Name: Date:

Problem Set # 3

Instructions: *Numerical answers require units and appropriate numbers of significant digits.* ***Remember to show your work!***

Review Problems:

R-1. (4 points) (a) Find the derivative and the integral of the function

$$f(x) = ae^{bx} \tag{1}$$

where a and b are arbitrary real constants.

(b) Show that this function satisfies the equation $\frac{df(x)}{dx} = bf(x)$

R-2. (6 points) The natural logarithm, $\ln x$ is defined as

$$x = e^{\ln x} \tag{2}$$

Show that

(a) $\int_a^b e^{\ln x} dx = \frac{1}{2}(b^2 - a^2)$
(b) $\ln(e^x) + \ln(e^{-x}) = 0$

R-3. (6 points) The Taylor series of a function around a given value, a, is given by

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \tag{3}$$

where $f^{(n)}$ represents the n-th derivative of a function. Assuming that $a = 0$,

(a) Derive the Taylor series for $f(x) = x^4 + 3x$.
(b) Derive the Taylor series for $f(x) = e^{ax}$ up to the 7^{th} term.
(c) What would be a good approximation for $x \ll 1$?

Name: Date:

1. (12 points) Nine seismic stations are aligned east to west, shown in Figure 1. In 2011 they detected a moderate sized earthquake. Seismic records of this event are shown in Figure 2. There are two marks as P wave arrivals on each record in Figure 2, which belong to P_g and P_n. S wave arrivals are also marked.

(a) Mark P_g and P_n arrivals on Figure 2. Can you give an approximate range for the cross-over distance?

(b) Find the P_g and P_n velocities for the crust and the mantle in the area.

(c) Assuming the crust is approximately 40 km thick in this area, calculate the cross-over distance using the velocities you obtained in (b).

(d) Find the S-wave velocity.

(e) Use the velocities you calculated in (b) and (d) to find the distance to the earthquake epicenter from the V40A station.

(f) Find the geographic coordinates of the earthquake epicenter. Mark the approximate location of the earthquake on the map in Figure 1.

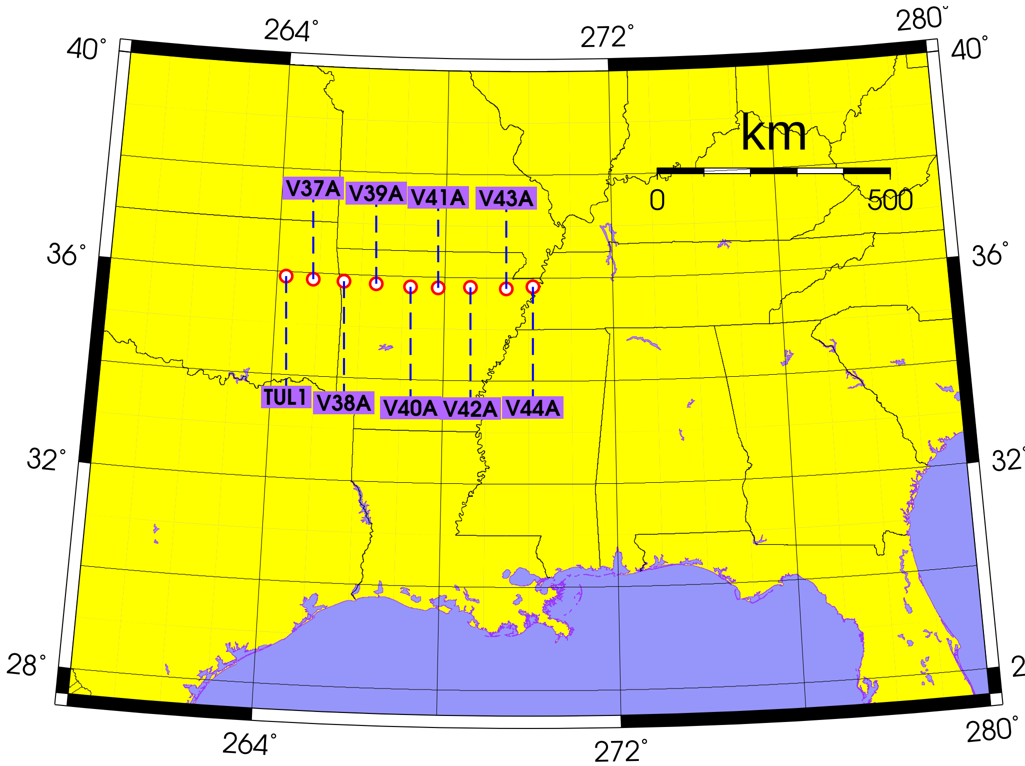


Figure 1: Stations the data from which are used in Figure 2.

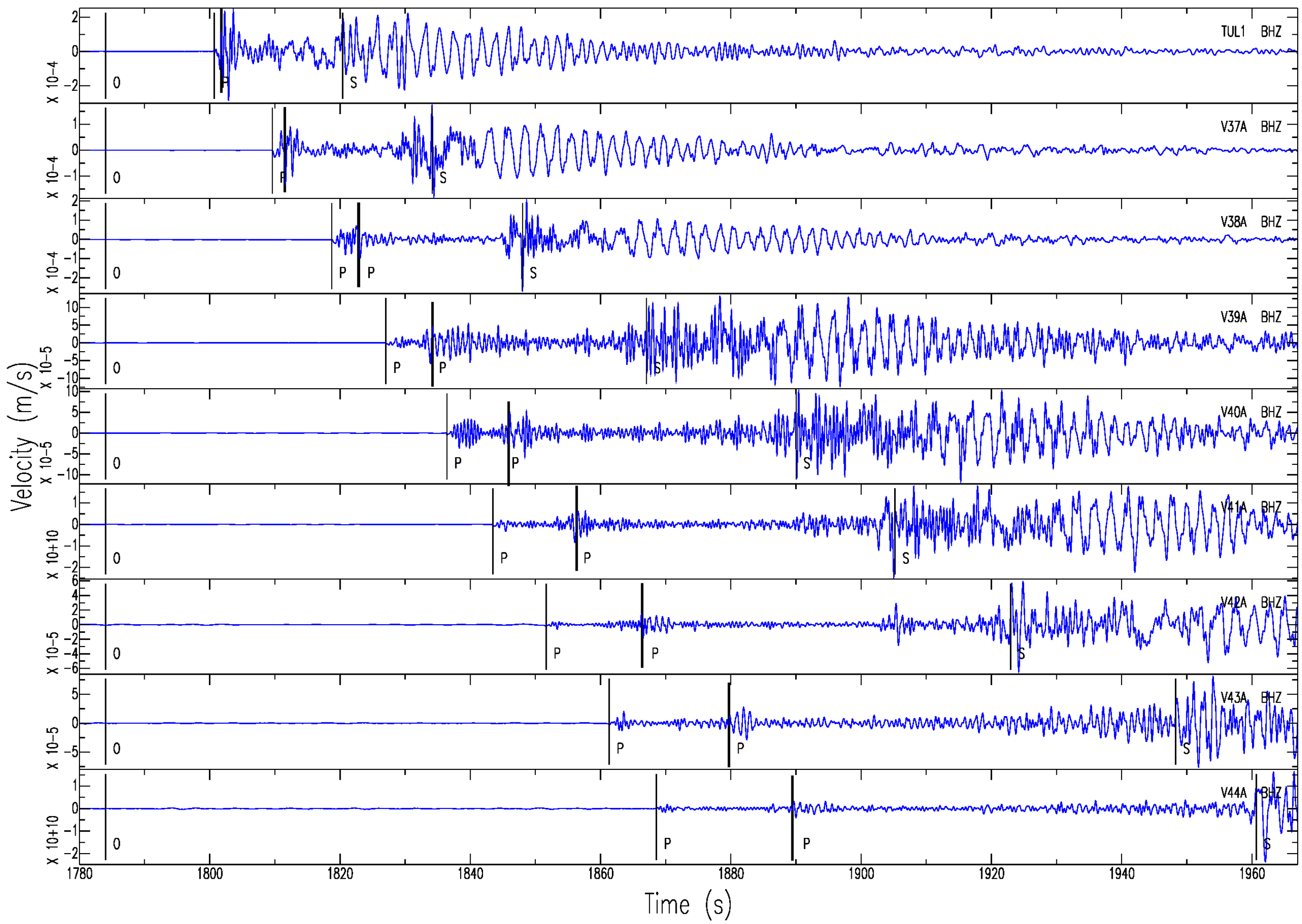


Figure 2: Seismic records of the stations in Figure 1. The two P wave arrivals belong to P_g and P_n. The records are sorted for stations from west to east (top to bottom in the figure).

2. (6 points) Calculate the pressure at the bottom of 6 km high layers of each of the following:

(a) ice ($\rho = 1$ g/cm^3)
(b) sediment ($\rho = 2.2$ g/cm^3)
(c) granite ($\rho = 2.8$ g/cm^3)

3. (12 points) Assume the earth is made up of a crust 30 km thick with average density of 3 g/cm^3, a mantle from a depth of 35 km to a depth of 2890 km with average density of 5 g/cm^3, an outer core from a depth of 2890 km to a depth of 5150 km with average density of 11 g/cm^3, and an inner core down to the center of the planet (6371 km) with an average density 13.2 g/cm^3. For each of these four regions determine their fractional volume and mass. Which one is the most voluminous?

4. (4 points) Figure 3 shows a phase diagram for diamond and graphite, and the geothermal gradient – the distribution of pressure and temperature in the earth. Given that diamonds, which are found at the surface, were formed at a depth in the mantle where diamond is stable, what is the minimum depth that diamonds could have come from?

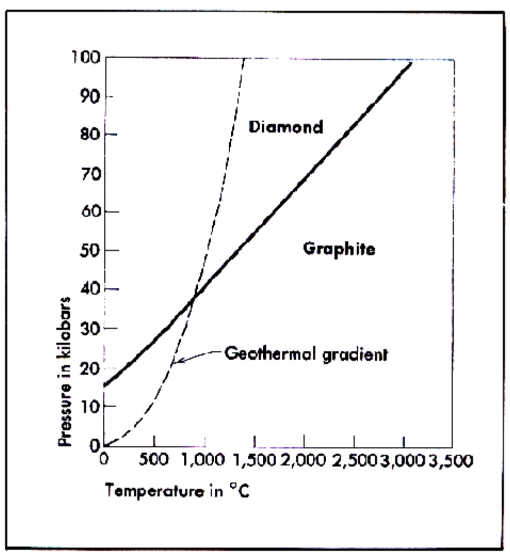
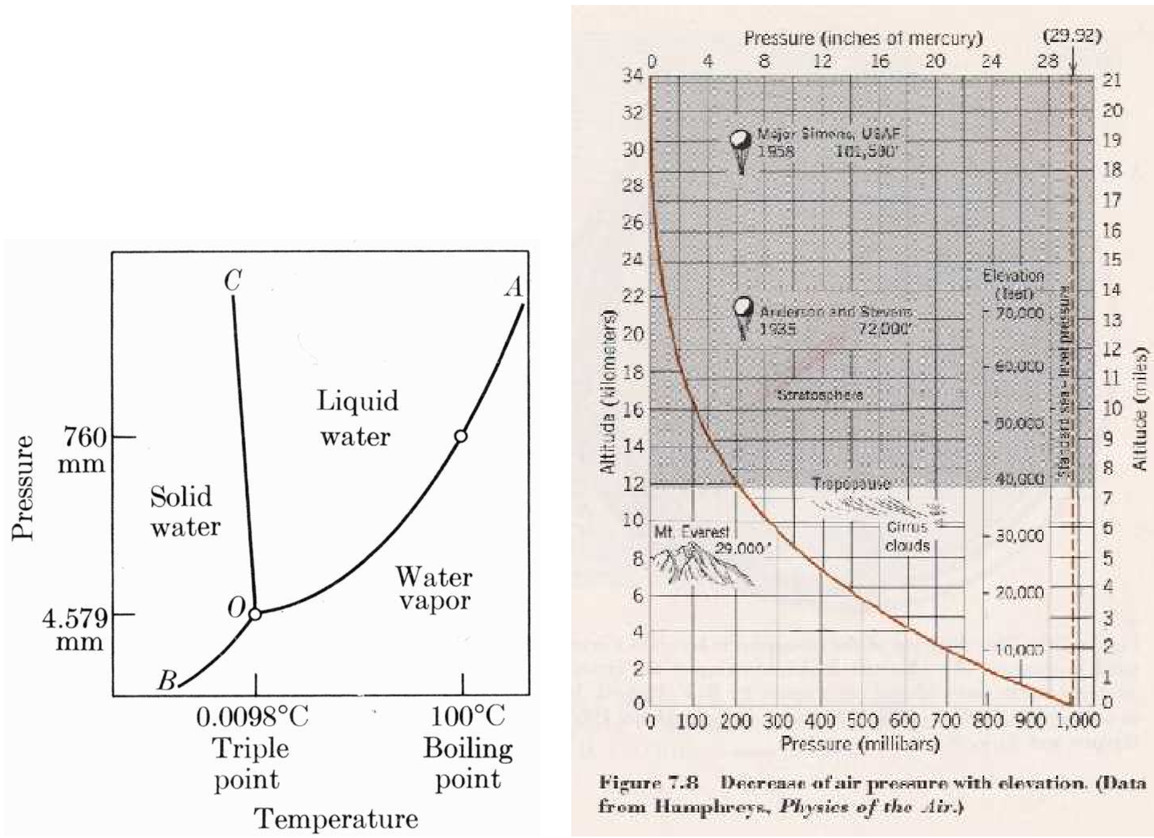


Figure 3: Carbon and graphite phase diagram.

5. (4 points) Mountain hikers find that at high altitudes, e.g. 14,000 ft., water boils at a temperature quite different from at sea level. This is because, as shown in the diagram below, atmospheric pressure decreases from 1 bar at sea level to less than 0.6 bars at 14,000 ft. Using the phase diagram for water, decide if boiling occurs at higher or lower temperatures at higher altitudes.



(a) (b)

Figure 4: (a) Water phase diagram; (b)Pressure-Altitude curve

Name: Date:

C-1 (16 points) Earth models give the values for density and P & S wave velocities as a function of depth.

(a) Download the PREM (Preliminary Reference Earth Model) model from

`http://www.earth.northwestern.edu/~amir/202/prem.dat`.[1]

The first 5 columns of this file are *depth*, *distance from Earth's center*, *P-wave velocity*, *S-wave velocity*, and *density*.

(b) What are the units of depth, density, P-wave velocity, and S-wave velocity?

(c) Plot the velocities and density against depth on the same plot using EXCEL or your favorite plotting software.

(d) Label the crust, the mantle, the outer core and the inner core on the plot.

(e) Label the mantle transition zones and specify their depths.

(f) How does the density of the inner core compare with that of the crust?

(g) How does the thickness of the crust compare with those of the mantle and the outer+inner core?

[1] This is an 8-column text file that can be viewed/modified using any text editor, EXCEL, etc.