidates, at $j \in \{45, 85, 90, 95\}$, but the small p-values occur for fewer distinct value of j than in 2012 or 2008. In this sense the prevalence or perhaps it's better to say the diversity of artificially augmented vote counts appears to increase over time.

*** Figures 9 and 10 about here ***

The pattern and amount of artificial vote augmentation in the 2011 and 2007 Duma elections seem to be similar to what happens in the 2012 and 2008 presidential elections. The relationship between turnout and United Russia's vote proportion in these Duma elections resembles the relationship between turnout and the leading candidate's vote proportion in the presidential elections. The smoothed density for 2011 has a dominant peak at turnout and United Russia proportion both equal to 1.0, and other local peaks occur when turnout is near .45, .6 or .8 and the United Russia proportion is near .4, .6 or .8, when turnout equals 1.0 or when the United Russia proportion equals .2 (Figure 11). The pattern in the smoothed density for 2007 is similar: local peaks that are apparent in

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2011 when the United Russia proportion equals .2 are not present in 2007, and local peaks occur when turnout is near .5, .65 or .8 and the United Russia proportion is near .5, .6 or .8 (Figure 12). Neither the pairs in the set $\{(i,j) : i = 45, 60, 80, j = 40, 60, 80\}$ in 2011 nor the pairs in the set $\{(i,j) : i = 50, 65, 80, j = 50, 60, 80\}$ in 2007 are always larger than all their "neighbors" in the joint distribution of the rounded turnout and United Russia percentage variables, but those values are "neighbors" of other local modes.

*** Figures 11 and 12 about here ***

The set of divisible-by-five values on which randomization tests suggest vote augmentation is concentrated is similar between the Duma and presidential election years, and again there is evidence of a progression over time. In 2011, as in 2012, among the turnout-based tests a single p-value is significant for all parties except KPRF and United Russia, at $j = 100$ (Figure 13). For United Russia in 2011, as for Putin in 2012, the significant p-values occur at slightly lower values, namely $j \in \{75, 85\}$. For the tests based on vote percentage, significant p-values occur about as often and for about the same values of j in both 2011 and 2012 for the parties that competed in both elections (Figure 14). In 2007 the turnout-based tests resemble the pattern in 2012 in having significant p-values at $j = 100$ for several small parties and a significant p-value for United Russia at a slightly smaller value, namely $j = 95$ (Figure 15). A few other p-values are also significant, and each party has at least a few small p-values. The tests based on vote percentage in 2007 feature slightly fewer significant p-values than in 2012, as well as slightly fewer small p-values for parties that competed in both elections (Figure 16).

*** Figures 13, 14, 15 and 16 about here ***

The pattern of votes being significantly augmented at divisible-by-five turnout and United Russia vote percentage values becomes more prevalent over time at least in the sense that significant augmentations are associated with more distinct values of those two

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variables over time. Moreover given the very different patterns observed in the 2003 Duma election, it is reasonable to say that the divisible-by-five patterning in vote augmentations began in the 2004 presidential election.

Both the distribution of divisible-by-five turnout and United Russia vote percentage values and the randomization test results differ in 2003 from the findings from subsequent years. The most immediate difference in the smoothed density for the proportional representation votes is that in 2003 the peak when both turnout and United Russia vote proportion equal 1.0 is no longer the highest mode: a mode near the turnout-proportion pair $(.55, .35)$ is higher (Figure 17). Inspecting the joint distribution of rounded turnout and United Russia vote percentage values shows the greatest count, which is 161, occurs for $(56, 34)$, while the count for $(55, 35)$ is only 121. In that joint distribution, the only divisible-by-five pairs that are local peaks among their "neighbors" are $(75, 60)$, $(95, 90)$ and a few pairs in which turnout rounds to 100 percent.[10] In fact the pair $(100, 100)$ is not a local peak—the count for $(100, 99)$ is one greater (135 versus 134). This suggests there is not substantial coordination on a few values that are divisible by five. The randomization test based on turnout and proportional representation votes finds three significant p-values, a result arguably not all that different from the pattern for turnout in 2012, but in the test based on United Russia percentage no p-values are significant (Figures 18 and 19). All the significant p-values in the turnout test are for turnout value $j = 100$ and affect votes for United Russia, LDPR and Yabloko, so there is some evidence of artificial vote augmentation. But the absence of significant results in the test based on United Russia percentage imply that whatever is going on in this election is unlike what happens in the later years.

*** Figures 17, 18 and 19 about here ***

The votes cast in the plurality rule elections in 2003 show even fewer signs of vote augmentation related to turnout or United Russia percentage values that are divisible by

[10]These pairs are $(100, 50)$, $(100, 60)$, $(100, 80)$ and $(100, 85)$.

five. The smoothed density of the two variables has many local peaks, but only eight of these are centered on a pair of divisible-by-five values in the joint distribution of the rounded variables (Figure 20).[11] None of those pairs occur very frequently—the largest count has the pair (55, 60) occurring in 28 UIKs—and the pair (100, 100) is not a local peak. The randomization test based on turnout produces three significant p-values, but none of these are for United Russia (Figure 21). The randomization test based on United Russia percentage produces no significant p-values (Figure 22). While KPRF has a single p-value that is less than .1, none of the p-values for United Russia or for LDPR are small. If there is vote augmentation in these elections, it occurs in patterns utterly unlike what happens in later years.

<div align="center">*** Figures 20, 21 and 22 about here ***</div>

3   Digit Tests

Many have argued there is strong evidence of widespread election frauds in Russian federal elections at least since 2004 (e.g. Myagkov, Ordeshook and Shaikin 2009; Klimek et al. 2012), and the tests discussed in the previous section have identified specific sets of UIKs in which vote counts are systematically greater than the counts in "neighboring" UIKs. The question for this section is whether any of the various tests that have been proposed that examine digits in the vote counts respond to the alleged frauds.

In addition to tests based on the second significant digits examined previously, of especial interest is the test proposed by Beber and Scacco (2012), which is based on the idea that if there are no frauds then the last digits of vote counts should be uniformly distributed. For the last digits to be uniformly distributed means that they should occur equally frequently. This is analogous to the second-digit equal frequency (2E) hypothesis considered in Mebane (2006), except the the expected frequency of $\frac{1}{10}$ is applied to the last

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[11]The pairs are (40, 50), (55, 60), (60, 75), (65, 15), (70, 30), (90, 50), (100, 40) and (100, 50).

digit of each vote count. To test whether the hypothsis that the last digits in a set of N_L vote counts is uniformly distributed, I let ℓ_i be the number of times the last digit is i in the set and define the Pearson chi-squared test statistic $X_{LU}^2 = \sum_{i=0}^{9} (\ell_i - N_L/10)^2/(N_L/10)$, where $N_L = \sum_{i=0}^{9} \ell_i$. To compare X_{LU}^2 to the chi-squared distribution with nine degrees of freedom, I use a significance probability of the form advocated by Pericchi and Torres (2011), $\hat{\alpha}_L = (1 + [-ep_L \log(p_L)]^{-1})^{-1}$ where where p_L is p-value of X_{LU}^2.

Vote counts in UIK-level data may be small, and indeed Beber and Scacco (2012, 217) warn against using their test with small counts. Indeed, with small counts an expectation that the last digits should be uniformly distributed goes against the suggestion from Pericchi and Torres (2011) that 2BL should describe the distribution of the second-significant digits: if the counts are two-digit numbers then the last digit is the second digit. To avoid including small counts I restrict attention to vote counts greater than 99 by defining a modified statistic $X_{LU*}^2 = \sum_{i=0}^{9} (\ell_{i*} - N_{L*}/10)^2/(N_{L*}/10)$, where ℓ_{i*} is the number of times the last digit is i and the count is greater than 99 and $N_{L*} = \sum_{i=0}^{9} \ell_{i*}$, with significance probability $\hat{\alpha}_{L*} = (1 + [-ep_{L*} \log(p_{L*})]^{-1})^{-1}$ where p_{L*} is p-value of X_{LU*}^2. If the last-digits of all the counts are in fact uniformly distributed, then removing the counts with values between zero and 99 should not affect the test except by reducing its power due to the smaller number of cases.

The randomization test results for 2004–2012 suggest it is reasonable to consider not only digit tests for the entire set of UIKs in each election but also in subsets that roughly match the apparent occurrence of vote augmentation. UIKs that have rounded turnout and round Putin (or United Russia, etc.) vote percentages that are divisible by five should be considered separately from the others, and Figures 2, 8 and 14 suggest further separating the UIKs in which the Putin/United Russia percentage is greater than 60 from the rest. To this end I compute digit test statistics for four sets of UIKs, symbolized and defined as follows:

All the set of all UIKs;

D5$_{>60}$ UIKs in which rounded turnout and rounded Putin vote percentage are divisible by 5 and Putin vote percentage is greater than 60;

ND5$_{>60}$ UIKs in which neither rounded turnout nor rounded Putin vote percentage are divisible by 5 and rounded Putin vote percentage is greater than 60;

ND5$_{<60}$ UIKs in which neither rounded turnout nor rounded Putin vote percentage are divisible by 5 and rounded Putin vote percentage is less than or equal to 60.

Given the randomization tests, the UIKs in D5$_{>60}$ likely have augmented vote counts, while the UIKs in ND5$_{<60}$ do not have any of the divisible-by-five features that may be used to signal and also have relatively low precentages supporting Putin or United Russia. I use ND5$_{>60}$ to offer a contrast with D5$_{>60}$ with regard to the divisible-by-five values and a contrast with ND5$_{<60}$ on the Putin/United Russia percentage. Clearly, based on the signaling story and randomization test results, we should expect the vote counts for the UIKs in D5$_{>60}$ to show the clearest signs of being fraudulent. But obviously the other sets may show signs of fraud if fraud is prevalent but the signaling story is wrong or if, the randomization test results notwithstanding, vote augmentation is not the only device used to commit fraud.

All this supposes that the digit tests detect fraud. Mebane (2013) and the discussion of conditional second-digit tests already give ample reason to doubt that X^2_{2BL} is effective for that. Whether X^2_{LU} or X^2_{LU*} work to detect fraud in Russia remains to be seen.

It also supposes that those committing any frauds in Russia are unsophisticated. In fact, claims about election fraud were well known to officials in Russia, and there is every reason to believe they were familiar with digit tests among others.[12] One must credit the possibility that any faked vote counts are designed to pass various tests. Digit tests would be easy to pass with fake data if the fakery were sufficiently well coordinated. The

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[12]Key election officials attended workshops held to discuss findings regarding Russian election frauds and attempted to defend the elections.

institutional requirements for signaling (Kalinin and Mebane 2011) are at least one reason why other markers for fraud are nonetheless easy to detect.

The second-digit tests applied to data from the 2012 election show extensive deviations from 2BL, but it is unclear whether these deviations should be interpreted as diagnosing fraud. In the set of all UIKs, X^2_{2BL} is significant for all candidates except Zyuganov (Table 2). $\hat{j}$ differs significantly from $\bar{j}$ for Zhirinovskij, Mironov and Prohorov but not for Zyuganov or Putin.[13] Remarkably, X^2_{2BL} is not significant for any of the candidates in the set $D5_{>60}$, but X^2_{2BL} is significant for all candidates in both $ND5_{>60}$ and $ND5_{<60}$. $\hat{j}$ for Putin is significantly greater in $ND5_{<60}$ than in $ND5_{>60}$. In $ND5_{>60}$ $\hat{j} < \bar{j}$ while in $ND5_{<60}$ $\hat{j} \approx 4.35$. Simply applying the interpretation suggested by Mebane (2010) would suggest that Putin gained strategically switched votes in UIKs where he received a low proportion of the vote, while such gains did not occur in UIKs where he received most of the votes. Intuitively that seems like an odd result, but because no one expected Putin to lose the election and because the other candidates all received nontrivial and unequal shares of the vote,[14] it is difficult to say what if any strategy most voters might have been using. And it remains unclear to what extent the data are fraudulent. The most cynical interpretation, perhaps, is that in the UIKs in $D5_{>60}$, vote counts were substantially faked with an eye to passing the test set out by Pericchi and Torres (2004, 2011), while in the UIKs in $ND5_{<60}$ there was less fakery and the vote counts more reflect Putin's actual support.

*** Table 2 about here ***

The last-digit test applied to the 2012 data produces eerie results that also appear in a sense too good to be true. The least one can say is that the results are exceptionally good for Putin. X^2_{LU*} is never significant for Putin in any of the subsets, although the statistic in

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[13]$\hat{j}$ is the mean of the second digits. If the distribution of the counts' second-digits has frequencies r_j as given by Benford's Law, then the second digits' expectation is $\bar{j} = \sum_{j=0}^{9} jr_j = 4.187$.

[14]Based on Central Election Commission of the Russian Federation (2013), Putin received 45,602,075 votes, which is 64.4 percent of the valid votes, while the other candidates (Zhirinovskij, Zyuganov, Mironov and Prohorov) respectively received 6.3, 17.4, 3.9 and 8.1 percent. This outcome does not match the equilibria of, for example, Cox (1994).

the set of all UIKs comes close with $\hat{\alpha}_{LU*} = .06$ (Table 3). Remarkable is that in all sets $X^2_{LU} < X^2_{LU*}$ for Putin: even in the set of all UIKs, including the additional 12,367 UIKs that have values between zero and 99 increases the fit to a uniform last-digit distribution. X^2_{LU*} is not significant in D5$_{>60}$ for any candidate except Zyuganov, X^2_{LU*} is significant for every candidate except Putin in ND5$_{>60}$, and X^2_{LU*} is significant for every candidate except Putin and Zyuganov in ND5$_{<60}$.[15]

*** Table 3 about here ***

It is difficult to think of a natural electoral process in which there are no artificial elements that would produce a result where some candidates have vote counts with uniformly distributed last digits and others do not, and moreover some candidates have vote counts with uniformly distributed last digits in some places but not others. So following Beber and Scacco (2012), one might say the last-digit test diagnoses fraud. But then, oddly, the test would suggest that all the candidates had fraudulent vote totals except for Putin and occasionally except for Zyuganov, and except for all the candidates (except now Zyuganov) in those UIKs where the randomization test shows vote counts were augmented. Beber and Scacco (2012) describe psychological mechanisms to motivate their test, but it is difficult to understand how such mechanisms could possibly explain the patterns in Table 3. More likely is the cynical explanation that Putin's vote counts are generally faked everywhere—faked using sophisticated algorithms and coordination and not merely in the UIKs in D5$_{>60}$—and that the fakery's effect on the vote counts for other candidates is intricate. Those effects probably trace back to Putin's relationships with the other candidates and might well not be accidental.

The patterns in the digit tests in 2011 resemble 2012 in some ways, but there are some key differences. The second-digit tests show that in the set of all UIKs X^2_{2BL} is significant for all parties except KPRF (Table 4). KPRF is Zyuganov's party in 2012, and in the set

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[15]The two p-values that equal .05 are not significant if FDR adjustment is applied to the set of all p-values in D5$_{>60}$, ND5$_{>60}$ and ND5$_{<60}$.

of all UIKs Zyuganov's vote counts were the exceptional case for X^2_{2BL} in 2012. Again resembling 2012, X^2_{2BL} is significant for only one party (Right Cause) in the set D5$_{>60}$, and X^2_{2BL} is significant for all candidates in ND5$_{>60}$ and all but two parties (Just Russia and KPRF) in ND5$_{<60}$.[16] $\hat{j}$ for United Russia is significantly greater in ND5$_{<60}$ than in ND5$_{>60}$. In the sets D5$_{>60}$, ND5$_{>60}$ and ND5$_{<60}$, only KPRF and United Russia have $\hat{j} > \bar{j}$ significantly. Neither of these parties were close to the threshold of seven percent needed to gain a seat in the Duma,[17] so there is no obvious basis in voters' strategies that might be related to the thresholds to explain this result.

<center>*** Table 4 about here ***</center>

The last-digit test in 2011 has three key similarities with 2012. X^2_{LU*} is never significant for United Russia in any of the subsets, although the statistic in the set of all UIKs comes close with $\hat{\alpha}_{LU*} = .07$ (Table 5). In all sets $X^2_{LU} < X^2_{LU*}$ for United Russia. Ignoring the cases where $N_{L*} < 100$,[18] X^2_{LU*} is not significant in D5$_{>60}$ for any party except KPRF.[19] Differences from 2012 are that X^2_{LU*} is significant for only one party—KPRF—in ND5$_{>60}$, and X^2_{LU*} is significant for only one party—-LDPR—in ND5$_{<60}$. The similarities between 2011 and 2012 are the unexceptioned uniformity of the last digits in the United Russia vote counts, the unique significance in D5$_{>60}$ of X^2_{LU*} for KPRF, and the unusual distribution observed for single parties in the subsets which mostly test free of vote augmentation in the randomization tests. In 2011 the unusual results are that KPRF has a significant X^2_{LU*} value in ND5$_{>60}$ and LDPR has a significant X^2_{LU*} value in ND5$_{<60}$, while in 2012 the unusual result was the insignificant X^2_{LU*} value for Zyuganov in ND5$_{<60}$.

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[16]The p-value that equals .03 is not significant if FDR adjustment is applied to the set of all p-values in D5$_{>60}$, ND5$_{>60}$ and ND5$_{<60}$.

[17]Based on Central Election Commission of the Russian Federation (2013), United Russia received 32,371,737 votes, which is 49.3 percent, while the other parties (Just Russia, LDPR, Patriots, KPRF, Yabloko, Right Cause) respectively received 13.2, 11.7, .97, 19.2, 3.4 and .60 percent.

[18]While it may be reasonable even with fewer cases to assume the distribution of X^2_{LU*} under the null is the chi-squared distribution (Larntz 1978; Koehler and Larntz 1980; McCullagh 1986), in this case I avoid taking that assumption seriously. The exact distributions could be computed when N_{L*} is small. I haven't done that because not much of substance seems to be at stake with the very small parties.

[19]The p-value that equals .01 is not significant if FDR adjustment is applied to the set of all p-values in D5$_{>60}$, ND5$_{>60}$ and ND5$_{<60}$.

The patterns in the digit tests in 2008 also resemble those in 2012, with one interesting difference. The second-digit tests show that in the set of all UIKs X^2_{2BL} is significant for all candidates except Zyuganov (Table 6). In this set, $\hat{j}$ differs significantly from $\bar{j}$ for all candidates. X^2_{2BL} is significant for only one of the candidates (Zhirinovskij) in the set D5$_{>60}$, but X^2_{2BL} is not significant for only a single candidate in both ND5$_{>60}$ (Zyuganov) and ND5$_{<60}$ (Bogdanov). Again an important question is why there is such a large difference between D5$_{>60}$ and ND5$_{>60}$ in X^2_{2BL} for Medvedev and Bogdanov. In both sets Medvedev received pretty much the same range of percentages of the votes. Perhaps the difference comes down to sample size. $\hat{j}$ for Medvedev does not differ significantly between the two sets, and neither does $\hat{j}$ for Bogdanov. Perhaps the second digits in D5$_{>60}$ are not 2BL-distributed, but the sample size there—which is 24 or 35 times smaller than the sample in ND5$_{>60}$—is not big enough to trigger a significant X^2_{2BL} value. For Medvedev $\hat{j} < \bar{j}$ in D5$_{>60}$ and ND5$_{>60}$ but $\hat{j} > \bar{j}$ in ND5$_{<60}$, a pattern that echoes the pattern observed across the same sets for Putin in 2012.

The interesting variation in the last digit test results for 2008 is that the test suggests that in at least one subset of the data the last digit distribution for the United Russia candidate is not uniform. Table 7 shows that X^2_{LU*} is significant for Medvedev both in the set of all UIKs and in D5$_{>60}$.[20] X^2_{LU*} is not significant for Medvedev in ND5$_{>60}$ or ND5$_{<60}$. $X^2_{LU*} > X^2_{LU}$ in three of the sets of UIKs but not in ND5$_{<60}$. Ignoring the cases where $N_{L*} < 100$, X^2_{LU*} is significant for Zyuganov and Zhirinovskij in the set of all the UIKs and in ND5$_{>60}$ but not in D5$_{>60}$ or ND5$_{<60}$. Why the last digits of the vote counts for Zyuganov and Zhirinovskij are uniform in UIKs where the randomization tests show their votes are augmented (D5$_{>60}$) but not in the UIKs where they are not (ND5$_{>60}$) is as much

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[20]The two p-value that equal .03 (one in D5$_{>60}$ and one in ND5$_{>60}$) are not significant if FDR adjustment is applied to the set of all p-values in D5$_{>60}$, ND5$_{>60}$ and ND5$_{<60}$.

a mystery as it is when the same pattern occurs in other years. Given all the strange patterns in the tests, it hardly seems reasonable to conclude that the last digit tests show that votes for Medvedev in 2008 were manipulated while the votes for United Russia in 2011 or for Putin in 2012 were not.

*** Table 7 about here ***

The digit tests for 2004 data suggest that the distinctive features observed in 2008 are more about Medvedev (or about Putin) than about any trend over time. With regard to Putin, at least, last digit tests in 2004 resemble those in 2012: X^2_{LU*} is never significant for Putin in any of the subsets (Table 8). In the set of all UIKs X^2_{LU*} is significant for all candidates that have $N_{L*} > 100$ (including Against All), just as in 2012. In other respects the last digit tests for 2004 resemble those in 2012 only slightly. Ignoring the cases where $N_{L*} < 100$, in D5$_{>60}$ X^2_{LU*} is not significant for both of the other candidates while in 2012 X^2_{LU*} is significant for one out of three. In ND5$_{>60}$ X^2_{LU*} is not significant for two of the four other candidates (including Against All) whereas in 2012 it is significant for all four candidates. In ND5$_{<60}$ X^2_{LU*} is not significant for any of the four candidates (including Against All) while in 2012 X^2_{LU*} is significant for all but one of the four candidates. One can speak of a trend across presidential elections, perhaps, in which last digits of vote counts follow a uniform distribution with decreasing frequency as time progresses, but any such trend does not apply to the vote counts for Putin. The significant last digit test results for Medvedev are unique among the United Russia candidates.

*** Table 8 about here ***

Second-digit test statistics for 2004 somewhat resemble the results for 2012. In 2004 X^2_{2BL} is always significant in the set of all UIKs and in ND5$_{>60}$ (Table 9). In 2012 in these two sets only X^2_{2BL} for Zyuganov in the set of all UIKs is significant. In D5$_{>60}$ X^2_{2BL} is not significant for five of the seven candidates (including Against All), whereas in 2012 in this

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set X^2_{2BL} is not significant for all five candidates.[21] Despite the differences in X^2_{2BL} between D5$_{>60}$ and ND5$_{>60}$, $\hat{j}$ for each candidate does not differ significantly between the sets. Perhaps, as in 2008, the different results for X^2_{2BL} come down to the sample sizes: the second digits in D5$_{>60}$ may not be 2BL-distributed, and the sample size there is too small to detect the distribution difference. In ND5$_{<60}$ X^2_{2BL} is not significant for four out of seven candidates, which compares to X^2_{2BL} being significant in this set for all five candidates in 2012. In this set $\hat{j} > \bar{j}$ significantly for Putin and Haritonov despite the fact that X^2_{2BL} is not significant. This 2004 value of $\hat{j}$ for Putin is the same as $\hat{j}$ for Putin, Medvedev or United Russia in this same set in the other elections.

*** Table 9 about here ***

4 Discussion

Simply referring to the digit tests in the most straightforward and unconditional way, they produce results that hardly seem plausible. In light of the extensive evidence to the contrary, it is not possible to believe what the last digit tests in the set of all UIKs in each year would suggest, which is that counts of votes for Putin or for United Russia were not fraudulently manipulated while, uniquely among the set of choices affiliated with United Russia, the counts of votes for Medevev were fraudulently manipulated. The tests of second digits in the set of all UIKs would trigger a diagnosis of fraud for Putin, Medvedev and United Russia if interpreted as Pericchi and Torres (2011) would suggest. But even if we do not acknowledge the doubts about the unconditional second-digit tests discussed in Mebane (2013), in 2012 X^2_{2BL} says that the vote counts for Putin do not follow 2BL while $\hat{j}$ gives no reason to doubt that they do.

Unconditional digit tests suggest interesting conclusions about other candidates and parties, but one wonders how credible are these suggestions. Do Zyuganov and KPRF

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[21]In 2004, the p-value that equals .03 is not significant if FDR adjustment is applied to the set of all p-values in D5$_{>60}$, ND5$_{>60}$ and ND5$_{<60}$.

uniquely among the alternatives in 2008 and 2011 have unmanipulated vote counts? That is what both second-digit and last-digit tests suggest, moreover the second-digit tests suggest a similar conclusion for 2012.

The randomization tests suggest vote counts for many candidates are augmented in some UIKs relative to other "neighboring" UIKs, but introducing some conditioning in the digit tests by separating the data into sets based on the randomization test results produces a confusing picture. In every year, 2004–2011, both second-digit and last-digit tests are less likely to be triggered in the set of UIKs in which there is demonstrably more vote count augmentation (the set $D5_{>60}$) than in the set of those UIKs' "neighbors" (the set $ND5_{>60}$). It may be that the differences in test outcomes between the two sets are artifacts of their respective sample sizes: the latter sets tend to be at least ten times larger than the former sets. But the sets $D5_{>60}$ are often not all that small. The least one can say is that the results illuminate how little statistical power the chi-squared statistics have.

Regarding the last-digit tests, perhaps a prescription emerges from using them with the data from Russia. Perhaps the idea should be not to use the test to try to isolate which vote statistics are good and which are not, but instead to adopt a rule that any deviation from a uniform distribution for any candidate should discredit the entire election. Such a rule could be extended to say that fiding a significant deviation in any reasonable subset of large-enough precincts for any candidate should discredit the election, given suitable adjustments for multiple testing.

In every year except 2004, the second-digit test statistics have X^2_{2BL} significant and $\hat{j} \approx 4.35$ for United Russia in the set of UIKs ($ND5_{<60}$) that have turnout and United Russia (or Putin) vote percentage not divisible by five and vote percentages for United Russia less than 60 percent. In $ND5_{<60}$ in 2004 the value of $\hat{j}$ for Putin is similar even though X^2_{2BL} is not significant. These findings contrast with the results from UIKs in which United Russia's vote percentage is greater than 60 percent. In the sets of those UIKs, $\hat{j} \leq \bar{j}$. It is tempting to say that these values of $\hat{j}$ reflect there being smaller effects
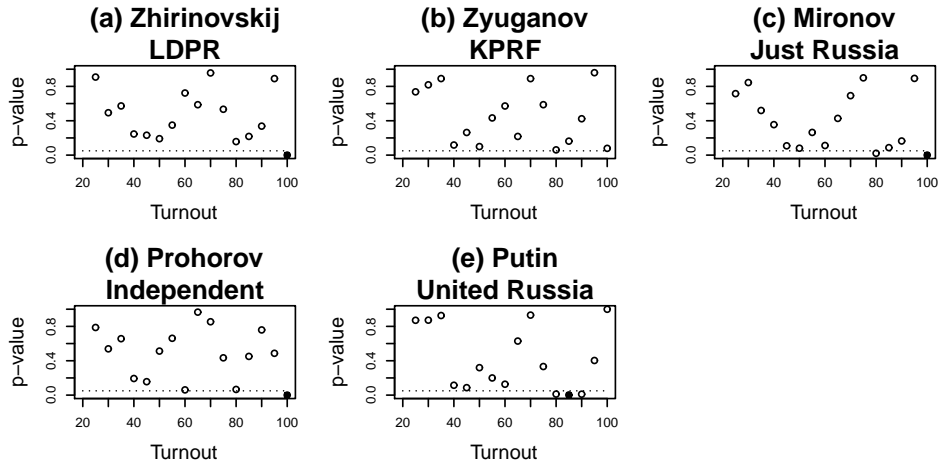
of fraud among the vote counts that give Putin or United Russia smaller shares of the vote, because having less fraud there allows evidence of votes being strategically added to Putin's votes to come through. A strong argument against this interpretation is that, because the other candidates receive significant numbers of votes, it is difficult to know what strategy the voters here might be using.

Is it plausible that votes have been concocted with an eye to passing the digit tests? It's certainly plausible that random number generators or other algorithms were used to adjust vote counts, rather than relying on the numbers individual humans spontaneously think up. If vote frauds are coordinated across all of Russia, whether through the signaling scheme described by Kalinin and Mebane (2011) or not, and the assigned or chosen targets are particular turnout or Putin/United Russia vote percentages and not particular totals for the vote counts, then the vote counts that result might be uniformly distributed even though many of them are faked. Whether such a mechanism can produce vote counts that pass the second-digit tests according to X^2_{2BL} is a question.

The results for 2004 pose the sharpest challenge to the preceding speculations. Neither Pericchi and Torres (2004) nor even the conference draft versions of Beber and Scacco (2012) existed at the time of the 2004 election. The election was in the month of May and Pericchi and Torres (2004) did not appear until October of that year. The conference-paper version of Beber and Scacco (2012) did not exist until at least two years later. It's of course wonderful to think that social science and statistical research is taken so seriously that malefactors in one of the world's superpowers actively respond to it. If that idea is to be granted, then we can perhaps cite the forerunners of Pericchi and Torres (2004) and Beber and Scacco (2012) that did exist in time to warn the alleged perpetrators. As Pericchi and Torres (2004) and Beber and Scacco (2012) respectively acknowledge, Nigrini (1996) suggested using Benford's Law to detect tax cheating and Mosimann, Wiseman and Edelman (1995) suggested checking the last digits to detect research fraud. And for the general idea to deploy Benford's Law against fraud, Varian (1972) comes even earlier. None

of the precursors had votes or specifically 2BL in mind, but there they were.

Figure 1: Turnout Divisible-by-five Tests: Russian 2012 Presidential Election

(a) Zhirinovskij LDPR

(b) Zyuganov KPRF

(c) Mironov Just Russia

(d) Prohorov Independent

(e) Putin United Russia

Note: ○, randomization test *p*-value; ●, *p*-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation.

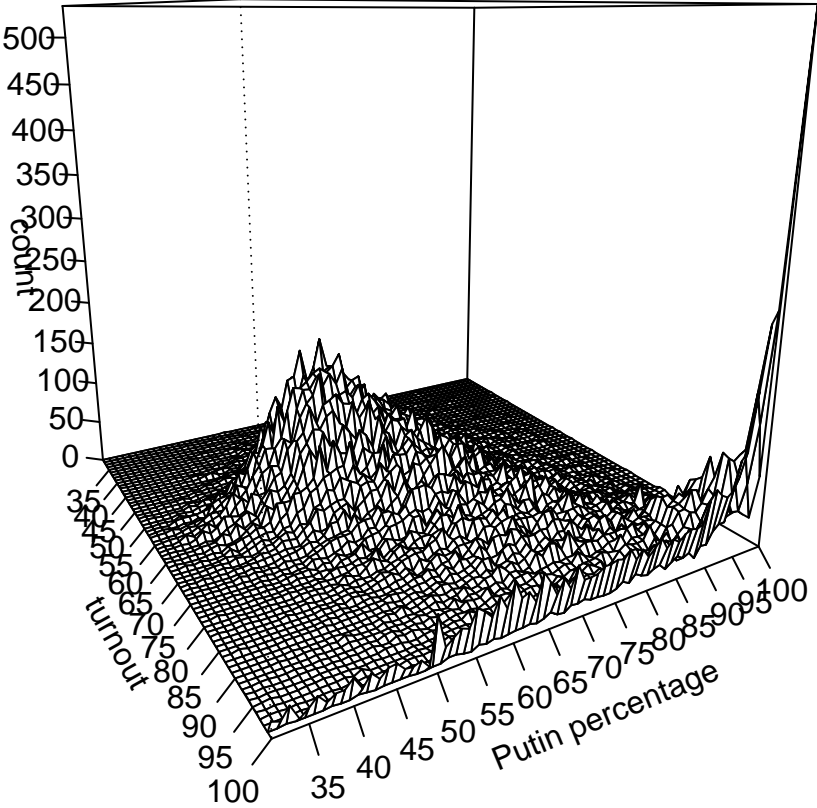Figure 2: Putin Vote Percentage Divisible-by-five Tests: Russian 2012 Presidential Election
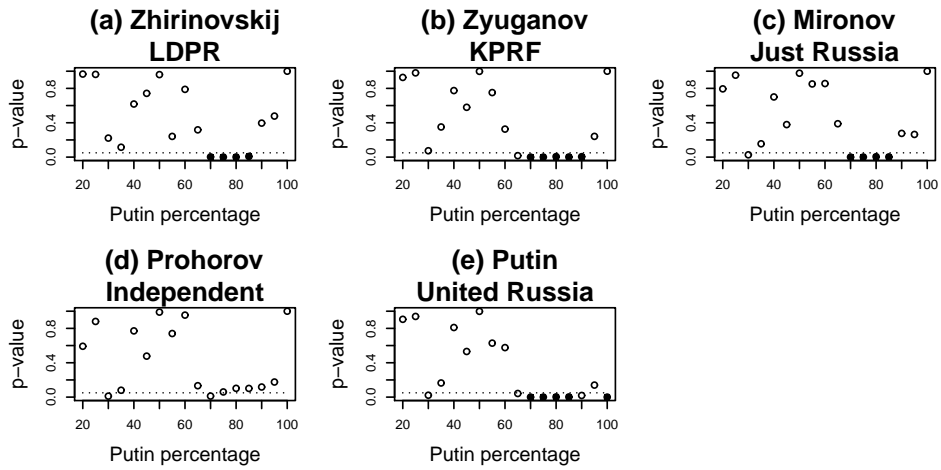
Note: ○, randomization test p-value; ●, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation.

Figure 3: Joint Distribution of Rounded Turnout and Putin Vote Percentage, 2012 Elections
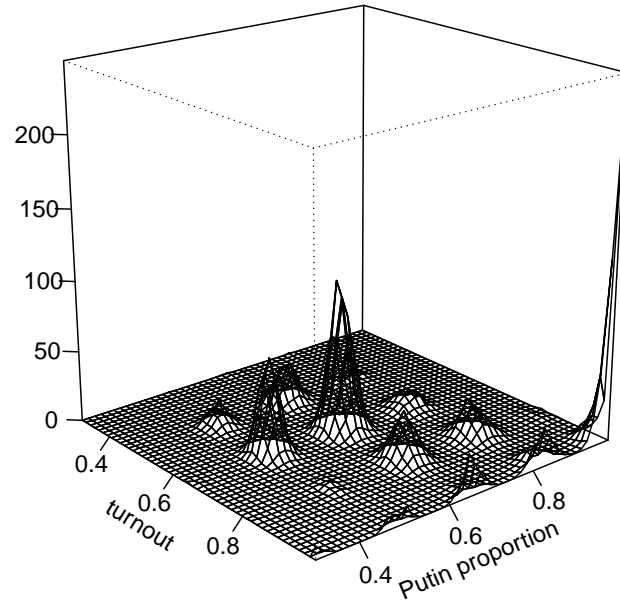

Note: joint density of UIK-level rounded turnout and rounded United Russia vote percentage.

Table 1: Divisible-by-five Local Peaks: Russian 2012 Presidential Election

turnout	Putin percentage	turnout	Putin percentage
50	55	85	95
50	70	90	85
60	80	95	85
65	65	100	25
70	75	100	40
70	80	100	45
80	80	100	50
85	80	100	55
85	85	100	60
85	90	100	100

Note: combinations of rounded turnout and Putin vote percentage values that occur more frequently than combinations of turnout and vote percentage that differ by two or less in each value; for instance, the pair $(50, 55)$ occurs more frequently than any of the pairs $\{(i, j) : i = 48, 49, 51, 52, j = 53, 54, 56, 57\}$.

Figure 4: Turnout and Putin Vote Proportion Density, 2012 Election

Note: smoothed joint density of UIK-level turnout and United Russia vote proportion.

Figure 5: Turnout and Medvedev Vote Proportion Density, 2008 Election
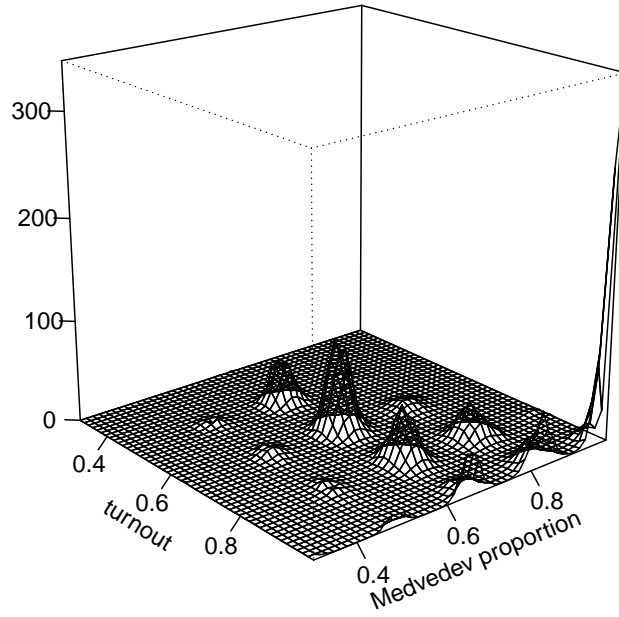
Note: smoothed joint density of UIK-level turnout and Medvedev vote proportion.

Figure 6: Turnout and Putin Vote Proportion Density, 2004 Election
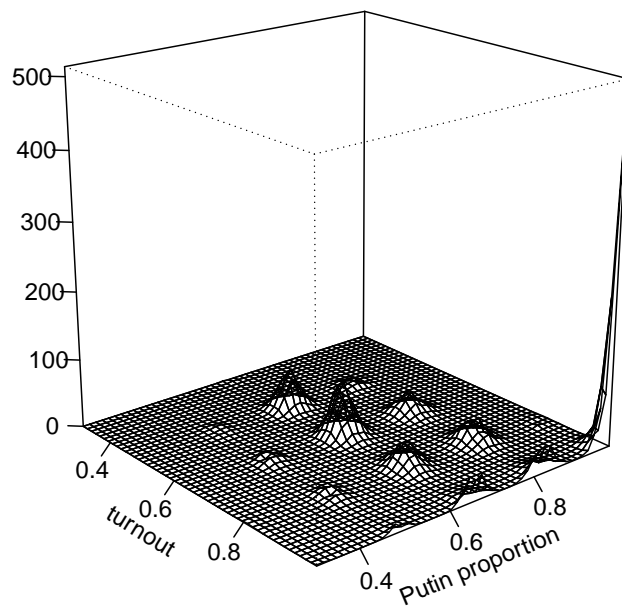
Note: smoothed joint density of UIK-level turnout and Putin vote proportion.

Figure 7: Turnout Divisible-by-five Tests: Russian 2008 Presidential Election
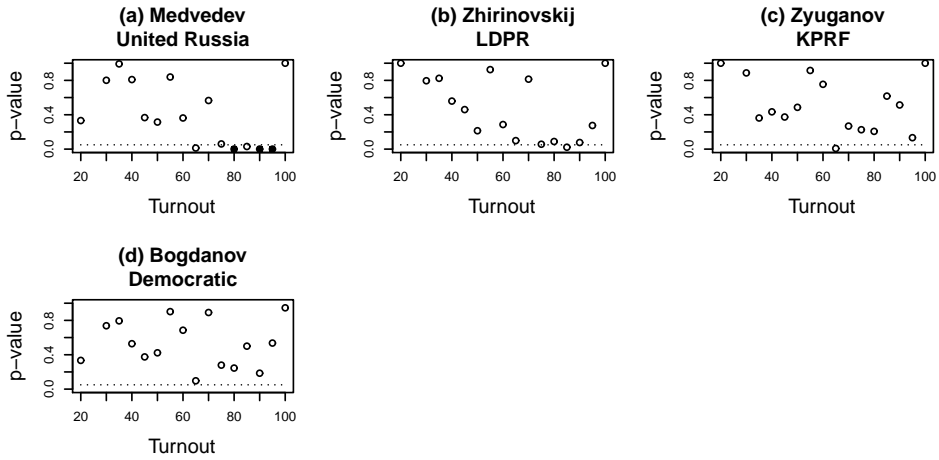
Note: ○, randomization test *p*-value; ●, *p*-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation.

Figure 8: Medvedev Vote Percentage Divisible-by-five Tests: Russian 2008 Presidential Election
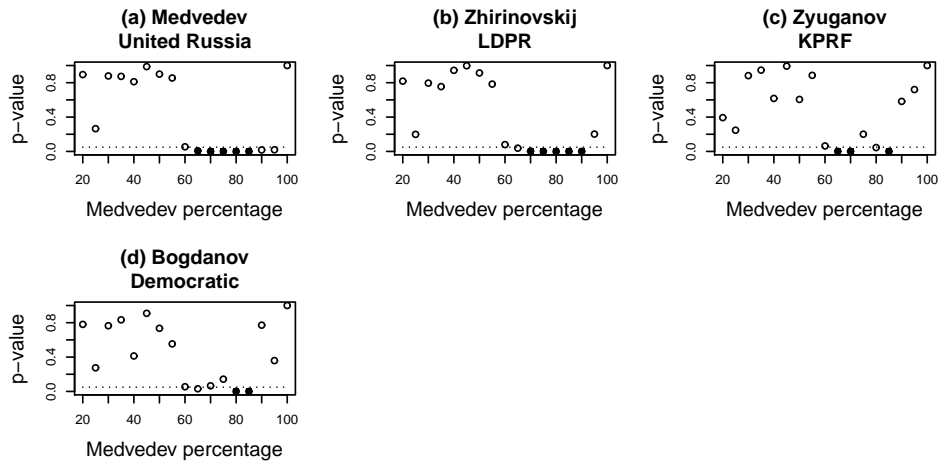
Note: ○, randomization test *p*-value; ●, *p*-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation.

Figure 9: Turnout Divisible-by-five Tests: Russian 2004 Presidential Election

Note: ○, randomization test p-value; ●, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; Life, Russian Party of Life.

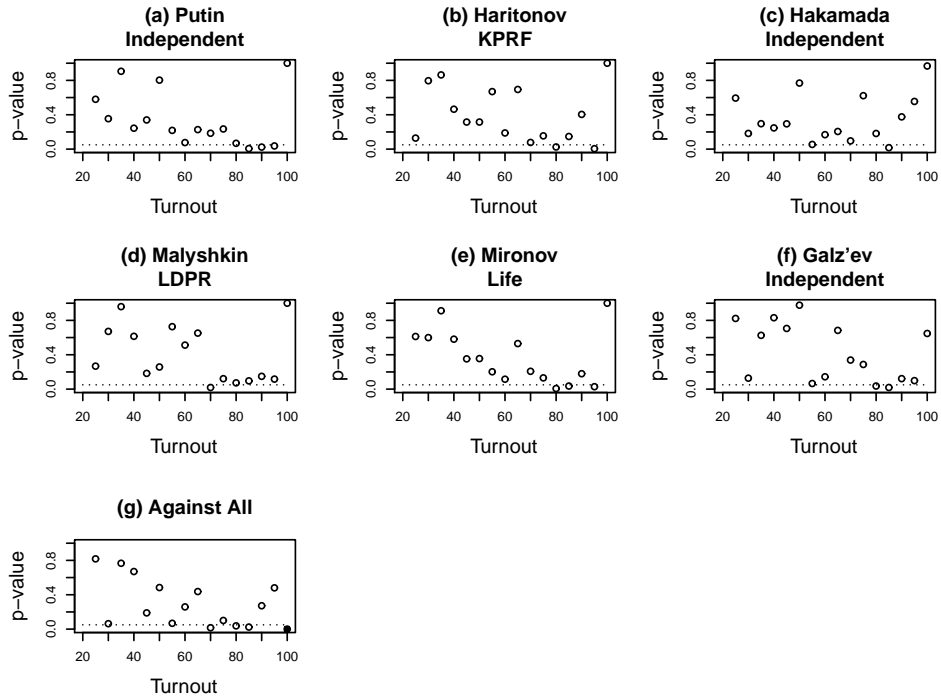Figure 10: Putin Vote Percentage Divisible-by-five Tests: Russian 2004 Presidential Election

Note: ○, randomization test p-value; ●, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; Life, Russian Party of Life.

Figure 11: Turnout and United Russia Vote Proportion Density, 2011 Election

Note: smoothed joint density of UIK-level turnout and United Russia vote proportion.

Figure 12: Turnout and United Russia Vote Proportion Density, 2007 Election

Note: smoothed joint density of UIK-level turnout and United Russia vote proportion.

Figure 13: Turnout Divisible-by-five Tests: Russian 2011 Duma Election
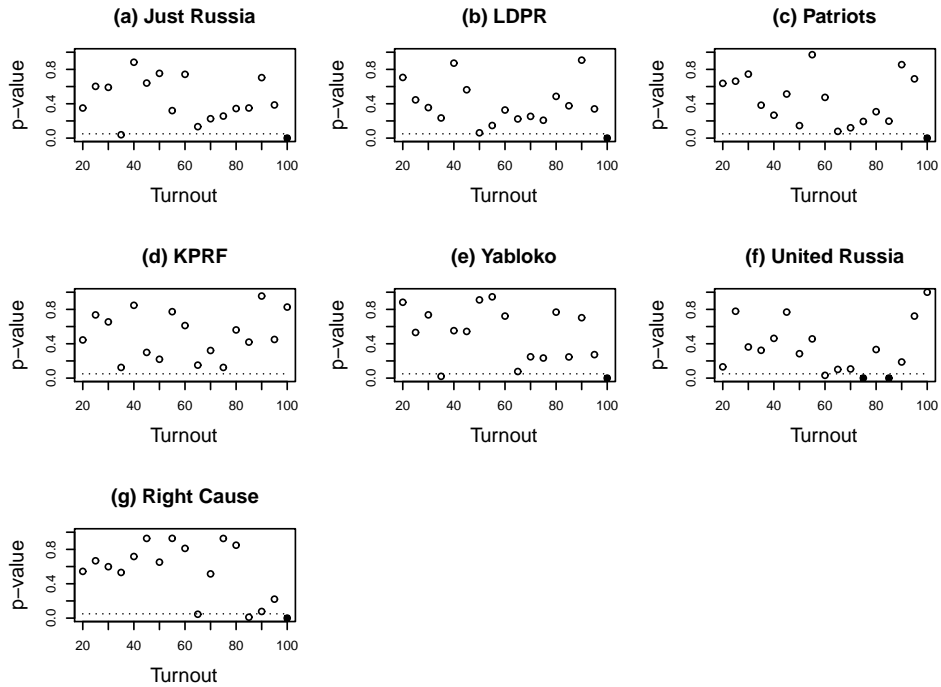
(a) Just Russia

(b) LDPR

(c) Patriots

(d) KPRF

(e) Yabloko

(f) United Russia

(g) Right Cause

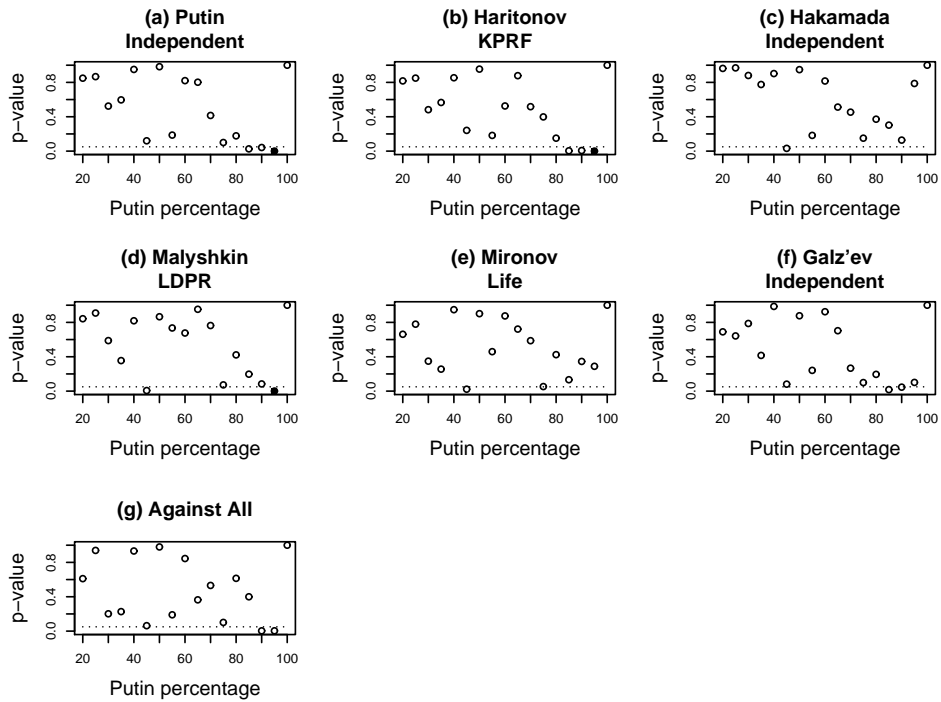Note: ∘, randomization test p-value; •, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation.

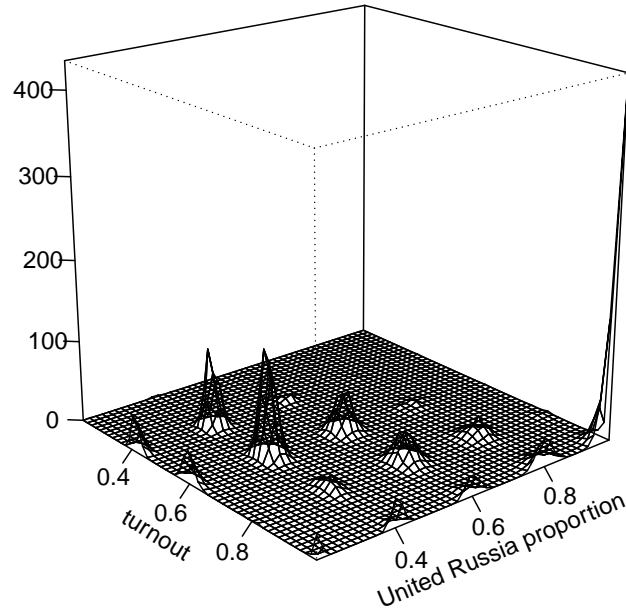Figure 14: United Russia Vote Percentage Divisible-by-five Randomization Tests: Russian 2011 Duma Election
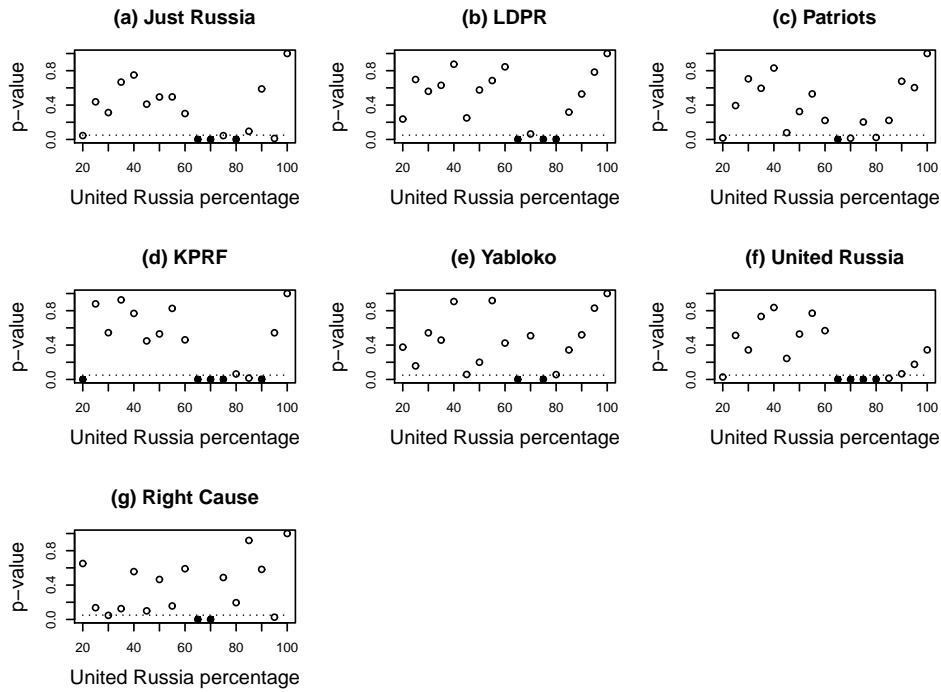
Note: ○, randomization test *p*-value; ●, *p*-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation.

Figure 15: Turnout Divisible-by-five Tests: Russian 2007 Duma Election
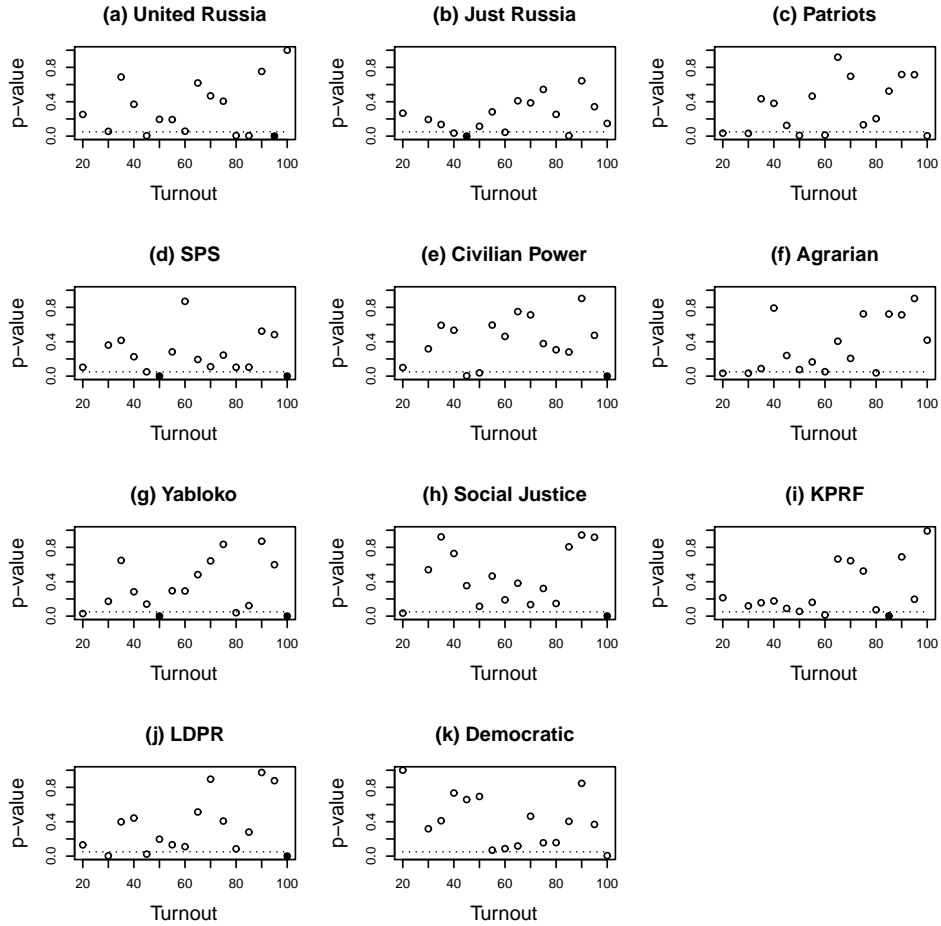
Note: ○, randomization test *p*-value; ●, *p*-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; SPS, Union of Rightist Forces.

Figure 16: United Russia Vote Percentage Divisible-by-five Randomization Tests: Russian 2007 Duma Election
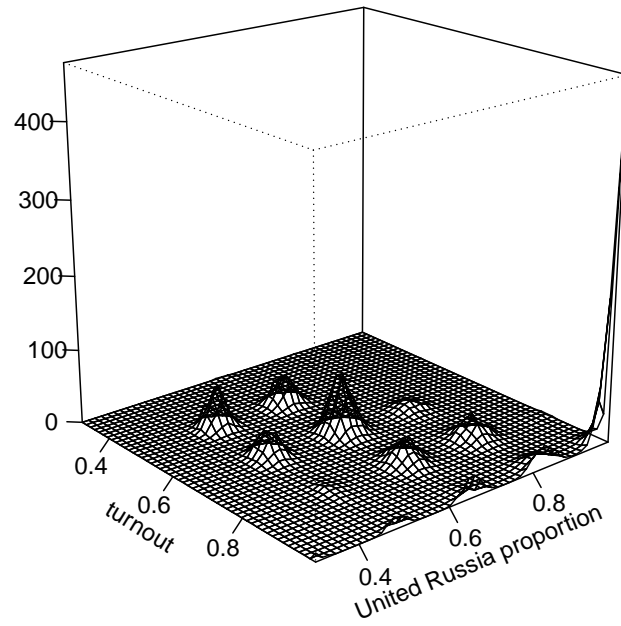

Note: ∘, randomization test p-value; •, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; SPS, Union of Rightist Forces.

Figure 17: Turnout and United Russia Vote Proportion Density, 2003 Election, PR Votes


Note: smoothed joint density of UIK-level turnout and United Russia vote proportion.

Figure 18: Turnout Divisible-by-five Tests: Russian 2003 Duma Election, Proprotional Votes

(a) Against All

(b) United Russia

(c) KPRF

(d) LDPR

(e) Rodina

(f) Yabloko

(g) SPS

(h) Agrarian

(i) Pensioners

(j) Rebirth

(k) People's

Note: ○, randomization test p-value; ●, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; SPS, Union of Rightist Forces; Rebirth, Party of Russia's Rebirth-Russian Party of Life; People's, People's Party of the Russian Federation.

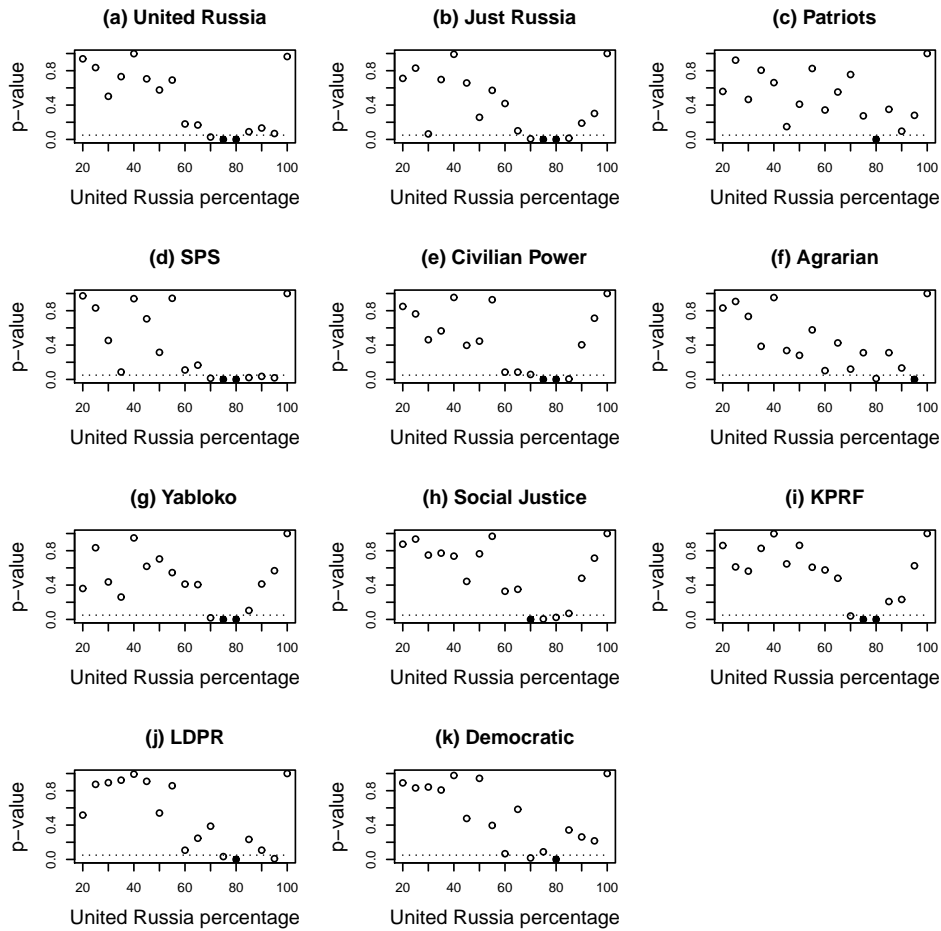Figure 19: United Russia Vote Percentage Divisible-by-five Randomization Tests: Russian 2003 Duma Election, Proprotional Votes

Note: ○, randomization test p-value; ●, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown; dotted line locates the value .05. Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; SPS, Union of Rightist Forces; Rebirth, Party of Russia's Rebirth-Russian Party of Life; People's, People's Party of the Russian Federation.

Figure 20: Turnout and United Russia Vote Proportion Density, 2003 Election, Plurality Votes
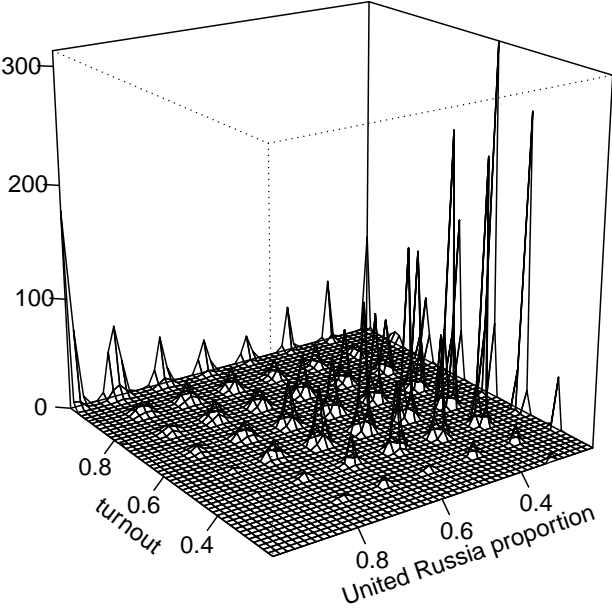
Note: smoothed joint density of UIK-level turnout and United Russia vote proportion.

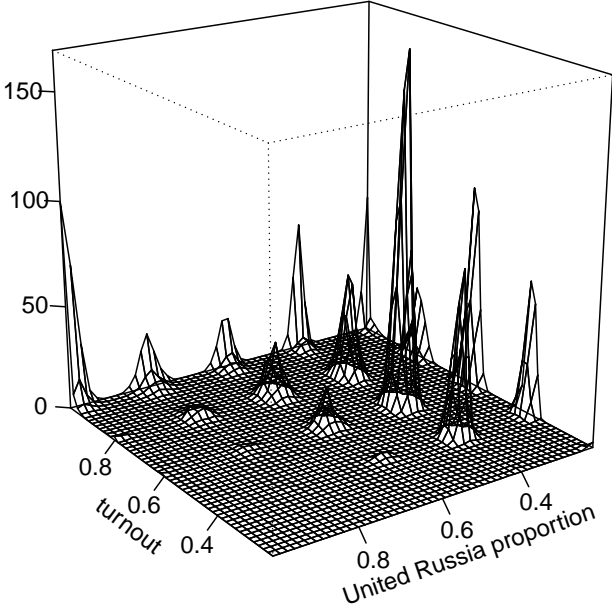Figure 21: Turnout Divisible-by-five Tests: Russian 2003 Duma Election, Plurality Votes
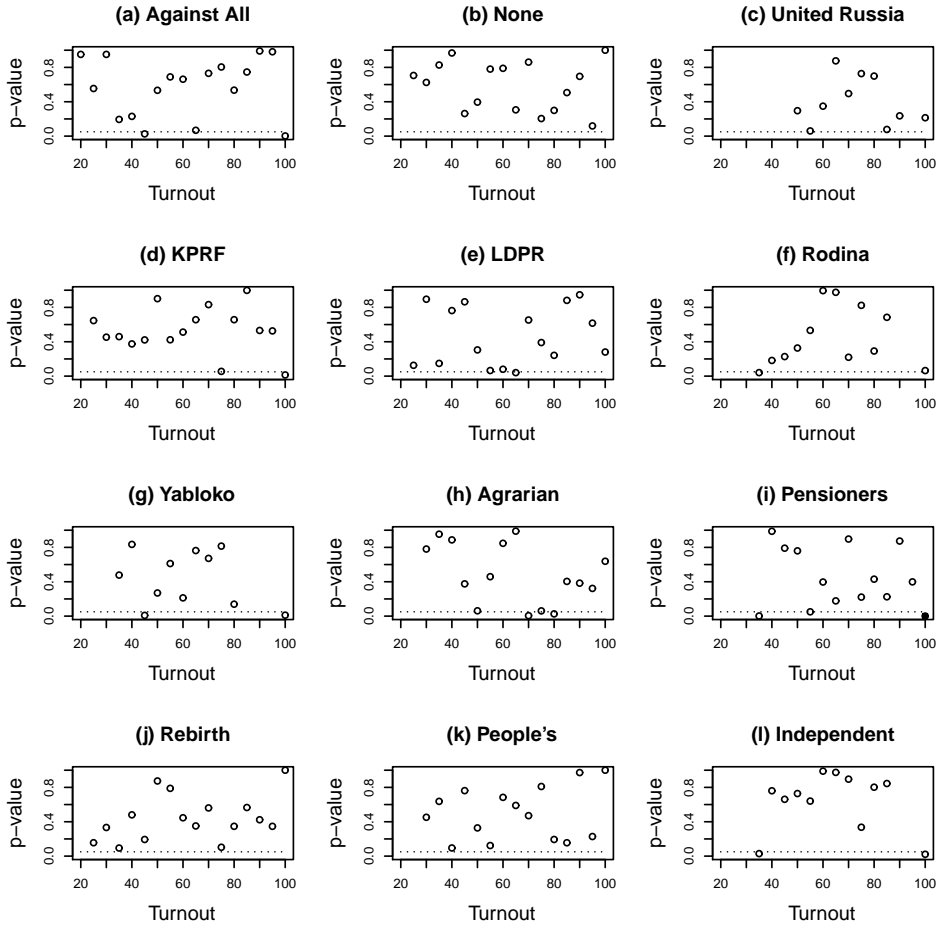
Note: ○, randomization test p-value; ●, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown plus 204 tests for other parties; dotted line locates the value .05.

Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; SPS, Union of Rightist Forces; Rebirth, Party of Russia's Rebirth-Russian Party of Life; People's, People's Party of the Russian Federation.

Figure 22: Putin Vote Percentage Divisible-by-five Tests: Russian 2003 Duma Election, Plurality Votes

(a) Against All

(b) None

(c) United Russia

(d) KPRF

(e) LDPR

(f) Rodina

(g) Yabloko

(h) Agrarian

(i) Pensioners

(j) Rebirth

(k) People's

(l) Independent
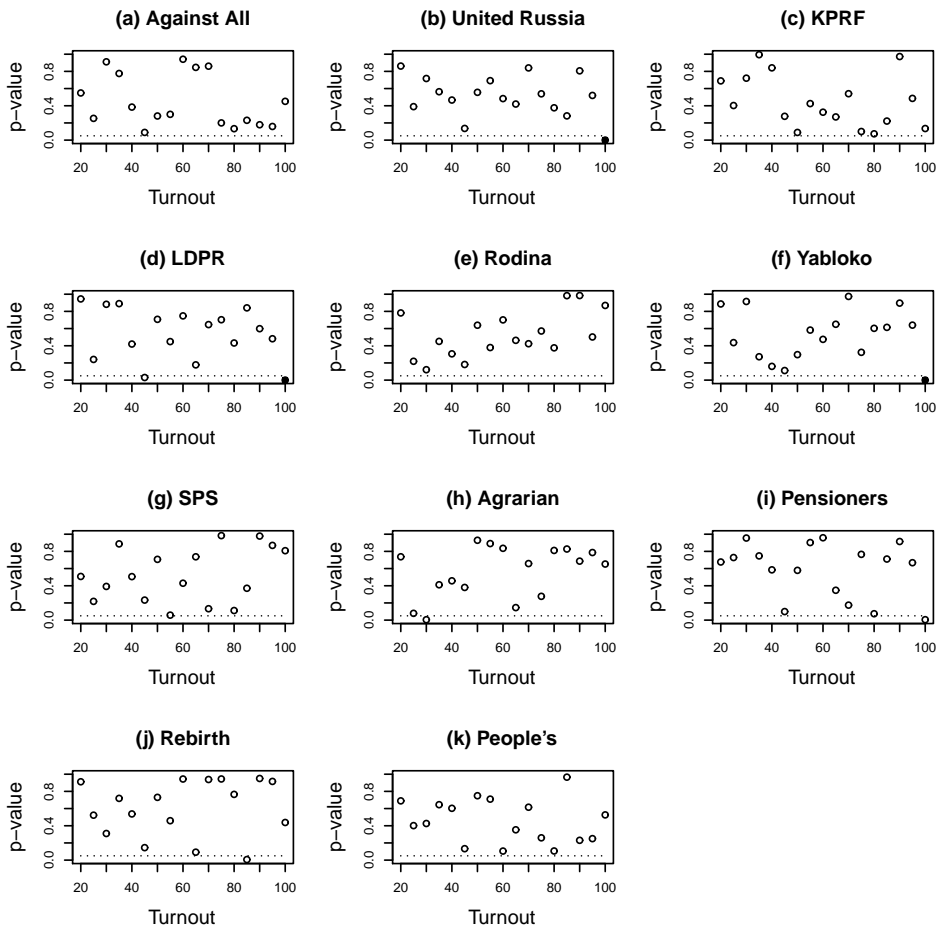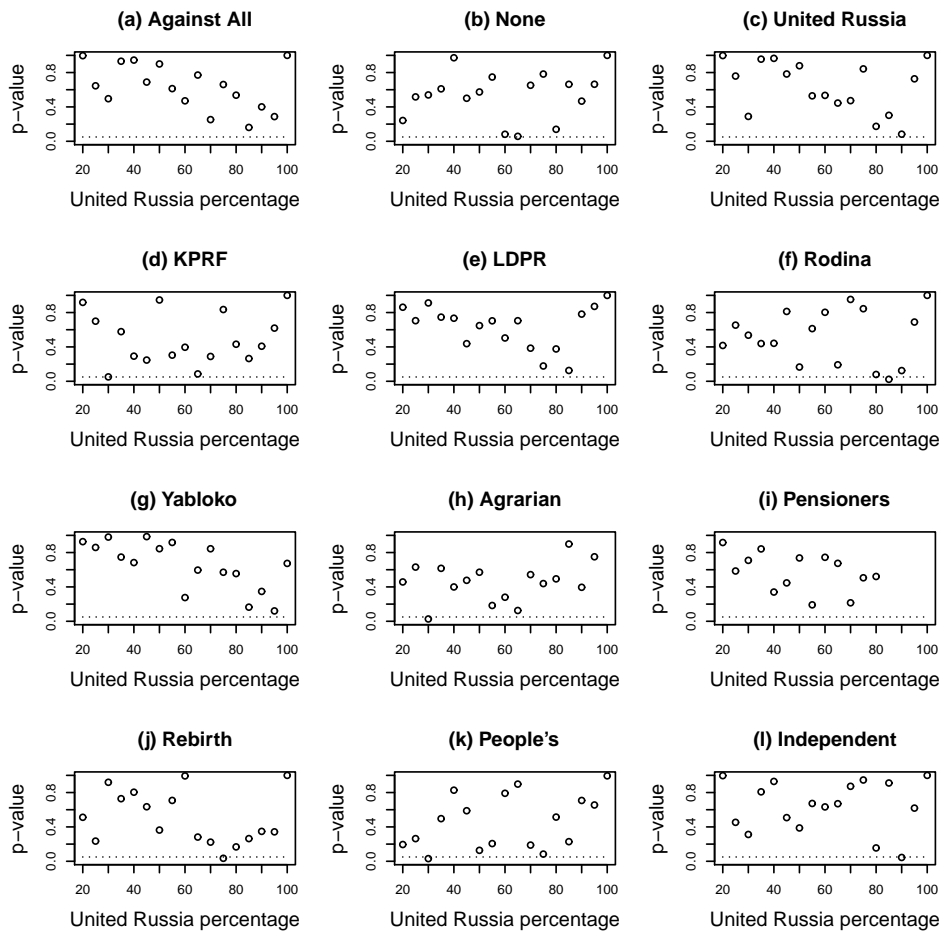
Note: ∘, randomization test p-value; •, p-value that is significant at test level $\alpha = .05$ given FDR correction across all tests for all candidates shown plus 169 tests for other parties; dotted line locates the value .05.
Party abbreviation legend: LDPR, Liberal Democratic Party of Russia; KPRF, Communist Party of the Russian Federation; SPS, Union of Rightist Forces; Rebirth, Party of Russia's Rebirth-Russian Party of Life; People's, People's Party of the Russian Federation.

Table 2: Second-digit Tests, Russian Presidential Elections, 2012

Set	Party	N	χ^2_{2BL}	$\hat{\alpha}$	$\hat{j}$	$\hat{j}_{\text{lo}}$	$\hat{j}_{\text{hi}}$
All	Zhirinovskij	74,808	608.80	.00	3.96	3.94	3.98
	Zyuganov	86,742	13.67	.42	4.19	4.17	4.21
	Mironov	60,290	52.61	.00	4.13	4.10	4.15
	Prohorov	63,895	275.53	.00	4.00	3.98	4.02
	Putin	94,937	67.47	.00	4.21	4.19	4.22
$\text{D5}_{>60}{}^{a}$	Zhirinovskij	1,724	7.35	.45	4.06	3.92	4.19
	Zyuganov	2,198	4.70	.26	4.12	3.99	4.24
	Mironov	1,178	12.63	.46	3.99	3.82	4.15
	Prohorov	1,314	17.38	.27	3.97	3.81	4.14
	Putin	3,119	9.55	.50	4.08	3.98	4.18
$\text{ND5}_{>60}{}^{b}$	Zhirinovskij	25,861	71.16	.00	4.06	4.03	4.10
	Zyuganov	32,354	24.73	.05	4.12	4.09	4.15
	Mironov	18,448	85.12	.00	4.00	3.95	4.04
	Prohorov	20,171	183.21	.00	3.92	3.88	3.96
	Putin	36,252	57.47	.00	4.11	4.08	4.14
$\text{ND5}_{<60}{}^{c}$	Zhirinovskij	20,933	481.69	.00	3.81	3.77	3.86
	Zyuganov	21,797	45.61	.00	4.31	4.27	4.35
	Mironov	19,326	25.04	.04	4.22	4.18	4.27
	Prohorov	19,743	35.52	.00	4.08	4.04	4.12
	Putin	21,917	134.61	.00	4.38	4.34	4.42

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage ≤ 60. Statistics for UIK vote counts. N is the number of UIKs with a vote count > 9. $\hat{\alpha} = (1 + [-ep \log(p)]^{-1})^{-1}$ where p is the p-value of X^2_{2BL}. $\hat{j}_{\text{lo}}$ and $\hat{j}_{\text{hi}}$ are the lower and upper bounds of the 95% confidence interval for $\hat{j}$.
Data source: .

Table 3: Last-digit Tests, Russian Presidential Elections, 2012

Set	Party	χ^2_{LU}	$\hat{\alpha}_{LU}$	χ^2_{LU*}	$\hat{\alpha}_{LU*}$	N_{L*}
All	Zhirinovskij	1007.16	.00	32.47	.00	11,507
	Zyuganov	156.70	.00	36.68	.00	47,090
	Mironov	2979.84	.00	61.16	.00	1,899
	Prohorov	2607.31	.00	43.23	.00	21,097
	Putin	13.48	.43	23.75	.06	83,047
D5$_{>60}$[a]	Zhirinovskij	1010.64	.00	14.89	.38	122
	Zyuganov	658.90	.00	27.94	.02	700
	Mironov	1087.72	.00	20.82	.14	11
	Prohorov	1250.80	.00	11.32	.49	118
	Putin	5.79	.36	6.58	.42	2,597
ND5$_{>60}$[b]	Zhirinovskij	452.23	.00	37.05	.00	2,074
	Zyuganov	49.10	.00	49.91	.00	11,752
	Mironov	1790.97	.00	24.56	.05	225
	Prohorov	1444.44	.00	24.35	.05	2,655
	Putin	4.90	.28	12.34	.46	31,018
ND5$_{<60}$[c]	Zhirinovskij	11.48	.48	79.19	.00	5,249
	Zyuganov	4.02	.19	5.00	.29	17,939
	Mironov	32.72	.00	40.53	.00	984
	Prohorov	32.30	.00	26.09	.03	10,840
	Putin	8.52	.49	8.71	.49	20,141

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage $\leq$ 60. Statistics for UIK vote counts. $\hat{\alpha}_k = (1 + [-ep_k \log(p_k)]^{-1})^{-1}$ where p_k is the p-value of X^2_k, $k \in \{LU, LU*\}$. N_L: All, 95,414; D5$_{>60}$, 3,138; ND5$_{>60}$, 36,257; ND5$_{<60}$, 21,960.
Data source: .

Table 4: Second-digit Digit Tests, Russian Duma Elections, 2011

Set	Party	N	χ^2_{2BL}	$\hat{\alpha}$	$\hat{j}$	$\hat{j}_{\text{lo}}$	$\hat{j}_{\text{hi}}$
All	Just Russia	79,165	65.54	.00	4.11	4.09	4.13
	LDPR	79,539	181.59	.00	4.06	4.04	4.08
	Patriots	25,943	1260.31	.00	3.56	3.52	3.59
	KPRF	86,282	20.92	.13	4.19	4.17	4.21
	Yabloko	41,499	108.16	.00	4.05	4.02	4.08
	United Russia	94,575	35.02	.00	4.24	4.22	4.26
	Right Cause	12,982	2225.36	.00	3.02	2.97	3.06
D5$_{>60}$[a]	Just Russia	793	19.58	.18	3.90	3.69	4.11
	LDPR	782	15.93	.33	3.80	3.60	4.01
	Patriots	99	20.46	.15	3.38	2.82	3.94
	KPRF	1,105	15.69	.34	4.01	3.83	4.19
	Yabloko	205	22.01	.10	3.43	3.04	3.83
	United Russia	1,826	21.70	.11	4.07	3.94	4.20
	Right Cause	64	32.44	.00	2.39	1.76	3.02
ND5$_{>60}$[b]	Just Russia	9,449	104.50	.00	3.90	3.84	3.96
	LDPR	9,374	88.73	.00	3.95	3.89	4.01
	Patriots	1,205	127.41	.00	3.26	3.11	3.42
	KPRF	12,755	47.88	.00	4.07	4.02	4.12
	Yabloko	2,216	76.86	.00	3.71	3.59	3.83
	United Russia	15,964	26.02	.03	4.10	4.05	4.14
	Right Cause	580	97.14	.00	3.09	2.87	3.31
ND5$_{<60}$[c]	Just Russia	39,817	13.69	.42	4.16	4.13	4.19
	LDPR	40,132	77.45	.00	4.09	4.07	4.12
	Patriots	15,143	713.68	.00	3.57	3.52	3.61
	KPRF	40,785	23.72	.06	4.25	4.22	4.28
	Yabloko	23,813	54.68	.00	4.06	4.03	4.10
	United Russia	41,624	80.94	.00	4.30	4.27	4.33
	Right Cause	7,542	1240.75	.00	3.03	2.97	3.09

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage ≤ 60. Statistics for UIK vote counts. N is the number of UIKs with a vote count > 9. $\hat{\alpha} = (1 + [-ep \log(p)]^{-1})^{-1}$ where p is the p-value of X^2_{2BL}. $\hat{j}_{\text{lo}}$ and $\hat{j}_{\text{hi}}$ are the lower and upper bounds of the 95% confidence interval for $\hat{j}$.

Data source: .

Table 5: Last-digit Tests, Russian Duma Elections, 2011

Set	Party	χ^2_{LU}	$\hat{\alpha}_{LU}$	χ^2_{LU*}	$\hat{\alpha}_{LU*}$	N_{L*}
All	Just Russia	1140.98	.00	43.13	.00	35,730
	LDPR	1574.35	.00	53.64	.00	33,714
	Patriots	30168.79	.00	6.70	.42	27
	KPRF	202.03	.00	59.02	.00	46,404
	Yabloko	27166.87	.00	28.24	.02	5,132
	United Russia	14.57	.39	23.34	.07	76,835
	Right Cause	67106.01	.00	4.33	.22	15
$D5_{>60}{}^a$	Just Russia	1097.72	.00	16.92	.29	123
	LDPR	1410.80	.00	10.27	.50	132
	Patriots	4361.95	.00	9.00	.50	1
	KPRF	564.67	.00	29.96	.01	394
	Yabloko	3945.29	.00	8.71	.49	7
	United Russia	5.53	.34	12.22	.47	1,516
	Right Cause	5562.26	.00	—	—	0
$ND5_{>60}{}^b$	Just Russia	1348.39	.00	6.55	.41	1,432
	LDPR	2014.72	.00	18.45	.22	1,311
	Patriots	21248.80	.00	9.00	.50	1
	KPRF	95.08	.00	53.70	.00	3,625
	Yabloko	21146.29	.00	16.71	.30	62
	United Russia	10.78	.49	18.33	.23	13,545
	Right Cause	35240.75	.00	8.00	.48	2
$ND5_{<60}{}^c$	Just Russia	9.15	.50	23.67	.07	21,154
	LDPR	17.23	.28	38.24	.00	19,976
	Patriots	4023.05	.00	11.00	.49	15
	KPRF	7.92	.47	21.23	.12	25,480
	Yabloko	2983.74	.00	17.81	.25	3,132
	United Russia	8.50	.49	14.96	.37	33,529
	Right Cause	12925.08	.00	5.00	.29	5

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage ≤ 60. Statistics for UIK vote counts. $\hat{\alpha}_k = (1 + [-ep_k \log(p_k)]^{-1})^{-1}$ where p_k is the p-value of X^2_k, $k \in \{LU, LU*\}$. N_L: All, 95,168; $D5_{>60}$, 1,839; $ND5_{>60}$, 15,966; $ND5_{<60}$, 41,716.
Data source: .

Table 6: Second-digit Tests, Russian Presidential Elections, 2008

Set	Party	N	χ^2_{2BL}	$\hat{\alpha}$	$\hat{j}$	$\hat{j}_{\text{lo}}$	$\hat{j}_{\text{hi}}$
All	Medvedev	95,883	124.22	.00	4.11	4.09	4.13
	Zyuganov	87,298	18.48	.22	4.22	4.20	4.24
	Bogdanov	35,308	212.55	.00	3.97	3.94	4.00
	Zhirinovskij	79,650	434.69	.00	3.98	3.96	4.00
$\text{D5}_{>60}{}^{a}$	Medvedev	3,864	18.95	.20	4.04	3.95	4.14
	Zyuganov	3,025	13.20	.44	4.09	3.98	4.19
	Bogdanov	1,020	20.20	.16	3.81	3.63	3.98
	Zhirinovskij	2,638	45.48	.00	3.82	3.71	3.93
$\text{ND5}_{>60}{}^{b}$	Medvedev	92,019	115.61	.00	4.11	4.09	4.13
	Zyuganov	84,273	23.02	.08	4.23	4.21	4.25
	Bogdanov	34,288	198.53	.00	3.97	3.94	4.00
	Zhirinovskij	77,012	403.19	.00	3.99	3.96	4.01
$\text{ND5}_{<60}{}^{c}$	Medvedev	10,448	38.36	.00	4.27	4.21	4.33
	Zyuganov	10,372	27.65	.02	4.33	4.27	4.38
	Bogdanov	5,596	16.19	.32	4.05	3.98	4.13
	Zhirinovskij	10,003	58.44	.00	4.00	3.94	4.06

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage ≤ 60. Statistics for UIK vote counts. N is the number of UIKs with a vote count > 9. $\hat{\alpha} = (1 + [-ep\log(p)]^{-1})^{-1}$ where p is the p-value of X^2_{2BL}. $\hat{j}_{\text{lo}}$ and $\hat{j}_{\text{hi}}$ are the lower and upper bounds of the 95% confidence interval for $\hat{j}$. Number of UIKs used for χ^2_{LU}: All, 95,414; D5$_{>60}$, 3,138; ND5$_{>60}$, 36,257; ND5$_{<60}$, 21,960.
Data source: .

Table 7: Last-digit Tests, Russian Presidential Elections, 2008

Set	Party	χ^2_{LU}	$\hat{\alpha}_{LU}$	χ^2_{LU*}	$\hat{\alpha}_{LU*}$	N_{L*}
All	Medvedev	27.27	.02	37.46	.00	85,259
	Zyuganov	120.58	.00	29.57	.01	48,095
	Bogdanov	24268.18	.00	9.33	.50	84
	Zhirinovskij	727.71	.00	69.70	.00	29,479
$D5_{>60}{}^a$	Medvedev	25.40	.04	26.44	.03	3,289
	Zyuganov	371.80	.00	11.39	.49	1,422
	Bogdanov	2685.60	.00	8.00	.48	2
	Zhirinovskij	605.96	.00	9.82	.50	808
$ND5_{>60}{}^b$	Medvedev	20.69	.14	21.03	.13	42,926
	Zyuganov	83.07	.00	25.89	.03	21,874
	Bogdanov	22134.04	.00	7.71	.47	28
	Zhirinovskij	542.12	.00	45.96	.00	13,260
$ND5_{<60}{}^c$	Medvedev	10.35	.50	10.01	.50	8,867
	Zyuganov	13.64	.42	14.37	.40	7,731
	Bogdanov	787.69	.00	12.81	.45	21
	Zhirinovskij	10.07	.50	16.49	.31	5,144

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage ≤ 60. Statistics for UIK vote counts. $\hat{\alpha}_k = (1 + [-ep_k \log(p_k)]^{-1})^{-1}$ where p_k is the p-value of X^2_k, $k \in \{LU, LU*\}$. N_L: All, 95,414; $D5_{>60}$, 3,138; $ND5_{>60}$, 36,257; $ND5_{<60}$, 21,960.
Data source: .

Table 8: Last-digit Tests, Russian Presidential Elections, 2004

Set	Party	χ^2_{LU}	$\hat{\alpha}_{LU}$	χ^2_{LU*}	$\hat{\alpha}_{LU*}$	N_{L*}
All	Putin	14.99	.37	21.67	.11	83,603
	Haritonov	394.37	.00	29.63	.01	39,011
	Hakamada	8108.85	.00	46.51	.00	5,508
	Malyshkin	6063.29	.00	13.75	.42	67
	Mironov	27563.77	.00	15.46	.35	13
	Glaz'ev	6935.32	.00	53.25	.00	4,346
	Against All	9663.77	.00	46.61	.00	2,565
D5$_{>60}$[a]	Putin	13.47	.43	18.64	.22	3,547
	Haritonov	1235.88	.00	16.48	.31	1,101
	Hakamada	2671.71	.00	14.14	.41	169
	Malyshkin	2925.17	.00	—	—	0
	Mironov	4632.69	.00	9.00	.50	1
	Glaz'ev	2692.00	.00	12.06	.47	85
	Against All	2680.84	.00	13.12	.44	82
ND5$_{>60}$[b]	Putin	12.47	.46	18.86	.21	44,380
	Haritonov	27.43	.02	19.69	.18	18,898
	Hakamada	3491.45	.00	38.44	.00	2,883
	Malyshkin	2185.61	.00	14.18	.40	17
	Mironov	11832.53	.00	20.14	.16	7
	Glaz'ev	2580.90	.00	42.59	.00	1,695
	Against All	4101.41	.00	15.28	.36	1,141
ND5$_{<60}$[c]	Putin	7.01	.44	8.62	.49	7,116
	Haritonov	2.55	.05	9.91	.50	5,908
	Hakamada	264.01	.00	2.72	.06	633
	Malyshkin	78.94	.00	8.67	.49	18
	Mironov	1659.16	.00	8.00	.48	2
	Glaz'ev	173.18	.00	8.36	.49	1,109
	Against All	408.32	.00	15.43	.35	460

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage ≤ 60. Statistics for UIK vote counts. $\hat{\alpha}_k = (1 + [-ep_k \log(p_k)]^{-1})^{-1}$ where p_k is the p-value of X^2_k, $k \in \{LU, LU*\}$. N_L: All, 95,426; D5$_{>60}$, 4,311; ND5$_{>60}$, 91,115; ND5$_{<60}$, 8,922. Data source: .

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Table 9: Second-digit Tests, Russian Presidential Elections, 2004

Set	Party	N	χ^2_{2BL}	$\hat{\alpha}$	$\hat{j}$	$\hat{j}_{\text{lo}}$	$\hat{j}_{\text{hi}}$
All	Putin	95,128	292.03	0	4.10	4.08	4.12
	Haritonov	84,626	139.59	0	4.08	4.06	4.09
	Hakamada	50,087	243.15	0	4.01	3.99	4.04
	Malyshkin	52,131	162.21	0	4.04	4.01	4.06
	Mironov	18,963	2854.57	0	3.08	3.05	3.12
	Glaz'ev	54,879	154.59	0	4.07	4.04	4.09
	Against All	50,627	109.93	0	4.11	4.08	4.13
D5$_{>60}$[a]	Putin	4,267	16.52	.31	4.06	3.97	4.15
	Haritonov	2,914	16.80	.29	4.06	3.95	4.16
	Hakamada	1,682	26.10	.03	3.97	3.83	4.11
	Malyshkin	1,682	18.18	.23	3.96	3.82	4.10
	Mironov	623	105.24	.00	3.09	2.88	3.30
	Glaz'ev	1,801	11.64	.48	4.12	3.98	4.26
	Against All	1,696	9.42	.50	4.09	3.95	4.23
ND5$_{>60}$[b]	Putin	48,824	205.21	0	4.09	4.06	4.12
	Haritonov	44,188	120.88	0	4.04	4.01	4.07
	Hakamada	26,541	160.32	0	3.99	3.95	4.02
	Malyshkin	27,069	90.73	0	4.03	4.00	4.07
	Mironov	10,045	1483.83	0	3.10	3.05	3.15
	Glaz'ev	28,748	50.66	0	4.10	4.06	4.13
	Against All	26,594	40.44	0	4.13	4.10	4.17
ND5$_{<60}$[c]	Putin	8,909	14.00	.41	4.28	4.22	4.34
	Haritonov	8,853	20.94	.13	4.29	4.23	4.36
	Hakamada	4,998	20.58	.14	4.09	4.01	4.17
	Malyshkin	5,801	20.34	.15	4.10	4.02	4.17
	Mironov	1,982	404.65	.00	2.93	2.81	3.04
	Glaz'ev	5,857	39.94	.00	4.01	3.94	4.09
	Against All	5,243	47.60	.00	4.00	3.92	4.08

Notes: [a] rounded turnout and rounded Putin vote percentage divisible by 5 and Putin vote percentage > 60. [b] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage > 60. [c] neither rounded turnout nor rounded Putin vote percentage divisible by 5 and rounded Putin vote percentage ≤ 60. Statistics for UIK vote counts. N is the number of UIKs with a vote count > 9. $\hat{\alpha} = (1 + [-ep \log(p)]^{-1})^{-1}$ where p is the p-value of X^2_{2BL}. $\hat{j}_{\text{lo}}$ and $\hat{j}_{\text{hi}}$ are the lower and upper bounds of the 95% confidence interval for $\hat{j}$.

Data source: .