Secrets to a Happy Marriage
between Applied and Theoretical Mathematics

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Disclaimer: I am more theoretical than applied.
Science is becoming more *specialized*.

Starting from 19 century, and much more so now.

More *fragmented*.

**Mathematics:** similar fate.
Universal mathematicians have become a rarity.

Henri Poincare

David Hilbert
The most visible separation in Mathematics:

Theoretical ("pure") vs. Applied
Observation (Pessimistic)

**Students** are more eager than faculty to separate. They tend to have stronger *theoretical/applied* identity.
Applied: “We do not prove theorems, we do not need that rigor”.

Theoretical: “We are not concerned about applications”. Take pride in ‘purity’, absence of applications.

Is that for real? What if ...?
Observation (Optimistic)

Mathematics on the whole became more interdisciplinary.
Mathematicians are more willing to explore beyond their area.
Is separation of Mathematics good or bad?

What about it is real (and useful) and what artificial (and should be overcome)?
Secrets to a happy marriage between Theoretical and Applied?

Secret 1. Strive for elegance.

Beauty is the first test: there is no permanent place in the world for ugly mathematics.

(Godfrey Harold Hardy)

Does that apply only to “Pure” Math?

Elegance not necessary / not realistic in the “real world” (applied)?

“Perhaps my solution is not elegant, but it works for all practical purposes.”
Example: Suppose you have a car problem.

The car is O.K, but the door won’t latch.
You bring the car to a mechanic.
The mechanic cuts a hole through the door.
The mechanic **cuts a hole** through the door.

Then he fixes the latch, and ...
... he puts a patch on the hole he made, and hands you back the keys.
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Is this acceptable? You look **perplexed**.
That patch looks awful.

But the door opens and closes.

Perhaps my solution is not elegant, but it works for all practical purposes.
"That patch looks awful"

"But the door opens and closes. Perhaps my solution is not elegant, but it works for all practical purposes."

Moral: appreciation of elegance goes beyond “pure” math (and beyond math).
Toward elegance in both theoretical and applied Mathematics.
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Concrete suggestions:

1. State theorems and present arguments as simply as possible.

*If you can’t explain it *simply*, you don’t understand it well enough.*

– Albert Einstein
Toward elegance in both theoretical and applied Mathematics.

Concrete suggestions:

1. State theorems and present arguments as simply as possible.

2. Explain your method heuristically in a separate section; illustrate with examples.

2. Strategy of the proofs

Let us present the heuristics of the proofs of Theorems 1.1 and 1.3. Both proofs are based on the idea to use local and global structures of the Lie groups $U(n)$ and ...
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3. Do not move arguments to the appendix. Methodology is important.
Secret 2. From application-specific to conceptual mathematics.

Sometimes just one step away.

Can my method work in a general setting?

Look for similarities, connections to other problems, areas.

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coding, quantization, wireless communications, machine learning, high-dimensional statistics, data streaming, data compression, analog-to-information conversion, computational biology, geophysics, medical imaging, astronomy, acoustics, remote sensing, computer graphics, robotics and control, neuroscience, optics and holography, ...
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![Image](image.png)

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**Irony:** the solution does not apply to the original problem!
Secret 3. Talk to theoretical mathematicians.

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Example: “I am modeling the effect of OGP on BND. The equations here are:

\[
P_1 = \frac{1}{(2\pi)^3} \frac{1}{3} \int_0^{k_c} \frac{\hbar k (d\omega / dk) 4\pi k^2 dk}{\exp (\hbar \omega / k_B T) - 1} - \frac{k_B T}{6\pi^2} k_c^3 \ln \left\{ 1 - \exp \left[ -\frac{\hbar \omega (k_c^*)}{k_B T} \right] \right\},
\]

\[
s_i = \frac{2}{(2\pi)^3} \frac{1}{T} \int_0^{\infty} \frac{\hbar \omega_i 4\pi k^2 dk}{\exp (\hbar \omega_i / k_B T) - 1} + \frac{2}{(2\pi)^3} \frac{1}{3T} \int_0^{\infty} \frac{\hbar k (d\omega_i / dk) 4\pi k^2 dk}{\exp (\hbar \omega_i / k_B T) - 1},
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\]

I made some simulations:

\[
I guess something theoretical can be proved about it. What do you think?”
The likely reaction is
Right: specific, rigorous, concise.
Ideal: a three-line question with yes/no answer.

Example: I am studying the differential equation
\[ x^2 y'' + xy' + (x^2 - n^2)y = x^{n+1}(2n-1)!! \]
Is it true that the solution \( y(x) \) is monotone for all \( n > 2 \)?

Now the theoretical mathematician is on the hook.
Then you can go on with supporting evidence, practical motivation, etc.
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Sometimes challenge them.

“Why do you care about this problem/object?”
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“Why do you care about this problem/object?”

“Suppose you had an infinite mental power, and you solved the problem completely. Then what? What difference would it make?”
Secret 5. Keep the **history** clean.

What *was* known before you; what *was not*; what *you* are contributing.
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*Avoid:* making an impression that the paper solves all problems.
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Transparent history helps outsiders (incl. theoretical mathematicians) to enter, appreciate the area.
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Unfortunately, there is

Topical Bias in Generalist Mathematics Journals

Joseph F. Grcar

Positive Bias of Trans. AMS

Negative Bias of Trans. AMS

Pure

Applied
Secret 7. Write surveys and educational papers.
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Example: Notices of AMS.
Summary of secrets

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Secret 2. From application-specific to conceptual mathematics.
Secret 3. Talk to theoretical mathematicians.
Secret 4. Listen to theoretical mathematicians.
Secret 5. Keep the history clean.
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Many others I do not know. They are yours to discover...