12.1

Project Scheduling by Critical Path Method (CPM)

Katta G. Murty Lecture slides

CPM is application of DP to problems in Construction (project) Management.

Many large construction projects going on around world.

The recently completed Hong Kong Airport (Chek Lap Kok) and associated rail and bridge work, > US $ 10 Billion

The more than US $ 1 billion Shanghai World Financial Center (95 floors, 460 meters tall, 3.5 million feet^2, expected to be world's tallest building when completed).

CPM is a method for planning, scheduling, and controlling such projects.

Basic Steps in CPM

1 Decompose project into Individual Jobs or Activities.

2 Determine Precedence Relationships among jobs. They come from technological constraints. For example:

- Suspension bridge road units cannot be lifted into place until pillars are prepared.
- "Painting the walls" cannot be started until "erecting walls" completed.

3 Draw the Arrow Diagram, or AOA Diagram, or Project Network representing precedence relationships.

4 Estimate job durations.

5 Use CPM to schedule activities over time to minimize project duration (DP used here).

6 In operation, if any job delayed, use CPM to revise schedule for remaining jobs.

Precedence Relationships

Predecessor or Ancestor	Job 1 is a Predecessor or Ancestor of job 2 if job 2 cannot be started until job 1 completed.
Successor or Descendent	In above job 2 is a Successor or descendent of job 1.
Immediate Predecessor	If job 1 is a predecessor of job 2, and no other job is a predecessor to job 2 and successor to job 1; job 1 is an Immediate Predecessor of job 2
Immediate Successor	If job 1 is an immediate predecessor of job 2, then job 2 is an Immediate Successor of job 1.

A job may have many immediate predecessors, and many immediate successors. It can only be started after all its immediate predecessors have been completed.

Transitivity

Job 1 predecessor of job 2, and job 2 predecessor of job 3 $\Rightarrow$ job 1 predecessor of job 3. This property called Transitivity. Given immediate predecessors of a job, we can obtain all its predecessors by transitivity.

Precedence relationships define a Partial Ordering among jobs.

If transitivity establishes that a job is a predecessor of itself, there is an Inconsistency in the data that should be corrected.

If a distant predecessor of a job is shown as an immediate predecessor, there is Redundancy in the predecessor data, which should be corrected.

Example: Constructing an Office Bldg.

Job	IPs	Duration*
1. Site selection & purchase		30
2. Architectural plans (AP)		45
3. Revisions to AP		30
4. Arrange Financing		60
5. Construct outer frame		90
6. Plans for interior		25
7. Order furnishings (desks, etc.)		120
8. Order Computer equipment		120
9. Computer Network Design		70
10. Interior Construction		70
11. Instal computer equipment		90
12. Move-In		5

*Days

Construct single family home

Job	IPs	Duration*
0. Purchase site		16
1. Prepare blueprint	0	6
2. Excavation	1	4
3. Foundations & Basement walls	2	3
4. Frame	3	5
5. Electrical wiring	4	2
6. Insulate	4	1
7. Roofing	4	2
8. Ductwork & Plumbing	6	4
9. Drywall installation	5, 7, 8	6
10. Instal siding	7	4
11. Exterior painting	10	3
12. Interior painting	9	4
13. Carpeting	12	2
14.Instal interior fixtures	13	7
15. Instal Exterior fixtures	11	2
16. Move-in	14, 15	1

*weeks

Hydroelectric Power Station Building Project

No.	Job	IPs	Duration*
1.	Site Ecological survey		6.2
2.	Get EPA approval	1	9.1
3.	Economic feasibility study	1	7.3
4.	Prel. design and cost est.	3	4.2
5.	Project approval	2, 4	10.2
6.	Call quotations for elec. equip. (turbines, generators, ...)	5	4.3
7.	Select suppliers for elec. equip.	6	3.1
8.	Final design	5	6.5
9.	Select construction contractors	5	2.7
10.	Arrange materials supply	8, 9	5.2
11.	Dam building	10	24.8
12.	Power station building	10	18.4
13.	Power lines erection	7, 8	20.3
14.	Elec. equip. installation	7, 12	6.8
15.	Build up reservoir water level	11	2.1
16.	Commission the generators	14, 15	1.2
17.	Start supplying power[182]	13, 16	1.1

*months

Project Networks

2 ways to represent predecessor relationships in network form.

AON (Activity On Node) network easy to draw, not very useful.

We discuss Activity On Arc (AOA) diagram or Arrow diagram. Here, jobs correspond to arcs. Each node corresponds to an Event, "jobs corresponding to all arcs incident into it, and all their predecessors, have been completed".

To portray predecessor relationships correctly in arrow diagram, sometimes necessary to introduce Dummy arcs corresponding to Dummy jobs (which always have 0 duration).

PROPERTIES SATISFIED BY ARROW DIAGRAM:

1 If job 1 is a predecessor of job 2, there will be a directed path in arrow diagram from the head node of job 1 to the tail node of job 2 (i.e. predecessor relationship represented through directed paths).

2 If job 1 is an immediate predecessor of job 2,

Either head node of job 1 is tail node of job 2

Or there is a directed path from head node of job 1 to the tail node of job 2 consisting of dummy arcs only.

Need For Dummy Arcs

Dummy arcs necessary whenever there is a subset of jobs A that have some but not all IPs in common. In this case we let arcs corresponding to common IPs of jobs in A to have same head node, and then add dummy arcs from that node to the tail nodes of arcs corresponding to jobs in A.

Example

Job	IPs
e_1	
e_2	
e_3	
e_4	
e_5	e_4
e_6	e_1, e_2, e_5
e_7	e_2, e_3, e_5

To avoid confusion, we follow convention that there should be no parallel arcs. So, if there are several jobs with same set of IPs, and same set of immediate successors, we represent them using dummy arcs.

How to Draw Arrow Diagram?

Put a Start node representing event of commencing project. Represent each job with no predecessor by an arc incident out of this start node.

Same way, at end, represent all jobs with no successors by arcs incident into a single final node Finish Node representing event of project completion.

If a job b has a single IP a, then b can be represented by an arc incident out of the head node of a.

If a job b has more than one IP, let $p_1, \ldots, p_r$ be the head nodes of IPs of job b:

either represent b by an arc incident out of one of the nodes

$p_1, \ldots, p_r$, say p_1, and include dummy arcs from $p_2, \ldots, p_r$ into p_1.

or represent b by an arc incident out of a new node Q and include dummy arcs $(p_1, Q), \ldots, (p_r, Q)$.

If some jobs have identical sets of immediate successors, make the head node of arcs representing them the same.

If several jobs have some common IPs, represent these common IPs by arcs with the same head node, and include dummy arcs coming out of this node as necessary. Check:

- Precedence relationships not implied by original data not introduced.
- Precedence relationships implied by original data not omitted.
- See if no. of nodes & no. of dummy nodes can be reduced. For example, if there is a node with a single arc (which is a dummy arc) incident at it, then the 2 nodes on this dummy arc can be merged, and that dummy arc eliminated.

Since no job can be its own predecessor, project network cannot contain any directed cycles, so project network must be an acyclic network.

Acyclic Networks

A directed network that contains no circuits (directed cycles):

A circuit or directed cycle

A cycle that is not a circuit.

Theorem: A Directed network acyclic iff nodes can be numbered s. th. on each arc

number of tail < number of head

Such numbering of nodes called **acyclic numbering** or **topological ordering**.

How to check if network acyclic, & find acyclic numbering of its nodes?

1. Number all nodes with no arcs incident into them, serially beginning with 1.

Strike off all these nodes & arcs incident at them.

2. Now continue numbering all nodes with no arcs that are not struck off, incident into them, serially.

Strike off all newly numbered nodes & arcs incident at them.

3. Repeat Step 2 until either:

all nodes numbered (in this case network acyclic, you have acyclic numbering, terminate)

or some unnumbered nodes, but every one has an arc that is not struck off, incident into it (in this case, network not acyclic, terminate).

Examples

Project Scheduling

Find acyclic numbering of nodes in project network. Node 1 = start node, node n = finish node.

To lay out jobs on time axis to minimize project duration. To answer questions like:

- Start time = 0. How soon can project be completed?
- How early would event corresp. to each node materialize?
- What is earliest time at which each job can be started (called Early Start Time of Job)?
- For each job, what is latest time at which it can be started, without delaying project completion?
- Etc.

The Data

For each arc (i, j) in the project network:

$$\begin{aligned} t_{ij} &= \text{Duration of job represented by arc } (i, j) \\ &= \text{Length of arc } (i, j) \end{aligned}$$

Earliest Event Times

Define:

$$t_i = \text{ Length of longest chain from node 1 to } i.$$

Clearly t_i = earliest time at which event corresponding to node i can materialize.

t_n = length of longest chain from start node 1 to finish node n = Minimum Project Duration.

A longest chain from node 1 to node n is called a Critical Path. There may be several critical paths.

Arcs on critical paths called Critical arcs, they represent Critical jobs.

To complete project in min. time, every critical job should be started at earliest time. Any delay in a critical job delays project completion.

Jobs not on any critical path called Slack jobs.

Early start time of job $(i,j) = t_i$

Early Finish Time of job $(i,j) = t_i + t_{ij}$.

Forward pass DP Algorithm to find Early Start Times t_is

Data Structures

The Forward label on node i will be (L_i, t_i) where L_i called Predecessor Index of node i, is the previous node to i on a longest chain from 1 to i.

Boundary Conditions: $t_1 = 0$. Forward label node 1 with $(\emptyset, 0)$. This label indicates it is Origin node.

In this pass nodes labeled in the order 1 to n. The recurrence eq. for node r is:

$$t_r = \max\{t_i + t_{ir} : \quad i \text{ s. th. } (i,r) \text{ is an arc}\}$$

L_r in the label for node r is the i that attains the maximum in the above eq.

The longest path from node 1 to i can be traced using the predecessor indices backwards beginning with node i.

The longest path from 1 to n is one critical path.

To find all critical jobs & Latest Start Times:

The forward pass gives only one critical path. To find all critical arcs, we define:

$\mu_i =$ Latest point of time at which the event corresponding to node i has to materialize for the project to finish in minimum duration.

$\mu_n = t_n$ (Boundary Condition)

$\mu_i = \mu_n -$ length of the longest chain from node i to node n.

The Backward Pass

This pass determines the μ_i by DP. They are determined in the order $i = n, n-1, \ldots, 2, 1$.

$\mu_n = t_n$ Boundary condition.

$\mu_i = \min\{\mu_j - t_{ij}: \quad j \text{ s. th. } (i,j) \text{ is an arc}\}$

The Late Finish of job $(i,j) = \mu_j$

The Late Staret of job $(i,j) = \mu_j - t_{ij}$

The Total Slack of job (i,j) = Late start - early start = $\mu_j - t_{ij} - t_i$.

Job (i,j) is critical iff its total slack $= 0$.

HOMEWORK PROBLEMS:

1.Draw the arrow diagram and determine the early and late start and finish times of the various jobs in the following projects (A. Kanda and N. Singh [1988]).

Project: Setting Up a Fossil Fuel Power Plant

No.	Job	IPs	Job duration (months)
1.	Land acquisition		6
2.	Identi. trained personnel	1	3
3.	Land dev. & infrastructure	1	2
4.	Control room eng.	1	12
5.	Lag in turbine civil works	1	8
6.	Delivery of TG	1	12
7.	Delivery of boiler	1	10
8.	Joining time for personnel	2	3
9.	Boiler prel. civil works	3	2
10.	Control room civil works	4	5
11.	TG civil works	5	9
12.	Training	8	6
13.	Boiler final civil works	9	9
14.	Erection of control room	10	8
15.	Erection of TG	6, 11	10
16.	Boiler erection	7, 13	12
17.	Hydraulic test	16	2
18.	Boiler light up	14, 17	1.5
19.	Box up of turbine	15	3
20.	Steam blowing, safety valve floating	18, 19	2.5
21.	Turbine rolling	20	1.5
22.	Trial run	21	1
23.	Synchronization	22	1

IP = Immediate predecessors

2. Draw an arrow diagram for each of the following projects. Prepare a schedule for the various jobs in each project to complete it in minimum time. (R. Visweswara Rao).

(a) Sewer and Waste System Design for a Power Plant

No.	Activity	IPs	Duration (days)
1.	Collection system outline		40
2..	Final design & approval	1	30
3.	Issue construction drawings	2	30
4.	Get sewer pipe & manholes	1	145
5.	Fabricate & ship	3,4	45
6.	Treat. system drawings & approval		70
7.	Issue treat. system construction drawings	6	30
8.	Award contract	7	60
9.	Final construction	8, 5	300

IP = Immediate predecessors

(b) Data Process and Collection System Design for a Power Plant

No.	Activity	IPs	Duration (days)
1.	Prel. Syst. description		40
2.	Develop specs.	1	100
3.	Client approval & place order	2	50

contd.

Data Process and Collection System Design contd.

No.	Activity	IPs	Duration (days)
4.	Develop I/O summary	2	60
5.	Develop alarm list	4	40
6.	Develop log formats	3, 5	40
7.	Software def.	3	35
8.	Hardware requirements	3	35
9.	Finalize I/O summary	5, 6	60
10.	Anal. performance calculation	9	70
11.	Auto. turbine startup anal.	9	60
12.	Boiler guides anal.	9	30
13.	Fabricate & ship	10, 11, 12	400
14.	Software preparation	7, 10, 11	80
15.	Install & check	13, 14	130
16.	Termination & wiring lists	9	30
17.	Schematic wiring lists	16	60
18.	Pulling & term. of cables	15, 17	60
19.	Operational test	18	125
20.	First firing	19	1

IP = Immediate predecessors

(c) Electrical Auxiliary System Design for a Nuclear Plant

No.	Activity	IPs	Duration (days)
1.	Aux. load list		120
2.	13.8 switchgear load ident.	1	190
3.	4.16kv & 480 v. switchgear load ident.	1	45
4.	Vital AC load determination	1	300
5.	DC load determ.	1	165
6.	Voltage drop study	2	84
7.	Diesel gen. sizing	3	77
8.	Inventer sizing	4	20
9.	Battery sizing	5, 8	40
10.	DC fault study	9	80
11.	Prel. AC fault current study	6, 7	20
12.	Power transformer sizing	2, 11	80
13.	Composite oneline diagram	2,3	72
14.	Safety (class 1E) system design	13	200
15.	Non-class 1E system design	13	190
16.	Relaying oneline & metering dia.	13	80
17.	3-line diagram	14, 15, 16	150
18.	Synchronizing & phasing diagrams	17	100
19.	Client review	10, 18	25
20.	Equipment purchase & installation	19	800