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Problem Set # 4

Instructions: Numerical answers require units and appropriate numbers of significant digits. **Show your work!**

Review Problems:

R-C-1. (12 points) A *good* approximation for a function.

An efficient way to approximate the value of functions at a given point, as commonly needed in science, is by using the Taylor expansion for that function *around* that point. You will remember from problem set 3 that Taylor series of a function around “zero” is given by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad (1)$$

which can be easily calculated using computers.

In an EXCEL spreadsheet try to find a good approximation for the function $f(x) = e^x$ at $x=0.01$ by following through these steps:

- In the first column, list the integers from 0 to 20.
- In the second column, calculate the factorial of the values on the first column by using the =FACT function.
- In the first cell of the third column, type in the number “ 0.01”.
- Set the value of the rest of the third column equal to the value of the first row using “ =C1”.
- In the 4th column, calculate the value of x^n (n being the row number from 0 to 20) at $x = 0.01$ by typing =C1^A1.
- In the 5th column, divide the 4th by the 2nd column.
- Using the =SUM function, for each cell the 6th column, calculate the sum of the numbers above that row from the 5th column.

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- Finally, in the 7th column, subtract value of every cell from column 6 from actual value of the function (e^x) at your point ($x = 0.01$).

Now, using your results, answer these questions:

1. Why is the Taylor expansion for $f(x) = e^x$ particularly simple?
2. Plot the results of the Taylor expansion (values in the 6th column) against the “term number” (=row number).
3. Using your plot or the values on the 7th column, try to find a limit at which the series starts to converge. At which “term” does this happen?
4. Make other plots by replacing the first cell of the third column with 0.1, 0.5, 0.9, 1, and 10. At each case, does the series converge? If so, approximately at which term?

Turn in hard copies of your plots and also submit the files onto Canvas.

R-2. (3 points) Slope of a function

Recall that for a function $y = f(x)$, its slope at the point $x = x_0$ is the derivative $y = f'(x) = df/dx$ evaluated at the point $x = x_0$.

- a. Plot the function $y(x) = 2x + 2$ and its derivative over the range $x = -5$ to $x = 5$.
- b. Plot the function $y(x) = 2x^2 + 2$ and its derivative over the range $x = -5$ to $x = 5$.
- c. Find the slope of a line passing through the points (0, 3) and (2, 11).

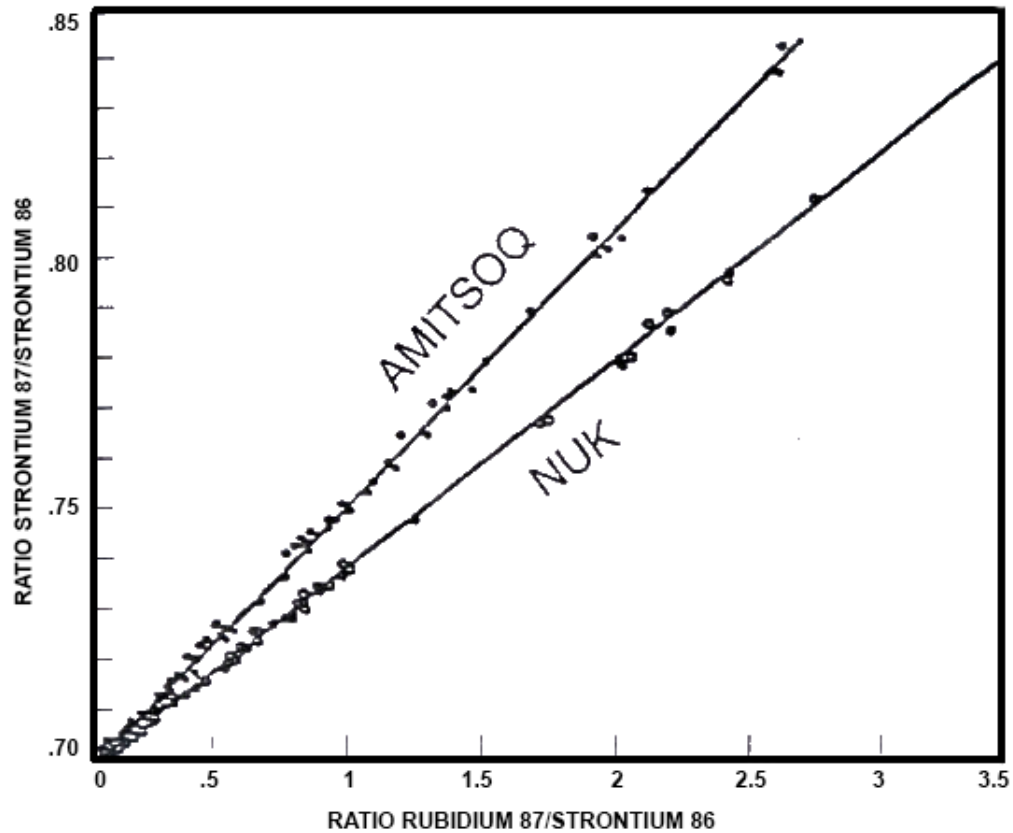
Turn in hard copies of your plots and also submit the files onto Canvas.

1. (6 points) The data below are radioactive age dating results for two of the oldest rocks on earth, the Amitsoq gneiss (upper line) and the Nuk gneiss (lower line), both from Greenland.

- (a) Find the decay constant λ for the *Rb-Sr* method, given that the half-life is 49 billion years.
- (b) Find the age of each of the two rocks.

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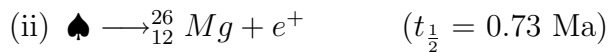


2. (3 points) Such determinations of the age of the earth have caused some political controversy. Go to the class homepage and click on the topic “Radiometric Age Dating.” (<http://www.earth.northwestern.edu/people/seth/202/#AGE>). Click on the link *How do we know how old the earth is?* Reading the text, you’ll come to a section discussing several arguments for the earth being much younger. Pick one of them and briefly summarize the arguments for and against this idea.

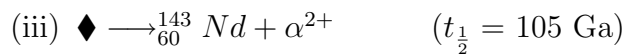
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3. (3 points) There are several important radioactive decay schemes we did not consider in class, two of which are important heat sources in the early history of the solar system. These are:



A third important decay is



What are isotopes \clubsuit , \spadesuit and \blacklozenge ? You may wish to consult a periodic table of elements. Recall that,

e^{-} : electron (negative charge)

e^{+} : positron (positive charge, mass equal to that of electron)

α^{2+} : alpha particle (2 protons, 2 neutrons)

4. (5 points) Uranium isotopes ${}^{235}\text{U}$ (half-life = 7.2×10^8 years) and ${}^{238}\text{U}$ (half-life = 4.8×10^9 years) were both produced in supernova eruptions by the r-process. If they were created in the ratio ${}^{235}\text{U}/{}^{238}\text{U} = 1.5$, and the current ratio is 0.007, when were they created? Compare this age to the age of the solar system.

5. (18 points) A challenge in thinking about the history of earth or other planets is the very long time scales. To help you, mark key dates and events on two time lines corresponding to the life of the sun. Turn one in and keep the other to use and update during the quarter. **Ga** is billion years ago, **Ma** is millions of years ago, **ka** is thousands of years ago.

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4.6 Ga

Label the following with heavy tick marks and brief text.

- 3.8 Ga: End of the Hadean Eon. The Hadean (named after Hades, Greek god of the underworld) began with the accretion of the Earth and spans the period of differentiation, heavy bombardment, and formation of the early atmosphere.
- 2.5 Ga: End of the Archean, by the end of which most of the continents formed and life appeared in the form of archaea and bacteria.
- 0.542 Ga: End of the Proterozoic Eon, during which multicellular organisms appeared and the atmosphere became rich in oxygen. The Hadean, Archean, and Proterozoic are collectively called Precambrian.
- 0 Ga (the present). Present end of the Phanerozoic Eon, during which life diversified into complex forms, first in the sea and then on land.

Label the following events with smaller tick marks and brief text. These ages are still under investigation, but approximate ones are fine for our purposes.

- 4 Ga: oldest rocks.
- 3.2 Ga: oldest microfossils: single cells (prokaryotes)
- 2.3 Ga: change from oxygen-poor to oxygen-rich atmosphere
- 1.1 Ga: assembly of Rodinia supercontinent
- 750 Ma: breakup of Rodinia supercontinent
- 475 Ma: first land plants
- 340 Ma: first reptiles
- 300 Ma: assembly of Pangaea supercontinent
- 230 Ma: first dinosaurs
- 200 Ma: breakup of Pangaea supercontinents
- 195 Ma: first mammals
- 65 Ma: death of dinosaurs
- 200 ka: homo sapiens
- 1 billion years in future: earth no longer in habitable zone where liquid water exists.

**6.4
billion
years
from
today**

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C-2. (10 points) Computer programs are excellent means of simulating natural phenomena. One of the most important tools in geology is radioactive decay. When a radioactive isotope (parent) decays, it creates a decay product (daughter). During one half-life, half of the parent elements decay into daughter elements. Each individual parent atom has a 50% chance of decaying to a daughter element in one half-life. This can be easily simulated using a coin. Each time the coin is flipped, there is a 50% chance the coin will land on heads. In this Problem, starting with 50 atoms, you will simulate coin flipping using an excel spreadsheet by means of a random number generator and an if-statement.

1. In the first column, enter a one in each cell because all the atoms are parent atoms. Each number one represents a parent atom. Only do this for the top 50 rows.
2. In the second column, enter in a random number generator. The random number generator, which is the function `=rand()` in excel, acts as the coin flip. If the random number is less than or equal to 0.5, the atom decayed. If the number is greater than 0.5, the parent atom did not decay.
3. In the third column, enter an if-statement to show which atoms decayed and which did not decay. The if statement is `=if(# <= 0.5,0,1)` where the random number in the second column is substituted for the `#` symbol. The atoms that list a 0 have decayed, and atoms that list a 1 have not decayed. At the bottom of the column, the number of ones are added together to count the number of parent atoms that have not decayed during the first half-life.
4. In the fourth column, enter another if statement that generates a random number for all the parent atoms that did not decay during the first half-life. The equation is `=if(# >0, (rand()), 0)`. This function will list a 0 for all atoms that have already decayed to daughter atoms.
5. In the fifth column, enter another if statement to show which atoms decayed and which did not decay. This is the same if statement as used in the third column. At the bottom of the column, the number of ones are added together to count the number of parent atoms that have not decayed during the first or second half lives.
6. Repeat the fourth and fifth columns three more times for the 3rd, 4th, and 5th half-lives.
7. Graph the number of parent atoms on a bar graph to show how the number of parent atoms changes from zero half-lives through 5 half-lives. Please hand in both your spreadsheet and your graph.
 - *Solutions done in a program besides EXCEL will be accepted, but must be formatted to look like the output described in EXCEL. Remember to submit the digital copies of your answers to the computer questions on Canvas.*