Powering of matrices
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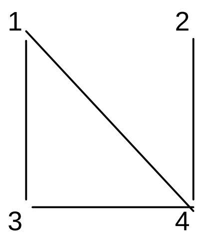
Example:

Consider the following graph:



Adjacency Matrix, X

To → / From ↓	1	2	3	4
1	0	0	1	1
2	0	0	0	1
X = 3	1	0	0	1
4	1	1	1	0

This matrix is symmetric about its main diagonal (upper left to lower right diagonal) reflecting the symmetry

Second power of adjacency matrix counts the number of two way paths from one node to another.

X^2 =

To → / From ↓	1	2	3	4
1	0	0	1	1
2	0	0	0	1
3	1	0	0	1
4	1	1	1	0

times

To → / From ↓	1	2	3	4
1	0	0	1	1
2	0	0	0	1
3	1	0	0	1
4	1	1	1	0

=

To → / From ↓	1	2	3	4
1	2	1	1	1
2	1	1	1	0
3	1	1	2	1
4	1	0	1	3

A bit more generally

a(11)	a(12)	a(13)	a(14)		a(11)	a(12)	a(13)	a(14)
a(21)	a(22)	a(23)	a(24)		a(21)	a(22)	a(23)	a(24)
a(31)	a(32)	a(33)	a(34)	*	a(31)	a(32)	a(33)	a(34)
a(41)	a(42)	a(43)	a(44)		a(41)	a(42)	a(43)	a(44)

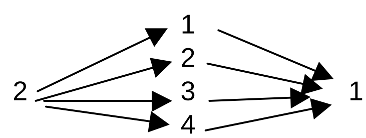
is a standard way to visualize the multiplication.

The (3,3) entry (third row, third column in the product) is: paths from 3 to 3 of length 2

a(31)a(13)+a(32)a(23)+a(33)a(33)+a(34)a(43)
each summand considers a path from 3 to 3 through each of 1, 2, 3, 4 as intervening node along the way.
Each considers a path of length two from 3 to 3.

a(11)	a(12)	a(13)	a(14)		a(11)	a(12)	a(13)	a(14)
a(21)	a(22)	a(23)	a(24)		a(21)	a(22)	a(23)	a(24)
a(31)	a(32)	a(33)	a(34)	*	a(31)	a(32)	a(33)	a(34)
a(41)	a(42)	a(43)	a(44)		a(41)	a(42)	a(43)	a(44)

The (2,1) entry (second row, first column of the product matrix) is: paths from 2 to 1 of length 2
a(21)a(11)+a(22)a(21)+a(23)a(31)+a(24)a(41)
each summand considers a path from 2 to 1 through each of 1, 2, 3, 4 as intervening node along the way.
Each considers a path of length two from 2 to 1.



in two-way flow