

## Homework Assignment #11 — Due Thursday, December 6

Textbook problems: Ch. 7: 7.3, 7.4, 7.6, 7.8

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- 7.3 Two plane semi-infinite slabs of the same uniform, isotropic, nonpermeable, lossless dielectric with index of refraction  $n$  are parallel and separated by an air gap ( $n = 1$ ) of width  $d$ . A plane electromagnetic wave of frequency  $\omega$  is incident on the gap from one of the slabs with angle of incidence  $i$ . For linear polarization *both* parallel to *and* perpendicular to the plane of incidence,
- calculate the ratio of power transmitted into the second slab to the incident power and the ratio of reflected to incident power;
  - for  $i$  greater than the critical angle for total internal reflection, sketch the ratio of transmitted power to incident power as a function of  $d$  measured in units of wavelength in the gap.
- 7.4 A plane-polarized electromagnetic wave of frequency  $\omega$  in free space is incident normally on the flat surface of a nonpermeable medium of conductivity  $\sigma$  and dielectric constant  $\epsilon$ .

- Calculate the amplitude and phase of the reflected wave relative to the incident wave for arbitrary  $\sigma$  and  $\epsilon$ .
- Discuss the limiting cases of a very poor and a very good conductor, and show that for a good conductor the reflection coefficient (ratio of reflected to incident intensity) is approximately

$$R \approx 1 - 2\frac{\omega}{c}\delta$$

where  $\delta$  is the skin depth.

- 7.6 A plane wave of frequency  $\omega$  is incident normally from vacuum on a semi-infinite slab of material with a *complex* index of refraction  $n(\omega)$  [ $n^2(\omega) = \epsilon(\omega)/\epsilon_0$ ].

- Show that the ratio of reflected power to incident power is

$$R = \left| \frac{1 - n(\omega)}{1 + n(\omega)} \right|^2$$

while the ratio of power transmitted into the medium to the incident power is

$$T = \frac{4\Re n(\omega)}{|1 + n(\omega)|^2}$$

- Evaluate  $\Re[i\omega(\vec{E} \cdot \vec{D}^* - \vec{B} \cdot \vec{H}^*)/2]$  as a function of  $(x, y, z)$ . Show that this rate of change of energy per unit volume accounts for the relative transmitted power  $T$ .

- c) For a conductor, with  $n^2 = 1 + i(\sigma/\omega\epsilon_0)$ ,  $\sigma$  real, write out the results of parts a and b in the limit  $\epsilon_0\omega \ll \sigma$ . Express your answer in terms of  $\delta$  as much as possible. Calculate  $\frac{1}{2}\Re(\vec{J}^* \cdot \vec{E})$  and compare with the result of part b. Do both enter the complex form of Poynting's theorem?

7.8 A monochromatic plane wave of frequency  $\omega$  is incident normally on a stack of layers of various thicknesses  $t_j$  and lossless indices of refraction  $n_j$ . Inside the stack, the wave has both forward and backward moving components. The change in the wave through any interface and also from one side of a layer to the other can be described by means of  $2 \times 2$  transfer matrices. If the electric field is written as

$$E = E_+e^{ikx} + E_-e^{-ikx}$$

in each layer, the transfer matrix equation  $E' = TE$  is explicitly

$$\begin{pmatrix} E'_+ \\ E'_- \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \begin{pmatrix} E_+ \\ E_- \end{pmatrix}$$

- a) Show that the transfer matrix for propagation inside, but across, a layer of index of refraction  $n_j$  and thickness  $t_j$  is

$$T_{\text{layer}}(n_j, t_j) = \begin{pmatrix} e^{ik_j t_j} & 0 \\ 0 & e^{-ik_j t_j} \end{pmatrix} = I \cos(k_j t_j) + i\sigma_3 \sin(k_j t_j)$$

where  $k_j = n_j\omega/c$ ,  $I$  is the unit matrix, and  $\sigma_k$  are the Pauli spin matrices of quantum mechanics. Show that the inverse matrix is  $T^*$ .

- b) Show that the transfer matrix to cross an interface from  $n_1$  ( $x < x_0$ ) to  $n_2$  ( $x > x_0$ ) is

$$T_{\text{interface}}(2, 1) = \frac{1}{2} \begin{pmatrix} n+1 & -(n-1) \\ -(n-1) & n+1 \end{pmatrix} = I \frac{(n+1)}{2} - \sigma_1 \frac{(n-1)}{2}$$

where  $n = n_1/n_2$ .

- c) Show that for a complete stack, the incident, reflected, and transmitted waves are related by

$$E_{\text{trans}} = \frac{\det(T)}{t_{22}} E_{\text{inc}}, \quad E_{\text{refl}} = -\frac{t_{21}}{t_{22}} E_{\text{inc}}$$

where  $t_{ij}$  are the elements of  $T$ , the product of the forward-going transfer matrices, including from the material filling space on the incident side into the first layer and from the last layer into the medium filling the space on the transmitted side.