Corrections for
"An Introduction to the Theory of Mechanism Design"
by Tilman Börgers, with a chapter by Roland Strausz and
Daniel Krähmer

Last updated April 26, 2019

- On page 13 the first sentence of the proof of Lemma 2.3 is incorrect. Corollary 17 in Chapter 6 of Royden and Fitzpatrick (2010) only implies that f is absolutely continuous on $(\underline{\theta}, \bar{\theta})$, which is not enough for the second sentence in the proof. Instead, Theorem 14.13 on page 333 of Yeh, *Real Analysis* (third edition, World Scientific, Singapore etc., 2014) can be used: "Let f be a convex function on $[a, b]$. If f is right-continuous at a and left-continuous at b, then f is absolutely continuous on $[a, b]$." As we already know that u is convex on $[\underline{\theta}, \bar{\theta}]$, we can use this result if we prove that u is right-continuous at $\underline{\theta}$ and left-continuous at $\bar{\theta}$. Because we know that u increasing, it is sufficient to show that $u(\underline{\theta}) \geq \inf_{\theta > \bar{\theta}} u(\theta)$ and that $\sup_{\theta < \bar{\theta}} u(\theta) \geq u(\bar{\theta})$. The first inequality follows from convexity: $(1-\lambda)u(\underline{\theta}) + \lambda u(\bar{\theta}) \geq u((1-\lambda)\underline{\theta} + \lambda\bar{\theta})$ for all $\lambda \in (0, 1)$. As λ tends to zero, the left hand side converges to $u(\underline{\theta})$, the right hand side converges to $\inf_{\theta > \bar{\theta}} u(\theta)$, and the claim follows. The second inequality follows from incentive compatibility: $\theta q(\theta) - t(\theta) \geq \theta q(\bar{\theta}) - t(\bar{\theta})$ for all $\theta \in (\underline{\theta}, \bar{\theta})$. As θ tends to $\bar{\theta}$ the left hand side tends to $\sup_{\theta < \bar{\theta}} u(\theta)$, whereas the right hand side converges to $u(\bar{\theta})$, and the second inequality follows. (I am very grateful to Mark Voorneveld for pointing out this error, and for showing me how to repair the proof.)

- On page 17, in the second to last line of the proof of Lemma 2.7, it should be "$q + \hat{q} \in \mathcal{M}$" instead of "$q \in \mathcal{M}$." (Thanks to Seungwon (Eugene) Jeong for pointing this out.)

- On page 42, Proposition 3.5, although correct as stated, is not phrased in the best possible way. If I could re-write it, I would formulate condition (ii) as: $T_i(\theta_i) = \theta_i Q_i(\theta_i) - \int_{\underline{\theta}}^{\theta_i} Q_i(x)dx - k_i$ where $k_i \geq 0$ is a constant. This would make condition (ii) in Proposition 3.5 exactly parallel to condition (ii) in Proposition 3.4, and thus would make it easier to compare the two results. (I would like to thank Tangren Feng for a discussion about the phrasing of Proposition 3.5.)

- On page 50, Proposition 3.7 should say: "An incentive-compatible, individually rational, and ex ante budget balanced first best mechanism exists if and only if either $N\underline{\theta} \geq c$ or $N\bar{\theta} \leq c$." The current version does not mention the condition "ex ante budget balanced." Relatedly, the sentence preceding Proposition 3.7 should mention ex ante budget balance, i.e. it should read: "The following impossibility result shows that in all nontrivial cases no first best mechanism is incentive-compatible, individually rational, and ex ante budget balanced." (Thanks to Seungwon (Eugene) Jeong for pointing this out.)

- On page 60, equation (3.50) should read: $-\frac{1}{2}+\frac{c}{2}-\frac{1}{2}\left(\frac{c}{2}\right)^2 = \frac{c}{2}\left(1-\frac{c}{4}\right)-\frac{1}{2} < \frac{1}{2}-\frac{1}{2} = 0$. (I am grateful to Nghiem Quang Huynh for pointing out this error.)

- On page 67, 5th line from the top, it should be "the highest type of the seller and the lowest type of the buyer" rather than "the lowest type of the seller and the highest type of the buyer." (Thanks to Daehyun Kim for noticing this error.)

- On page 68 in the third line from the end of the proof of Lemma 3.11, the inequality should be $\theta_S < \theta_B$ rather than $\theta_S > \theta_B$. (I am grateful to Nghiem Quang Huynh for pointing out this typo.)

- On page 69, in the first sentence after equation (3.63), it should be: "... of the highest type of the seller, $U_S(\bar{\theta}_S)$, and of the lowest type of the buyer, $U_B(\underline{\theta}_B)$" rather than "... of the highest type of the seller, $U_B(\bar{\theta}_B)$, and of the lowest type of the buyer, $U_S(\underline{\theta}_S)$." (I am grateful to Nghiem Quang Huynh for pointing out this typo.)

- On pages 88-90, the proof of Proposition 4.8 contains several gaps and typos. The Proposition is true as stated. The first two paragraphs of the existing proof are also correct. After these two paragraphs, I would like to insert the following new paragraph:

  "Let us now assume that $q(\theta) = 1$ for at least one $\theta \in \Theta$. We begin by arguing that $q(\theta_1, \theta_2) = q(\theta_1', \theta_2') = 1$ implies $t_i(\theta_1, \theta_2) = t_i(\theta_1', \theta_2')$ for $i = 1, 2$. The proof is as follows. Without loss of generality assume $\theta_1' \geq \theta_1$. Using Proposition 4.5 we can then infer $q(\theta_1', \theta_2) = 1$, and $t_1(\theta_1, \theta_2) = t_1(\theta_1', \theta_2)$. Moreover, because $t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2) = c$, and $t_1(\theta_1', \theta_2) + t_2(\theta_1', \theta_2) = c$, we can conclude: $t_2(\theta_1, \theta_2) = t_2(\theta_1', \theta_2)$. Iterating this argument, now replacing θ_2 by θ_2', we obtain what we want to prove. (In this second iteration, unlike the first iteration, it does not matter whether θ_2' is larger or smaller than θ_2 because, by assumption, $q(\theta_1', \theta_2') = 1$.) We denote by τ_1 and τ_2 the type-vector independent payments that agents 1 and 2 make whenever the good is produced. By budget balance: $\tau_1 + \tau_2 = c$. For type vectors (θ_1, θ_2) for which the public good is not produced, $q(\theta_1, \theta_2) = 0$, individual rationality and budget balance immediately imply that payments are do not depend on the values of θ_1 and θ_2 either: $t_1(\theta_1, \theta_2) = t_2(\theta_1, \theta_2) = 0$."

Next, define $\hat{\theta}_i$ and $\tilde{\theta}_i$ for $i = 1, 2$ as in the existing proof. To show that $\tilde{\theta}_i = \hat{\theta}_i$ for $i = 1, 2$ we can now use the following simple argument, that replaces the one currently used in the proof. Note that $\tilde{\theta}_i \geq \hat{\theta}_i$ is true by definition. It remains to show that $\tilde{\theta}_i > \hat{\theta}_i$ leads to a contradiction. If $\tilde{\theta}_i > \hat{\theta}_i$, then Proposition 4.5 implies $\tau_i = \tilde{\theta}_i$. But then $q(\hat{\theta}_i, \theta_j) = 1$ for some θ_j contradicts individual rationality for agent i.

The final four paragraphs of the proof can now remain unchanged, although some of the arguments in the second to last paragraph are now redundant because what these arguments show is already implied by the new paragraph that I inserted above. (I am grateful to Jan-Henrik Steg for pointing out gaps in this proof, and for suggesting the above argument.)

- In the fourth line from the top on page 101, in the proof of Proposition 5.2, it should be "... elements of Θ ... " instead of "... elements of θ ...." On the same page, in the last line of text preceding equation (5.13), it should be: "$\kappa = 1, 2, \ldots, k$" instead of "$\kappa = 1, 2, \ldots, k - 1$." On the same page, in inequality (5.15), the terms in the sum on the left hand side should be: $(u(a^\kappa, \hat{\theta}^{\kappa+1}) - u(a^\kappa, \hat{\theta}^\kappa))$ (the "hats" on the "thetas" are missing), and it would have been worth pointing out that on the right hand side of (5.15) a^k equals $q(\theta)$, and that thus the right hand side of (5.15) is independent of the sequence of types that we are considering. In the first line on page 102 it should be "makes" instead of "make." In the third line on page 102 it should be "$u(q(\theta), \theta) - t(\theta)$" instead of "$u(q(\theta), \theta)$." In inequalities (5.16) and (5.17), there are two instances where "$u(q(\theta), \theta)$" has one extra bracket at the end that is not needed. Similarly, "$u(q(\theta'), \theta'))$" should have been "$u(q(\theta'), \theta')$" in inequality (5.17). Finally, throughout the proof of Proposition 5.2 the symbols "$\tilde{\theta}$" (\tilde \theta) and "$\widetilde{\theta}$" (\widetilde \theta) should be read as the same symbols. (I am grateful to Mark Voorneveld for his comments on the proof of Proposition 5.2.)
- On page 103, in the last line of Proposition 5.3, it should be: "U :

$\Theta \to \mathbb{R}$" instead of "$U : \theta \to \mathbb{R}$." On the same page, in the second line of text following the proof of Proposition 5.3, it should have been "... it is a subgradient ..." instead of "... it is subgradient ...." (I am grateful to Mark Voorneveld for pointing out these typos.)

- On page 106 the 5th line from the bottom includes the following sentence: "The case in which A is infinite is harder only in terms of notation." Mark Voorneveld has persuaded me that proving Proposition 5.6 for the case of an infinite set A poses not just notational but also conceptual and mathematical challenges.

- On page 111, 5th line from the top, it should be: "$q(\underline{\theta})Ia$" instead of "$q(\underline{\theta}) = a$." (Thank you to Mark Voorneveld for pointing out this error.)

- On page 135, Proposition 7.8 is not quite correct. Proposition 7.8 becomes correct if the condition is added that q satisfies the characterization in Proposition 7.7 with *strictly positive* constants k_i, i.e. $k_i > 0$ for all $i \in I$. The proof in the book implicitly assumes $k_i > 0$ for all i: in equation (7.2) we divide by k_i. If $k_i = 0$ for some i, then q need not be dominant-strategy incentive compatible. A counterexample is Example 3.2 in: Juan Carlos Carbajal, Andrew McLennan, and Rabee Tourky: Truthful implementation and preference aggregation in restricted domains. *Journal of Economic Theory* **148**: 1074-1101, 2013. Proposition 7.6 is, as a consequence, also not quite correct: flexible decisions rules q that satisfy positive association of differences, but for which some of the constants k_i in Proposition 7.7 are zero, may not be dominant strategy implementable. (I am grateful to John Weymark for pointing this out, and to Arunava Sen for further comments on this point.)

- On page 111, 5th line from the top, it should be: "$q(\underline{\theta})Ia$" instead of "$q(\underline{\theta}) = a$." (Thank you to Mark Voorneveld for pointing out this error.)

- On page 143, last line of Definition 8.2, it should say:

$$\text{“}f(R_i, R_{-i}) R_i f(R_i', R_{-i}) \text{ for all } R_{-i} \in \mathcal{R}^{N-1}.\text{”}$$

  (Thank you to Tangren Feng for pointing this out.)

- In the bibliography the first name of Allan Gibbard should be spelled "Allan," not "Alan." (I am grateful to John Weymark for pointing out this error.)