

# A multi-frequency eddy current inversion method for characterizing conductivity gradients on water jet peened components

Veeraraghavan Sundararaghavan<sup>a</sup>, Krishnan Balasubramaniam<sup>a,\*</sup>,  
Nimmagadda Ramesh Babu<sup>b</sup>, Nataraja Rajesh<sup>b</sup>

<sup>a</sup>Department of Mechanical Engineering, Center for NDE, Indian Institute of Technology—Madras, 301, Machine Design Section, Chennai 600036, India

<sup>b</sup>Manufacturing Engineering Section, Department of Mechanical Engineering, Indian Institute of Technology—Madras, Chennai 600036, India

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## Abstract

This paper describes a multi-frequency eddy current inversion procedure for characterizing specimens that are water jet peened. Multi-frequency inductance data was obtained by using well-characterized eddy current probes. The inversion uses a multi-layer axisymmetric finite element model as the forward model and the conductivity of each layer is found through interpolation of the inductance–conductivity data generated by the forward model. Skin depth approximation was used to isolate the integral effects of the conductivity variation on the inductance signal. Inverted conductivity profiles of the water jet peened specimens was found to resemble the predicted profiles. Information regarding the shape of residual stress gradients and relative intensities of peening were inferred from the conductivity profiles.

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**Keywords:** Multi-frequency eddy current testing; Inverse model; Water jet peening; Conductivity gradient measurements

## 1. Introduction

A method for assessing material conductivity involves measurement of the impedance of coils, driven by a constant amplitude alternating current, above a conductive metallic slab with a plane surface. The impedance can either be calibrated or matched to a particular conductivity value using theoretical models [1]. In such a configuration, the primary flux that penetrates the depth of the material produces induced currents (also known as eddy currents) on the sample surface. These eddy current produce secondary fields that interact with the primary field set up by the impressed current in the coil causing a measurable change in the impedance of the coil that can be calibrated for a constant material conductivity. However, when the material is subjected to thermal processing, peening or solidification processes, the conductivity continuously changes as a function of depth. Such variations occur due to factors

like change in chemical composition and the stress state of the material crystalline structure. The detection of spatial variations in the structure of the material is possible through the measurement of electrical conductivity profiles.

This paper presents an approach through which the conductivity gradients resulting from water jet peening of components can be non-destructively detected using multi-frequency eddy current impedance measurements. Peening increases the wear and fatigue resistance of samples and increases the fatigue life of components. The testing and evaluation of residual stress levels and gradients in peened components would help in assuring the quality of the process. Water jet peening causes compression and dislocation of grains in the material, which in turn effects a change in the near surface conductivity. Information regarding the shape of residual stress gradients and relative intensities of peening can then be inferred from these conductivity profiles.

Methods reported in literature for inverting measured inductance signals for conductivity profiles involve iterative adjustment of relevant parameters in a forward model until the measured signal value is reached [2,3]. Recently, feature based and neural network based methods for conductivity inversion have been reported [4,5]. But all these methods

\* Corresponding author. Tel.: +91 44 2257 8588; fax: +91 44 2257 0545.

E-mail address: [balas@iitm.ac.in](mailto:balas@iitm.ac.in) (K. Balasubramaniam).

incorporate the concepts of constructing a comprehensive look-up table and performing iterative error minimization at some stage of the inversion scheme. A skin depth based inversion approach using a finite element method based forward model formulation described in this paper eliminates the above problems and is capable of quickly estimating the conductivity profiles from multi-frequency eddy current inductance data measured over water jet peened samples.

## 2. Water jet peening

Fatigue cracks typically initiate from the surface because the stresses due to operating conditions are often maximum at the surface. One of the most popular methods to prevent crack initiation is to induce compressive residual stresses at the surface by means of peening. Water jet peening is a new cold working technique through which favorable near surface residual stresses are imparted in a metallic component by directing a high velocity water jet on the metal surface. These residual stresses are known to be axisymmetric, and a typical stress profile is shown in Fig. 1. The residual stresses occurring due to peening have been well studied and documented [6,7]. A result from finite element simulation of stationary impingement of a water jet on a thick body indicates that localized plastic yielding initiates below the surface, causing maximum compressive stresses in the sub-surface region [6]. Increasing the water jet pressure increases the magnitude of compressive residual stress at the surface. A mode change is noticed at a higher depth wherein the stresses in the bulk of the material go from compressive to tensile mode. A method to assess the changes occurring in the material due to peening is through the measurement of its electrical conductivity profiles. Eddy current sensor inductance can, in principle, be used to calibrate bulk stresses as demonstrated by Ricken et al. [8]. However, inversion of multi-frequency inductance data needs to be performed for the measurement of conductivity variations over depth, which results from water jet peening operation.

## 3. Eddy current forward model

In order to study the effect of a given conductivity depth profile on the eddy current inductance signal taken on a multi-layered material, a finite element forward model

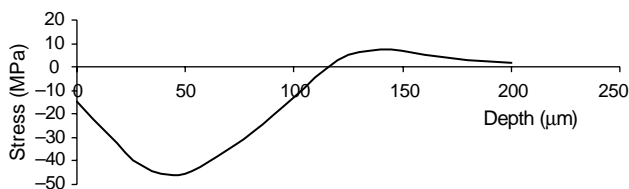


Fig. 1. Typical variation of residual stresses with depth in peened samples.

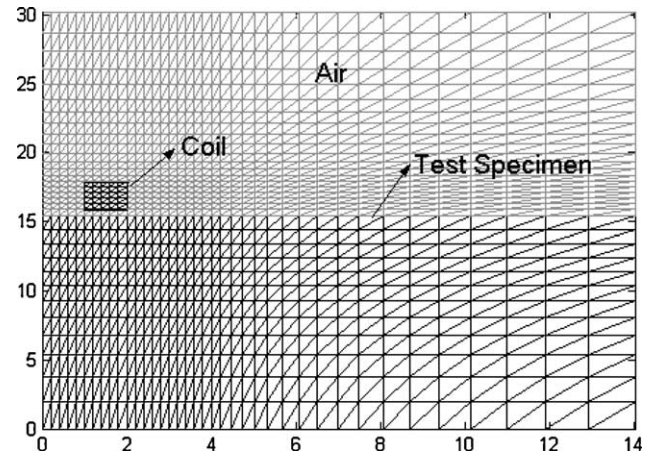


Fig. 2. Axisymmetric mesh used for finite element model.

based on Palanisamy [9] was developed. During the inversion process, the forward model calculates the effect of a change in conductivity of a particular layer of a multi-layered material on the measured impedance of the coil at every frequency. The model is briefly described in this section.

In the case of axisymmetric geometries, the eddy current governing equation in cylindrical coordinates ( $r, z$ ) is given as

$$\frac{1}{\mu} \left( \frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{A}{r^2} \right) = -J_s + \sigma \frac{\partial A}{\partial t} \quad (1)$$

where  $A$  is the magnetic vector potential (Wb/m),  $J_s$  is the source current density ( $A/m^2$ ),  $\mu$  and  $\sigma$  are the permeability (Wb/A m) and conductivity (S/m) of the specimen, respectively.

This is the linear diffusion equation for sinusoidal steady state condition and axisymmetric geometries. For a given situation, the magnetic vector potential can be found by solving the above equation using appropriate boundary conditions. Fig. 2 shows the axisymmetric mesh used for the forward model.

The region modeled in the mesh encloses the magnetic system consisting of test specimen, sensor coil and air. The region is discretized using first-order triangular elements and the energy balance within the entire region  $R$  was achieved by minimizing the energy functional at every node in the region [9]. Applying energy minimization, the final element matrix equation is obtained as

$$[S_e + jR_e]\{A\}_e = \{Q\}_e \quad (2)$$

where  $[S]_e$  and  $[R]_e$  are element matrices formed from the coordinates of the nodes of an element, permeability and conductivity values associated with the material pertaining to the element and the centroidal radius of the element from the axis of symmetry.  $\{Q\}_e$  represents the source term and  $\{A\}_e$  represents the unknown magnetic vector potentials at the nodes of the element. Element matrices as described

above are separately formed for all the elements in the discretized region and these individual element equations are combined into a single global matrix equation that can be solved for the magnetic potential in the problem space after applying the boundary conditions. Once the magnetic potentials are calculated, the total impedance of a circular coil ( $Z_{coil}$ ) whose cross-section is discretized into  $N$  triangular finite elements is calculated as

$$Z_{coil} = \frac{2\pi j\omega N_s}{I_s} \sum_{j=1}^N r_{cj} A_{cj} \Delta_j \quad (3)$$

where  $j$  is the complex operator,  $N_s$  is the turn density of the coil (turns/m<sup>2</sup>),  $I_s$  is the current in the coil,  $\omega$  is the angular frequency of the excitation current, and  $r_{cj}$ ,  $A_{cj}$ ,  $\Delta_j$  are the centroidal radius, centroidal magnetic vector potential and the area of the  $j$ th triangular element, respectively.

Verification of the model was done by comparing the results with the analytical model reported by Dodds et al. [1]. The self-inductance ( $L_0$ ) of a coil, whose height, outer radius, inner radius and number of turns are 6.35, 9.525, 3.175, and 200 mm, respectively, are calculated using both techniques. Analytical solution yields a self-inductance of  $3.217 \times 10^{-4}$  H while the finite element solution gives a self-inductance of  $3.216 \times 10^{-4}$  H, a net error of less than 0.1%.

#### 4. Skin depth effect

The solution (A) of the eddy current governing equation in a single dimension and with no source current ( $J_s$ ) and planar wave excitation yields an exponentially decreasing harmonic function. The skin depth ( $\delta$ ) is given by the distance at which the magnitude of eddy current decreases to  $1/e$  of the magnitude at the surface. Skin depth ( $\delta$ ) is given by

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \quad (4)$$

The skin depth formula given above is based on planar wave excitation, which does not occur in day to day testing situations. A method described in this paper optimizes the skin depth approximation for a particular testing situation through minimization of error between the inverted conductivity profiles and the actual conductivity profiles using the developed inverse model.

#### 5. Inverse model: incremental layer approach

The importance of the inversion process lies in the fact that it separates out the integral effects of conductivity profiles on eddy current signal. This is very useful in non-destructively gauging the effects of processes like surface

treatment, annealing, or peening on the material properties beneath the surface. The eddy current inversion technique [10] uses the finite element method based forward model and the skin depth effect as the basis for inversion.

The inverse model described is valid for axisymmetric testing situations and assumes layered and uniform distribution of conductivity over the width of the specimen. The assumption is valid in the cases of shot peened or water peened samples in which the stress profiles are inherently axisymmetric [6,7]. The data required for the inverse model are:

1. Coil parameters and the test specimen thickness.
2. Skin depth approximation value for the given testing situation (Section 6).
3. Multi-frequency eddy current inductance data over an annealed/unpeened specimen and the peened test specimen.

In the forward model, a multi-layered material is considered. The substrate is modeled as a single entity with known electrical properties and meshed with an exponentially increasing layer sizing with known conductivity. The sensor coil is kept over the peened sample and the inductance of the coil at various excitation frequencies are measured and given as an input to the inverse model. The frequency input is first sorted in the descending order. The highest frequency, corresponding to the least depth of penetration according to the skin depth rule, is used in the first solution step of the inverse model. Since, the substrate conductivity is known, a two-layer model (optimal skin depth at the highest frequency and the substrate) can be used to separate the conductivity of the topmost layer. Minimum of three sub-layers are required within the optimal skin depth layer for accuracy of the forward model. During this process, the finite element forward model assigns various values of conductivity to the topmost layer and calculates the impedances. The actual impedance value (from experiments) can then be matched to a particular value of conductivity by rational interpolation of the conductivity–inductance data. The selection of the top layer conductivity using the conductivity–inductance curve of an Aluminum alloy sample at a frequency of 200 kHz is indicated in Fig. 3.

In Fig. 3, the measured inductance at a frequency of 200 kHz is 71.4675  $\mu$ H, which corresponds to a conductivity

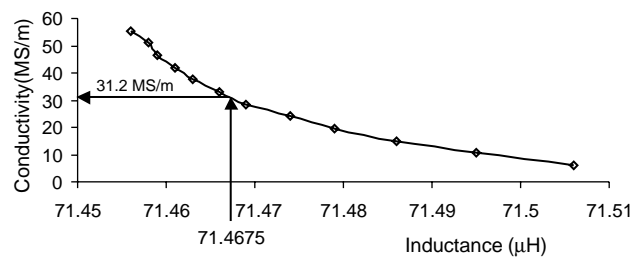


Fig. 3. Inductance–conductivity curve at 200 kHz for a top layer of unknown conductivity.

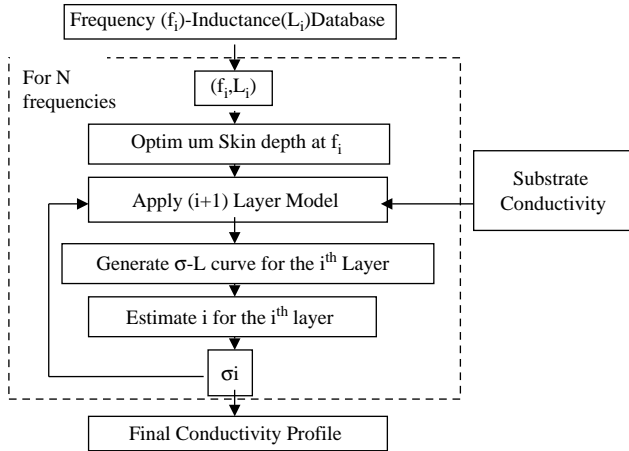


Fig. 4. Proposed multi-frequency inductance inversion methodology.

of 31.2 MS/m for the layer of 400 μm thickness from the conductivity–inductance curve generated by the forward model. In the subsequent step of the inversion scheme, a lower coil excitation frequency is used as the input to the inverse model. In this case, the depth of penetration is higher than that of the first frequency input, and the eddy current passes through the top layer whose conductivity was already found during the first step. The second layer is the one with unknown conductivity and the third layer is taken as the substrate. Hence, a three-layer model can be used to find the unknown conductivity by following a procedure similar to the first solution step of the inversion process. In the *N*th step, a *N* + 1 layered model is used. Hence, if data is taken over *n* frequencies, the inversion method generates the depth-conductivity profile within *n* steps. The inversion model is described in Fig. 4.

**6. Optimum skin depth selection**

In order to demonstrate the need for the selection of an optimal skin depth in the inverse model, reconstruction of a simulated conductivity profile of an Aluminum alloy 6063-T6 sample was employed at different skin depth approximations. The sample was created with three layers having thickness of 50 μm on top of a 15 mm substrate of conductivity 28 MS/m. The conductivity of the topmost layer was assigned as 35 MS/m, the conductivity of the middle layer and bottom layer were given as 31 and 29 MS/m, respectively. Frequencies of measurement were chosen based on the skin depth approximation and the layer thickness (*d*) value. For example, based on the substrate conductivity of  $\sigma$  MS/m, and a depth (*d*=50 μm) required to test the first layer, the required frequency (*f*) for testing at a penetration of *x* times the theoretical skin depth ( $\delta$ ) is found as

$$f = \frac{x^2}{\pi\mu\sigma d^2} \tag{5}$$

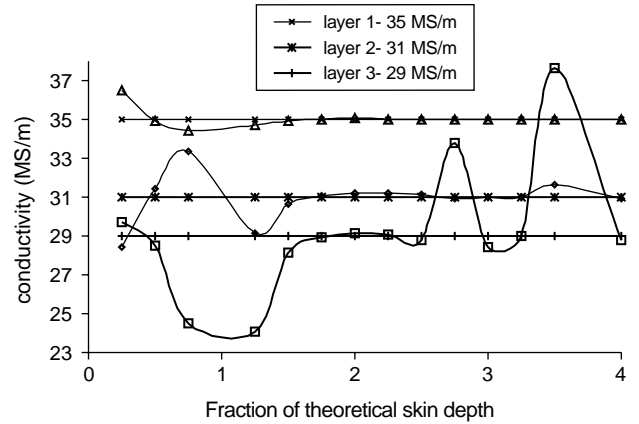


Fig. 5. Inverted layer conductivity values arising from various approximations of skin depths in the inverse model.

Inductance measurements were simulated at frequencies corresponding to maximum eddy current penetrations (*d*) of 50, 100 and 150 μm at various skin depth approximations (*x*) using the FEM forward model. This data was used in the inverse model for the reconstruction of the conductivity profiles. Fig. 5 shows the inverted layer conductivity values arising from various approximations of skin depth in the inverse model. Fig. 6 shows the correlation coefficient (*R*<sup>2</sup>)

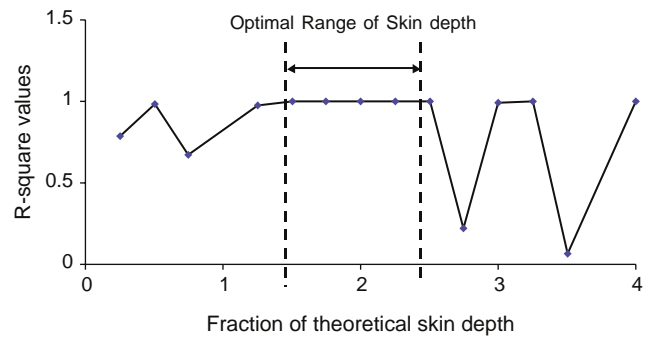


Fig. 6. Correlation coefficients for the inverted profile (vis-à-vis actual profile) at different skin depth approximations.

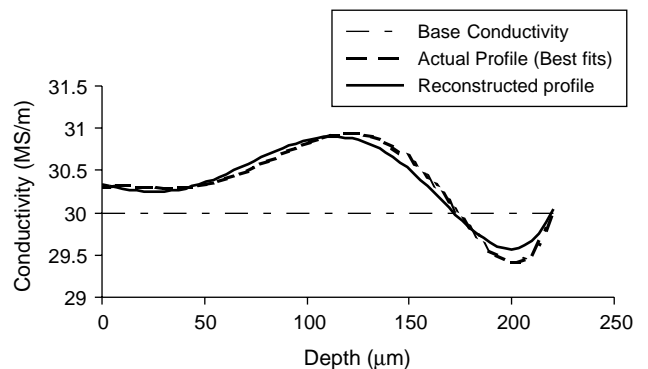


Fig. 7. Inversion of a simulated profile using the proposed methodology.

Table 1  
Properties of pancake coil used for multi-frequency inductance measurements

Property	Value
Number of turns	240
Wire diameter (AWG)	32
Outer diameter (mm)	10.2108
Inner diameter (mm)	5.1054
Lift off (mm)	2.0828
Thickness (mm)	14.732

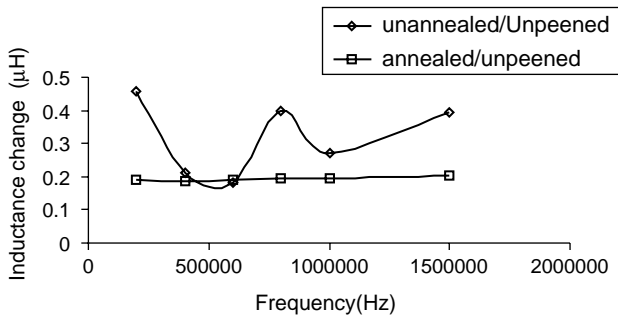


Fig. 8. Difference in eddy current inductance of annealed and unannealed specimens.

values for the inverted profile in comparison with the actual conductivity profile at different skin depth approximations. It is inferred from Figs. 5 and 6 that the minimum reconstruction error occurs within a range of 1.5–2.5 times the theoretical skin depth. There is a need to obtain the least skin depth approximation value that can accurately reconstruct a conductivity profile. A skin depth approximation of  $1.75\delta$  was found to give stable results for different simulated testing situations and was hence used for the reconstruction of experimental data.

### 7. Simulated results

In order to verify the validity of the inversion model, reconstruction of complex profiles similar to the expected stress profiles in water jet peened samples is shown in Fig. 7. An optimal skin depth of  $1.75\delta$  based on a substrate conductivity of 30 MS/m was used for the reconstruction. An  $R^2$  value of 0.9328 for the reconstructed profile shows a good correlation.

### 8. Experimental

Well-characterized 6063-T6 aluminum alloy was obtained in a sheet form of 1.2 cm thickness. The specimen was standardized to a rectangular shape of dimensions  $4 \times 3.5$  cm and subsequently finished using emery paper of different grades and belt grinding operations. In order to accurately find the change in residual stresses due to the peening operation on the sample, initial stresses existing in the material need to be relieved. Hence, the specimens were thermally treated at 623 K for 30 min with furnace cooling in order to induce total stress release [10]. A sensor coil was designed and calibrated using the method proposed by Sun et al. [3]. The sensor design parameters are listed in Table 1.

Impedance analyzer measurements were made on a thick Al 6063-T6 alloy (substrate conductivity: 30 MS/m) sample at various frequencies. Inductance change due to eddy currents was measured using Impedance analyzer HP-4192A on the annealed and unannealed specimens at different frequencies (Fig. 8). Each measurement involves reading the inductance of coil in air followed by inductance measured by placing the sensor coil over the sample at a particular frequency.

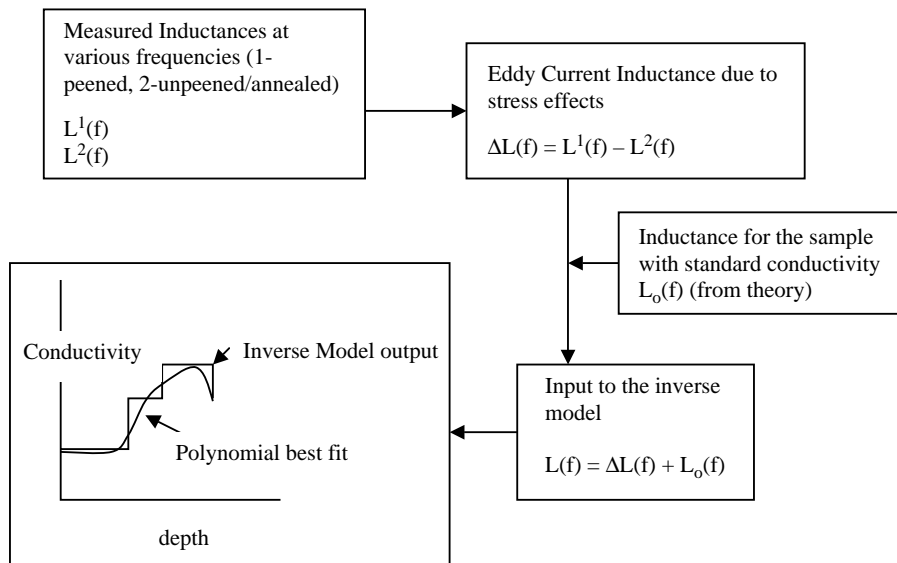


Fig. 9. Experimental approach for the inversion of conductivity profiles of peened samples.



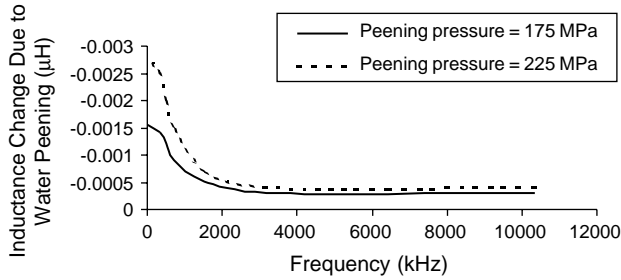


Fig. 10. Inductance change observed due to water jet peening at different frequencies.

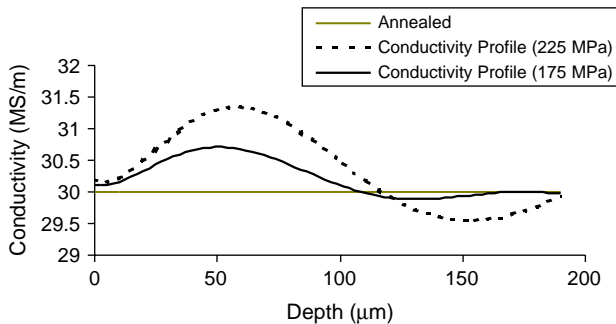


Fig. 11. Conductivity profiles obtained from inverse model (polynomial best fits).

From Fig. 8, it is noted that the inductance curve for the annealed sample is almost flat which shows that the residual stresses and work in the specimen are almost completely removed after annealing operation. Measurements were taken at frequencies of 0.72, 0.89, 1.15, 1.53, 2.14, 3.19, 5.28 and 10.34 MHz. These frequencies correspond to a depth of penetration of 190, 170, 150, 130, 110, 90, 70 and 50  $\mu\text{m}$ , respectively, for the aluminum alloy based on an optimum skin depth value of  $1.75\delta$ . The measured inductance over the annealed-peened and the annealed-unpeened samples are subtracted yielding the change in eddy current inductance due to stresses and other effects like cold work density in water peened sample. Inductances for the sample with standard conductivity (30 MS/m) are found at the measurement frequencies using finite element theory. The experimentally measured eddy current inductance changes are added to the theoretical inductance values and are used in the inverse model. The method is shown as a schematic in Fig. 9.

## 9. Results

Six standardized specimens were peened using water jets at pressures of 175 and 200 MPa employing a nozzle of 1.2 mm bore. The inductances of the coil placed over these specimens were measured and the inversion was performed. The inductance changes occurring due to peening are plotted in Fig. 10. The polynomial best fits of the inversion

results, i.e. the conductivity profiles of peened specimens are shown in Fig. 11.

## 10. Discussion

Compressive (tensile) stresses cause an increase (decrease) in the electrical conductivity [7]. Upto 5% increase in conductivity is obtained near the surface of water peened specimens. From the stress profiles of peened samples (Fig. 1), a stress reversal over depth was noted after peening wherein the mode changes from compressive in the top layer to a net tensile stress in the bulk of the material. Analogous to this effect, the conductivity of the specimen also changes its mode with respect to the substrate conductivity at depths of around 120  $\mu\text{m}$ . It is further noted that the reversal depth increases when the peening pressures are increased. The multi-frequency eddy current inversion technique is hence effective for assessing the near surface changes in conductivity due to peening and can be used to gauge information regarding the depth of reversal of residual stresses and relative intensities of peening between different samples. Future directions involve the study of the effect of dislocation density [7] on the eddy current signals and an attempt to isolate them as a step towards the calibration of conductivity profiles as the actual stress profiles.

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