Chapter 2

Diesel Locomotive Fueling Problem (LFP) in Railroad Operations

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1. **Introduction**

With many modes of transport (trucking, rail, and shipping where this option is available) now-a-days there is intense competition in the commodities transport industry, and in order to survive in the business, companies have to keep their charges low. Consequently they have to keep their operating costs low.

About 75% of transport by railroads in the world is based on diesel locomotives; the remaining 25% is mostly running on electrified track. Whereas, almost all the goods transported by rail in Europe is on electrified track, the situation in the US is the reverse with almost all of it powered by diesel locomotives. One of the major components in the operating cost of diesel powered rail transport industry is the cost of fuel. This case study deals with minimizing the cost of fuel and the cost of contracting trucks that deliver the fuel to the locomotives used in goods transport powered by diesel locomotives. The cost of fuel is highly location dependent (due to local taxes and transportation costs between supply and demand points), locomotive fueling problem (**LFP**) discussed in this paper is a critical problem in railroad operations. Given: the set of yards, the set of trains to operate, the locomotive assignments to trains, and the fuel cost and capacity data; this problem deals with finding the fueling plan for the various trains to minimize the total cost of fueling the locomotives.

The case study is a simplified real-life problem constructed and set up for the "**Problem Solving Competition-2010**" organized by the **Railway Applications Section (RAS)** of INFORMS (Institute for Operations Research and Management Science). The statement of the problem and all the data sets in it can be seen at http://www.informs.org/Community/RAS/Problem-Solving-Competition/2010-RAS-Competition. Kamalesh Somani (Kamalesh_Somani@CSX.com) of CSX Transportation, and Juan C. Morales (Juan.Morales@BNSF.com) of BNSF Railways contributed to this problem and the data sets in it.

Figure 1: Locomotive fuel tank being loaded with fuel

In this problem, as in most of the countries, cost/gallon of diesel varies from yard to yard. We describe 3 different algorithms that we used to solve this problem and highlight the summary of solutions obtained by each of them for comparisons of these algorithms.

2. **Brief Description of the problem in the Case Study**

The problem deals with *N*[=214] trains hauled by *L*[=214] locomotives on a railroad network consisting of *Y*[=73] yards over a two week **planning horizon**. The yard to yard distances, over the railroad network is given; the average yard to yard distance is 285.66 miles with standard deviation of 44.54 miles, median and mode of 300 miles. All locomotives are assumed to be identical in performance.

Each train visits a sequence of yards (referred to as **route** in the paper). For example, the route for the train T10 is the sequence (Y43,Y16,Y11,Y2,Y3, Y29,Y28,Y23) of yards, where Y43, Y23 are the origin, destination yard; and Y16, Y11, Y2, Y3, Y29, Y28 are all intermediate yards in that order in this route. A few characteristics of the trains included in the case are:

- All the 214 trains operate daily. Thus 214 trains originate every day from respective originating yards.
- 52 trains reach the destination yard the same day it leaves the originating yard. The remaining 162 trains reach the destination yard the next day.

- 135 trains ply between two yards only. Of these 135 trains, only 49 trains reach destination the same day (examples are trains T2 and T4); the remaining 86 trains reach destination the next day(examples are trains T1 and T3).
- 34 trains traverse only one intermediate yard between origin and destination yards. Of these 34 trains, 10 trains reach the intermediate and destination yards the next day (examples are trains T35 and T51) and 31 trains reach the intermediate yard on the starting day, but reach the destination yard the next day(examples are trains T35,T51,T27 and T25).
- 20 trains traverse two yards between origin and destination yards. Of these 20 trains, 14 trains reach the first intermediate yard the next day (examples are trains T13 and T14), 18 trains reach the second intermediate yard the next day (examples are trains T13,T14,T16 and T94) and all trains reach their destination the next day.
- 16 trains traverse three yards between origin and destination yards. Of these 16 trains, 7 trains reach the first intermediate yard the next day (examples are trains T7 and T8), 10 trains reach the second intermediate yard the next day (examples are trains T7,T8,T33 and T40), 15 trains reach the third intermediate yard the next day (examples are trains T7,T8,T33,T34 and T39) and all trains reach their destination the next day.

Each route may be hauled by a different locomotive on different days; the allocation of locomotives to routes is given as data. For hauling each train in this case study, only one locomotive is used. Each locomotive may haul different trains on different days.

All the routes operated in the case study problem can be grouped into a set of **pairs**, each pair operating between a pair of yards in the forward and reverse directions; but the set of yards visited in the two directions for a **route-pair** may be different. Every route pair has a dedicated pair of locomotives operating it. When we refer to a **yard** on a route, we mean either the origin or destination yards of the route, or an intermediate yard where the train has a scheduled stop. An example is locomotives L1, L2 hauling trains T1 on route (Y25,Y19) and T2 on route (Y19,Y25); with L1 , L2 hauling T1, T2 respectively on days 1, 3, 5, 7, 9, 11, 13; and L2, L1 hauling T2, T1 respectively on days 2, 4, 6, 8,10, 12, 14 of the planning horizon. Another example is locomotives L5, L6 hauling trains T5 on route (Y36, Y60, Y62), T6 on route (Y62, Y36) on alternate days.

The locomotive of each train can be refueled by fueling trucks positioned at any of the yards on its route except the destination yard, but the total number of refuelings on any route should be ≤ 2. All locomotives have the same fuel capacity of 4500 gallons; and the fuel consumption (3.5 gallons of fuel per mile) on any route is independent of the route and the locomotive operating the train. Fuel consumption for a locomotive travelling between any two yards can be determined using the given table of inter-yard distances. Fuel cost at different yards varies between $ 2.90 to $3.56/gallon (with average of $3.13 and standard deviation of $0.17). Each refueling incurs a setup cost of $250. Fuel is dispensed by fueling trucks positioned at yards, each having a maximum capacity of 25000 gallons/ day and involving a one-time contracting cost of $8000 for the two week planning horizon (the contracting cost is $4000 per week per truck). The problem statement allows each locomotive to start on the very first trip with any feasible amount of fuel (referred to as "**initial fuel"** in this paper) without any cost incurred; the locomotive should be left with the same amount of fuel after completing the last trip in the planning horizon.

Desired Outputs

We need to determine: (a) which yards will serve as fueling points for the locomotives (these yards, where refueling trucks are contracted are called "**committed yards**" in this paper), (b) which yard will be used to refuel the locomotive hauling each train, (c) the days and the amount of fuel loaded at each yard used as refueling point for each train, (d) the number of fueling trucks contracted at each yard, (e) and the amount of fuel(in gallons) in each locomotive tank at the beginning of the planning horizon. We need to minimize the total cost = fuel costs+ fueling truck contracting costs+ setup costs for refueling.

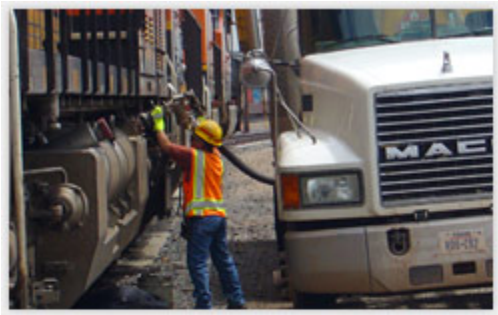

Figure 2: A locomotive being refueled by a fueling truck

3 Discussion on methods used for solving this problem

Typically when faced with problems like this, OR specialists will try to build a mathematical model for it (the appropriate model for this problem will be an MIP (mixed integer programming) model). For solving an MIP model, the company needs to have access to a software package like CPLEX, but many railroad companies may not have access to such packages.

So we started looking for an approach which is much simpler to solve the case study problem, gives comparable results, and scales up easily to problems of the size encountered in real world applications. We developed a greedy algorithm for the LFP, which meets all these requirements (see [1] for a description of greedy methods), which we discuss below. However, for the sake of comparison, we developed a mathematical model for this case study problem and solved it using CPLEX; we discuss this model and the results from it in Section 6.

The first step in developing a greedy method for solving this problem is to develop a "**greediness criterion**" for the decisions to be made in it. Keeping the objective function to be minimized in the problem in mind, there are two greediness criteria that we can use for selecting a yard *p* as a refueling yard for a route:

$GC1_p$ = fuel cost incurred at yard *p* for the route, if yard *p* is selected as a refueling yard

$GC2_p$ = (fuel cost incurred at yard *p* for the route) + (incremental truck contracting cost at this stage, if yard *p* is selected as a refueling yard for this route)

In the following section, we will discuss the greedy algorithm developed for the problem based on *GC1*.

4. **Description of the greedy algorithm, Algorithm1, using *GC1* as the greediness criterion**

The decisions in this algorithm are made in the specific order given below, one route at a time.

i. Partitioning the set of routes into various labeled categories

There are several pairs of yards $\{y_i, y_j\}$ such that (y_i,y_j) and (y_j,y_i) are both routes and this pair of routes are operated by a pair of locomotives dedicated to these routes only, each locomotive hauling one of these pairs alternately on alternate days. Route pairs of this type with no intermediate stops in either direction are classified into a set or

category labeled R_1. Routes (Y19,Y25) and (Y25,Y19) operated by locomotives L1 and L2 belong to category R_1. The set R_2 consists of the remaining routes with at least one scheduled stop in one or both the directions.

R_1 is again partitioned into:

-R_{11} (for these, full locomotive tank capacity is insufficient to cover the round-trip distance from origin to destination and back),

-R_{12} (for these a full locomotive tank capacity is sufficient to cover the round-trip distance from origin to destination and back).

In the case study problem, $R_{11}=\emptyset$, R_{12} contains 59% of all the routes with 33% of all the mileage in the problem.

R_2 consists of route pairs, with at least one stop at an intermediate yard in the forward or reverse direction or both, each operated by a dedicated pair of locomotives hauling in each direction alternately. R_2 is again partitioned into:

-R_{21} (for these, a full locomotive tank capacity is sufficient to cover the entire round trip pair; this contains 28% of all the routes covering 33% of all the mileage in the problem), and

-R_{22} are the remaining (in each route pair here, at least one refueling is needed in each direction; this set contains 12% of all the routes covering 34% of all the mileage in the problem).

Routes (Y10,Y7,Y2,Y12,Y30) and (Y30,Y12,Y10) are operated by locomotives L11 and L12. The round trip distance from yard Y10 back to yard Y10 through the yards Y7,Y2,Y12,Y30 and Y12 is 941 miles. Each locomotive thus requires 3293.5 gallons with fuel consumption at the rate of 3.5 gallons of fuel per mile. Since the locomotive tank capacity is 4500 gallons, the entire round trip pair can be covered with a single filling in the round trip. This route-pair belongs to the R_{21} category.

Routes (Y43,Y41,Y56,Y57,Y51) and (Y51, Y57,Y56,Y41,Y43) are operated by locomotives L7 and L8. The round trip distance from yard Y43 back to yard Y43 through the yards Y41,Y56,Y57,Y51,Y57,Y56 and Y41 is 2010 miles. Each locomotive thus requires 7035 gallons with fuel consumption at the rate of 3.5 gallons of fuel per mile. Since the locomotive tank capacity is 4500 gallons, the entire round trip pair cannot be covered with a single filling in the round trip. This route-pair belongs to the R_{22} category.

Therefore we will adopt the policy of refueling locomotives serving routes in categories R12 and R21 at most once in each round trip pair of routes and those in categories R11 and R22 at least once on each origin to destination route in this category.

ii. Identifying refueling yards for routes in each Category in Algorithm1

Here we discuss how this method selects the refueling yards to be used on each route; but not the actual refueling plan for each locomotive which will be discussed later in sub-section (iv). Refueling yards are selected using the greediness criterion *GC1* defined in Section 3 for the yards.

Clearly each route in R_1 ($= R_{12}$ in the case study problem) can only be refueled on any day at the origin yard of route hauled on that day, if we choose to refuel it on that day. Also, each locomotive on these routes in R_{12} need not be refueled every day.

We will now discuss how the refueling yards are selected on each route in this algorithm, for each category of yards separately.

Category R_{11}: For the routes belonging to set R_{11} in a general problem, there is no choice other than to refuel the locomotive at the origin yards of all trains hauled over the planning horizon on these routes. Thus the origin yards of all routes in set R_{11} will be refueling yards and refueling trucks are committed to be positioned at these yards.

Category R_{12}: Define a set called the ``current set R'_{12} initially $=$ R_{12}, S'_{12} = set of origin yards for routes in R'_{12} and Y'_{12} =yard with the cheapest fuel cost among those in S'_{12}. For each route in R'_{12} for which Y'_{12} is the origin yard, fix Y'_{12} as the committed refueling yard, and delete those routes from the set R'_{12} from further consideration. If R'_{12} is now $\emptyset$, go to the next category. Otherwise update S'_{12}, Y'_{12} using the current R'_{12} and repeat this step.

To illustrate, in the first iteration, S'_{12} ={Y1, Y2, Y3, Y6, Y10, Y13, Y14, Y15, Y17, Y18, Y19, Y21, Y22, Y23, Y24, Y25, Y26, Y27, Y30, Y32, Y34, Y36, Y37, Y38, Y40, Y41, Y43, Y44, Y45, Y46, Y47, Y48, Y49, Y50, Y52, Y53, Y54, Y56, Y57, Y58, Y59, Y60, Y62, Y63, Y64, Y65, Y66, Y67, Y69, Y70, Y73} and Y'_{12} =Y60. In the second iteration, S'_{12} ={ Y1 ,Y2 ,Y3 ,Y10 ,Y13 ,Y14 ,Y15 ,Y17 ,Y18 ,Y19 ,Y21 ,Y22 ,Y23 ,Y24 ,Y25 ,Y26 ,Y27 ,Y30 ,Y32 ,Y34 ,Y38 ,Y40 ,Y41 ,Y43 ,Y44 ,Y45 ,Y46 ,Y47 ,Y48 ,Y49 ,Y50 ,Y52 ,Y53 ,Y54 ,Y57 ,Y58 ,Y59 ,Y63 ,Y64 ,Y65 ,Y66 ,Y67 ,Y69 ,Y70 ,Y73} and Y'_{12} =Y32. Etc.

Proceeding in this manner, the other committed yards obtained in ascending order of fuel cost are Y34, Y25, Y17, Y53, Y54, Y52, Y64, Y1, Y38, Y44, Y23, Y13, Y46, Y50, Y49, Y41, Y24, Y66, Y47, Y59, Y15, Y22, Y30, Y3, Y10, Y63 and Y58. At least one of the yards of the 63 R_{12} route pairs are covered by these 29 committed yards. Both yards of a few route pairs associated with yards {Y47,Y15}, {Y3,Y30}, {Y23,Y66}, {Y60,Y41}, {Y41,Y32},{Y60,Y15}, {Y1,Y15}, {Y3, Y13}, {Y15, Y30}, {Y15,Y38}, and {Y34, Y59} are committed yards; in such cases, fueling is always done at the yard with lesser fuel cost (for example at yard Y30 for route pairs associated with yards Y3 and Y30).

Category R_{21}: For each route pair in this category, on every route if there are no previously committed yards amongst the set of yards visited by it, the yard having the cheapest fueling cost on it will be fixed as the committed refueling yard. There are no instances of this type in the case study; all R_{21} category route pairs have at least one previously committed yard amongst the set of yards visited by them.

If there is a committed yard Y_k on this route-pair (i.e. the yard Y_k has been committed as a refueling yard earlier and has the cheapest fuel cost amongst all the committed yards on this route-pair) and if Y_k has the cheapest fuel cost on this route, then commit this yard Y_k as the refueling yard for the route pair. On the other hand, if the cheapest fuel cost yard on this route-pair is Y_p not in the committed list, compute s_{kp} = saving in fuel cost by fueling this route-pair at Y_p rather than Y_k, = (fuel cost at yard Y_k - fuel cost at yard Y_p) x (fuel requirement for the roundtrip route pair over the planning horizon). If $s_{kp} \leq$ 8000, which is the total truck contracting cost, then this route will be fueled at the previously committed fuel station Y_k. Else it will be refueled at the yard Y_p which will now be added to the committed refueling yard list. This procedure is repeated for all category R_{21} route-pairs.

For example, the route pair (Y68, Y64, Y20, Y23) and (Y23, Y20, Y64, Y68) corresponding to trains T43 and T44, have previously committed yards Y23 and Y64. The fuel cost at the previously committed yards Y23 and Y64 are \$3.04 and \$2.98 respectively; thus Y64 is chosen for comparison. The cheapest fuel cost on this route is \$2.92 at Y68. The total distance covered in round trips by the route pair over the planning horizon is 12460 miles, for which the fuel requirement is 43610 gallons. With a difference of \$0.06 in fuel cost per gallon between yards Y64 and Y68, s_{kp} works out to \$2616.60. Since s_{kp}<8000, this route pair is refueled at previously committed yard Y64.

All the 30 R_{21} category route pairs in the case study are thus refueled at previously committed yards and there is no addition to the committed refueling yard list comprising of 29 yards.

Category R_{22}: In each of the route pairs in this set, at least one refueling is needed in each direction. We will now discuss how the refueling yards are selected in each direction of a general route pair in this category.

Fact 1 : *It turns out that one refueling in each direction is sufficient in the case study problem, hence greedy method selects just one refueling yard in each direction of the route pairs.*

We check the following for each couple of yards $\{y_i,y_k\}$ where y_i, y_k belong to different direction routes in this pair. If the mileage of the route from y_i to y_k or y_k to y_i is greater than what can be covered by one locomotive tank capacity, discard this pair. If the mileage of the portion of the route pair from y_i to y_k ,or y_k to y_i can both be covered by one locomotive tank capacity, then call this pair of yards as a **feasible couple** (Fact 1 implies that there will be at least one feasible couple of yards), and compute f_{ik} = (fuel consumption on the portion of the route-pair from y_i to y_k)(cost of fuel per gallon at y_k) + (fuel consumption on the portion of the route-pair from y_k to y_i)(cost of fuel per gallon at y_i). Among all feasible couples $\{y_i, y_k\}$, choose that optimal couple corresponding to the lowest value of f_{ik} as the pair of committed refueling yards for the route-pair under consideration. It may be noted here that the optimal couple may be $\{y_i, y_i\}$, which implies that re-fueling may be done at the same yard in different directions; for example, it is optimal to refuel at yards Y3 and Y3 for the route pairs (Y23,Y28,Y29,Y3,Y2,Y16,Y43) and (Y43,Y16,Y11,Y2,Y3,Y29,Y28,Y23) associated with trains T9 and T10. This procedure is repeated for all route-pairs in category R_{22}.

For example, fueling trucks positioned at yards Y41 and Y51 spaced 752 **route-miles** apart (from Y41 to Y56 to Y57 to Y51) can serve the route pairs (Y43,Y41,Y56,Y57,Y51) and (Y51,Y57,Y56,Y41, Y43) corresponding to trains T7 and T8. While Y41 is a previously committed yard, Y51 is added to the committed refueling yard list. Similarly yards Y29, Y11, Y33, Y8, Y20 and Y6 are added to the committed refueling yard list to cater the 14 R_{22} category route pairs in the case study.

iii. Determining initial fuel *FI*, at origin yards for routes, in Algorithm1

If the originating yard for a route at the beginning of the planning cycle is a committed yard, the initial fuel amount *FI* in the locomotive for that route is zero. If the originating yard for a route at the beginning of the planning cycle is not a committed yard, the initial fuel amount *FI* in the locomotive for that route is the fuel required to reach the first committed refueling yard on it from the origin.

iv. Fueling Plan of Locomotives & Number of trucks contracted at Committed Yards in Algorithm1

Once a setup cost for refueling the locomotive is incurred, the greedy method tries to take the full advantage of it by filling the locomotive to full capacity. This principle helps us to determine the fuel loaded at each refueling stop of each locomotive.

So fill up the locomotive tank at the first refueling yard on the route, to the full capacity of the locomotive fuel tank. At subsequent committed refueling yards on this route, refuel to the full capacity of the locomotive fuel tank for Category R_{11} and R_{22} routes. For category R_{12} and R_{21} routes, refuel to full tank capacity at every $\left\lceil \frac{Locomotive\ tank\ capacity}{(fuel\ consumption\ rate\ in\ gallons\ per\ mile)(round\ trip\ mileage)} \right\rceil$ round trips.

At the last refueling in the planning horizon, refuel each locomotive only to the extent that a balance fuel amount equal to the initial amount of fuel amount *FI* for this route will be left in its fuel tank at the end of that trip.

For example, for route pair (Y25,Y19) and (Y19,Y25), the fuel truck(s) is/are committed to be positioned at yard Y25. Counting the days of the planning horizon as days 1 to 14, locomotive L1 operates train T1 on route (Y25, Y19) on days 1, 3, 5, 7, 9; and train T2 on route (Y19, Y25) on days 2, 4, 6, 8, and 10 with refueling. On day 11 locomotive L1 will be starting at the origin yard Y25 to operate the route (Y25, Y19); it will be left with only 475 gallons of fuel in it, which is insufficient to cover the roundtrip consisting of the route (Y25, Y19) on this day and the route (Y19, Y25) the next day. So before starting at Y25 on day 11 locomotive L1 needs to refuel; and since only 4 more days are left in the planning horizon and the total fuel needed for the routes it has to cover in these days is 1610 gallons, fuel amount of 1610-475 =1135 gallons is refueled into this locomotive at yard 25 on day 11. Then it covers train T1 on days 11, 13; and train T2 on days 12, 14; and will be left with an an empty fuel tank (same as initial fuel) at the end of the planning horizon.

The other locomotive operating this route pair {(Y25, Y19), (Y19, Y25)} is L2 operatiing train T2 on the route (Y19, Y25) on day 1. Y19 is not a fueling yard, so this locomotive needs an initial fuel amount of 403 gallons = fuel required to reach the fueling yard Y25 on this route from its origin yard Y19 on day 1. Locomotive L2 gets 4500 gallons filled its fuel tank at yard Y25 on the 2nd day days 2, 4, 6, 8, 10 and on train T1 on route (Y25, Y19) on days 3, 5, 7, 9. On day 11 at the origin yard Y25 on train T1 on the route (Y25, Y19) it will have a fuel amount 475 gallons only left, not enough to cover the round trip {(Y25, Y19),(Y19, Y25)}, so it has to refuel on this day. Again since only 4 days are left in the planning horizon, and the fuel amount needed to cover the remaining trips in the planning horizon is 1135 gallons; on day 11 at the origin yard Y25 an amount of 1135 gallons is filled in its fuel tank. Locomotive L2 then operates trains T1 on the route (Y25, Y19) on days 11, 13, and train T2 on route (Y19, Y25) on days 12, 14; and will have 403 gallons of fuel at the end of the planning horizon (same as the initial fuel amount at the origin yard on day 1).

Using the fueling plan obtained above and the given information on yards visited on each route on each day of the planning horizon, the fuel dispensed at each committed yard on each day of the planning horizon is computed. The number of fueling trucks contracted at any committed yard y_i is determined by the formula $\left\lceil \frac{p_i}{q} \right\rceil$, where p_i is the maximum of the daily fuel dispensed at the committed yard y_i obtained in the above procedure, and q = 25000 gallons is the maximum capacity of each fueling truck.

4.1 Highlights of solution obtained by Algorithm1

The solution consists of 36 committed refueling yards, and the total number of trucks contracted at them is 43. Number of contracted trucks varies from 1 to 3 at different committed yards; number of contracted trucks is 1 at 30 committed yards and 2 at 5 committed yards. The total cost in the solution is \$11.5 million, of which the cost of fuel is \$10.84 million, operating costs of fueling trucks is \$0.34 million and fueling setup cost is \$0.32 million. The utilization of the fueling trucks over the planning horizon varies from a minimum of 3% to 60%. Algorithm1 was coded on C language, which takes about 0.5 seconds for compiling and solution on a 1.6 GHz computer.

Figure 3: A fueling truck refueling a locomotive.

Considering the mileage of each route, and the lowest fuel cost of the yards visited on each route, the total fuel cost in any solution is guaranteed to be ≥ \$10.7 million. The total fuel cost in the solution obtained in Algorithm1 is \$10.75 million. The fuel cost is 94% of the total cost in our solution. From this we can conclude that the solution obtained by Algorithm1 is nearly optimal.

5. **Description of the greedy algorithm, Algorithm2, using *GC2* as the greediness criterion**

Partitioning of set of routes into various categories of routes is the same as in Algorithm1, Section 4.

The aim of Algorithm2 is to minimize the total cost (cost of fuel dispensed +refueling setup cost + cost of contracting fuel trucks) as well as ensure maximum utilization (**utilization** is defined as the ratio of the mean amount of fuel dispensed by the truck to its maximum daily refueling capacity) of the fuel trucks at the previously committed yards. For example, a route pair involves two yards *P* and *Q*, where two fuel trucks are already committed at yard *P* and the utilization of the truck is only 28% (the trucks dispense 40500 gallons on the 1st day, 36000 gallons on 2nd day, 24192 gallons on 5th day, 24192 gallons on 6th day, 3864 gallons on 7th day, 3864 gallons on 8th day, 24192 gallons on 9th day, 24192 gallons on 10th day, 1625 gallons on 11th day, 5440 gallons on 13th day and 5440 gallons on 14th day of the planning horizon; the average fuel dispensed daily by the two trucks over the planning horizon is 13822 against the maximum daily dispensing capacity of 50000 gallons giving a utilization of 28%). Algorithm2 examines whether it is feasible to meet the refueling demands of the route pair using the existing two fuel trucks at *P* or it is cheaper to commit a fuel truck at *Q*. In case the refueling demands of the route pair is 44100 gallons over the planning horizon (4500 gallons on 1st day,9000 gallons on 2nd day, 4200 gallons on 5th day, 8400 gallons on 6th day, 4200 gallons on 9th day, 8400 gallons on 10th day, 1800 gallons on 13th day and 3600 gallons on 14th day), then refueling demands can be catered by daily dispensing capacity of the existing two fuel trucks at yard *P*; in such a case, it would not make any sense to commit a fuel truck at *Q* unless the fuel cost at *Q* is much lower than *P* (for example, if the fuel cost at Q is cheaper by \$0.20, the fuel cost savings for the 44100 gallons will be \$8820 and the net saving after considering the truck contracting cost of \$8000 would be \$820 by deciding to commit a fuel truck at Q) . Algorithm2 thus differs from Algorithm1, where the aim was to choose yards with cheapest fuel cost in order to minimize the cost of fuel alone.

This algorithm differs from Algorithm1 of Section 4 in the greediness criterion used for selecting refueling yards for various routes. Also in this algorithm, in contrast to Algorithm1, since the selection of the refueling yard on each route is based on the greediness criterion *GC2* defined in Section 3, it requires determining the amount of fuel to be dispensed at each yard on each day of the planning horizon on that route before the selection can be made. In Algorithm2 also, refueling yards are selected for route pairs one at a time sequentially, in the order R_{11}, R_{12}, R_{21} and R_{22}. At any stage, let T denote the set of committed refueling yards selected up to that stage.

For Category R_{11} the procedure for selection of refueling yards for routes is the same as in Algorithm1. Since the locomotive has to be refueled to full capacity at the origin yard of each route in R_{11}, T will be the set of all yards on routes in R_{11}.

Then prepare a two-way table G of fuel requirements at yards y in T on day $j \in \{1….14\}$ in the planning horizon, where entry g_{yj} in this table is the fuel dispensed at yard y on day j for the routes processed already. Prepare a new column $H= \ (h(y): \ y \in \{1,2,….,74\}$ = set of all yards) where the entry *h(y)* = cost of all fuel dispensed at this yard up to this stage +refueling setup cost + cost of contracting fuel trucks needed at this yard *y* to dispense this fuel over the planning horizon.

For Category R_{12}, order the route pairs in it in decreasing order of round-trip mileage; we will process the routes in this order.

We will select the refueling yard for each route pair *r* in R_{12} by the following procedure. For each of the terminal (i.e., origin or destination) yards *y* on this route, define $h_2(y)$ = the fuel + refueling setup cost +truck contracting cost of dispensing all the fuel commitments at this yard *y* for the route pair *r* under consideration. Choose the terminal yard *y* corresponding to the minimum value of $h_2(y)$ as the refueling yard for this route pair and update the column *H* and set *T* accordingly. A similar principle is used in processing routes in R_{21}, R_{22}.

For the case study, there are no R_{11} routes; thus *h(y)*=0 for all yards and T=∅. Hence we start with R_{12} category routes, ordering the route pairs in decreasing order of round-trip mileage. First, we consider the route pair (Y45,Y17) and (Y17,Y45) with 1200 round-trip miles corresponding to trains T64 and T65; the fuel cost is \$2.96 and \$3.16 at Y17 and Y45 respectively. Since $h_2(45)$=\$102906 and $h_2(17)$=\$97026, Y17 is chosen as the refueling yard for this route pair; thus *h(*17*)* is now updated to \$97026 and T={Y17}. Continuing in this manner, we obtain T={Y1, Y2, Y13, Y15, Y17, Y22, Y23,

Y24, Y25, Y30, Y32, Y34, Y41, Y44, Y48, Y49, Y50, Y57, Y60, Y64, Y66} for all the R_{12} category route pairs. R_{22} category route pairs are handled similarly.

Highlights of solution obtained by Algorithm2: The solution consists of 29 committed refueling yards and total number of trucks contracted at them is 37 (number of contracted trucks varies from 1 to 3 at different committed yards; number of contracted trucks is 1 at 31 committed yards and 2 at 4 committed yards). The total cost in the solution is $11.7 million, of which the cost of fuel is $11.07 million, operating costs of fueling trucks is $0.3 million, and fueling setup cost is $0.35 million. The utilization of the fueling trucks over the planning horizon varies from a minimum of 5% to 63%. Algorithm2 was coded on C language, which takes about 1.5 seconds for compiling and solution on a 1.6 GHz computer.

6. **The Mixed Integer Programming (MIP) Model for the case study problem, and its solution obtained using CPLEX software**

There are many different ways to model this problem using an MIP model. The main decisions to be made in this problem are: the set of yards which will serve as refueling yards, the yard(s) where each locomotive will refuel, the days on which this refueling will take place and the quantity of fuel loaded at each refueling, and the number of fuel trucks contracted at each refueling station. The problem of making all these decisions to minimize the total cost = fuel cost + setup costs for refueling + truck contracting costs can be modeled as an MIP (mixed integer programming) model, but it leads to a model with many 0-1 variables, and may take a long time to solve.

Thus MIP model can be simplified by using the partition of the 214 routes for the trains into the three categories R_{12}, R_{21} and R_{22} discussed above; and using the information:

- each locomotive serving the routes in category R_{12} needs to be refueled only periodically as discussed in the previous section,
- each locomotive serving routes in category R_{21} needs to be refueled only once on each round trip,
- each locomotive serving routes in the category R_{22} needs to be refueled only once in each direction of each roundtrip, and
- Each locomotive is filled to capacity at each refueling.

Also we notice that the fuel cost is the major element in the total cost; and all the other costs put together are only a small fraction of the fuel cost. If we take the fuel cost as the major part of the objective function to minimize, the model becomes even much simpler.

The solutions obtained in Algorithms 1, 2 had 36, 37 committed yards with 30, 31 among them having only one fueling truck contracted at them. So, it seems that we can approximate the total truck contracting cost at an optimum solution by 8000(number of committed yards in the solution). Using this, we consider the simpler problem of minimizing the [fuel costs + 8000(number of committed yards)] subject to all the constraints.

Here is the MIP model for the simplified version of this problem. In the following when we talk about “refueling on a route *p*”, it refers to “refueling of a locomotive serving route *p*”. Since each route pair is served by a unique pair of locomotives, given the planning horizon of 14 days, we conclude that both locomotives serving any route pair, will log equal distance by the end.

Using the fueling plan and the given information on yards visited on each route on each day of the planning horizon, we determine the (a) the days and the amount of fuel loaded at each yard used as refueling point for each train, (b) the number of fueling trucks contracted at each yard, (c) and the amount of fuel(in gallons) in each locomotive tank at the beginning of the planning horizon.

Indices used:

p: this index is used for routes, p=1 to 214

j: this index is used for yards, j =1 to 73

k: this index is used for routes in the forward or reverse direction between a pair of terminal yards for trains in R_{22} category, k=1,2

c: this index is used for feasible couples for trains in R_{22} category

Notation for data elements:

S_p: set of yards on route p

C_p: set of feasible couples on route p in R_{22} category; for example, there are 14 feasible couples for the locomotive serving route pairs (Y23,Y28,Y29,Y3,Y2,Y16,Y43) and (Y43,Y16,Y11, Y2,Y3, Y29, Y28, Y23): (Y23,Y43), (Y23,Y16), (Y28,Y43), (Y28,Y16), (Y28,Y11), (Y29,Y16), (Y29,Y11), (Y29,Y2), (Y3,Y2), (Y3,Y3), (Y2,Y3), (Y2,Y29), (Y16,Y29), (Y16,Y28)

F_{pc}: set of yards in feasible couple c on route p in R_{22} category (for example, (Y23, Y43) is the set of yards in feasible couple c=1)

t_j : fuel cost at yard j in terms of \$/mile= (\$/gallon)(3.5 gallons/mile)

m_p: total distance covered by the locomotives serving route p in entire 2-week planning horizon

Decision Variables: All the decision variables are binary:

$y12_{pj}$ =1, if refueling for category R_{12} route p is carried out at yard j
=0, otherwise
$y21_{pj}$ = 1, if refueling for category R_{21} route p is carried out at yard j
= 0, otherwise
$y22_{pjk}$ =1, if refueling for category R_{22} route p in direction k is carried out at feasible yard j
= 0, otherwise
$y22_{pcj}$= 1, if the yard j in a feasible couple c refuels category R_{22} route p
= 0, otherwise
w_{pc}=1, if refueling for category R_{22} route p is carried out at yards in feasible couple c
=0, otherwise
x_j = 1, if refueling trucks are committed at yard j
=0, otherwise

The Objective Function to minimize is:

Minimize $\sum_{p\in R12, j\in S_p} y12_{pj}t_j m_p + \sum_{p\in R21, j\in S_p} y21_{pj}t_j m_p + \sum_{p\in R22, c\in C_p} \sum_{j\epsilon F_{pc}} (w_{pc}y22_{pcj}t_j m_p) + \sum_{j=1}^{73} 8000x_j$

Subject to the Constraints:

(1) $\sum_{j\in s_p} y12_{pj} = 1, for\ all\ p \in R12$

(2) $y12_{pj} \leq x_j\ , for\ all\ p \in R12, j \in S_p$

(3) $\sum_{j\in s_p} y21_{pj} = 1, for\ all\ p \in R21$

(4) $y21_{pj} \leq x_j\ , for\ all\ p \in R21, j \in S_p$

(5) $\sum_{j\epsilon F_{pc}} y22_{pcj} \leq w_{pc} + 1, for\ all\ p\epsilon R22\ , c\epsilon C_p$

(6) $\sum_{c\epsilon C_p} w_{pc} = 1, for\ all\ p\epsilon R22$

(7) $\sum_{c\epsilon C_p} \sum_{j\epsilon F_{pc}} (w_{pc}y22_{pcj}) = 2, for\ all\ p\epsilon R22$

(8) $y22_{pcj} \leq x_j\ , for\ all\ p \in R22, c\epsilon C_p, j\epsilon F_{pc}$

Explanation of the constraints: Constraint (1) comes from the fact that each locomotive serving on a route in category R_{12} is to be refueled at one of the two terminal nodes between which the locomotive goes back and forth, that refueling trucks are to be committed at that yard for refueling. The binary variable $y12_{pj}$ decides the yard where refueling of the particular route *p* will take place even though refueling trucks may be located at both the terminal nodes. Constraint (2) ensures that refueling truck is actually available at the chosen terminal node. Constraints (3) and (4) come from the corresponding facts for locomotives serving routes in category R_{21}. Constraints (5), (6) and (7) ensures that for category R_{22} only a feasible couple of yards as defined in Section 4 is chosen as a pair of refueling stations for each route pair in R_{22}. Constraint (8) is similar to earlier constraints (2) and (4).

Highlights of solution obtained by MIP Model: The solution consists of 29 committed refueling yards and total number of trucks contracted at them is 40. Number of contracted trucks varies from 1 to 3 at different committed yards. The total cost in the solution is $11.56 million(which is only 0.5% higher than that obtained by Algorithm1), of which the cost of fuel is $10.92 million, operating costs of fueling trucks is $0.32 million and fueling setup cost is $0.32 million.

The utilization of the fueling trucks over the planning horizon varies from a minimum of 3 to 65%. The MIP model takes about 3 seconds for processing and solution using IBM ILOG CPLEX 12.1.0 on a 1.6 GHz computer.

7. Summary

As we have seen from the results obtained in Sections 5 & 6, the results obtained in the greedy method depend on the greediness criteria used for making the decisions. Here is a summary for comparing all three methods:

	Greedy Method with *GC1*	Greedy Method with *GC2*	MIP Model
No. of yards where refueling trucks are committed in the optimal solution	36	29	29

Total no. of trucks contracted in the optimal solution	43	37	40
Total truck contracting cost in the optimal solution(million $)	0.34	0.30	0.32
Total Fuel Cost in the optimal solution (million $)	10.84	11.07	10.92
Total refueling setup cost in the optimal solution(million $)	0.32	0.35	0.32
Total cost in the optimal solution (million $)	11.50	11.71	11.56

The solution obtained using Algorithm1 with *GC1*, is comparable to that obtained from the MIP model. Also Algorithm1 is very simple and easy to implement, and scales directly to large scale problems in real world applications.

The solution obtained using Algorithm1 is comparable to the solutions obtained by the three winners of the INFORMS-RAS competition[4]. Mor Kaspi and Tal Raviv of Tel-Aviv University obtained a total cost of 11.4 million using a MILP formulation (with specialized cuts and domination rules). Cristian Figueroa and Virot Chiraphadhanakul of MIT's Operations Research Center obtained a total cost of 11.4 million using a MIP model (with Lagrangean relaxation sub-models). Ed Ramsden of Lattice Semiconductor Corporation obtained a total cost of 11.5 million using a combination of heuristics and stochastic search algorithms.

In the case study problem, the greedy method based on the greediness criterion *GC1* gives better results than that based on *GC2*. Since fuel cost is over 94% of the overall costs in the optimal solution, *GC1* based on fuel cost as greediness criteria works better than *GC2*. Also since the case study problem is a realistic example of real world problems, we can expect the similar performance in general in practice.

In real world applications it is very likely that there will be more trains on tracks, and consequently most contracted fueling trucks tend to be used to full capacity; and the difference in the total costs associated with *GC1* and the MIP-based technique similar to the one discussed in Section 6 may not be much. The greedy method based on either the greediness criteria *GC1* or *GC2* are very easy to implement, do not need software packages like CPLEX, and take very little CPU time.

In most real world applications, partitioning the trains into categories like *R11*, *R12*, *R21*, *R22* like in the case study example; and identifying sections of the track where the locomotive of the train will undergo refueling possibly based on practical logistic considerations, may in fact be the preferred option of railroad managements. Then, if the company does not have access to an MIP software, the greedy method based on *GC1* may be the preferred approach for solving the problem. On the other hand if the company has access to MIP software, a model like the one in Section 6 can be used to model and solve this problem as explained in Chapter 3 of [2].

8. A Practical Exercise

Consider the same problem discussed in this chapter. Assume that all the data remains the same, except the fuel prices at the various yards, which have changed. The new prices at yards 1 to 73 are (2.93, 2.96, 2.99, 3.02, 3.05, 3.08, 3.11, 3.14, 3.17, 3.20, 3.23, 3.26, 3.29, 3.32, 3.35, 3.38, 3.41, 3.44, 3.47, 3.50, 3.56, 2.90, 2.93, 2.96, 2.99, 3.02, 3.05, 3.08, 3.11, 3.14, 3.17, 3.20, 3.23, 3.26, 3.29, 3.32, 3.35, 3.38, 3.41, 3.44, 3.47, 3.50, 3.53, 2.90, 2.93, 2.96, 2.99, 3.02, 3.05, 3.08, 3.11, 3.14, 3.17, 3.20, 3.23, 3.26, 3.29, 3.32, 3.35, 3.38, 3.41, 3.44, 3.47, 3.50, 3.53, 2.90, 2.93, 2.96, 2.99, 3.02, 3.05, 3.08, 3.11) respectively in that order. Solve the modified problem with this new data and discuss the output obtained.

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9.References:

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