MATH 632

Algebraic geometry II: Cohomology on algebraic varieties

• TuTh 11:30 am-1:00 pm, EH 3096

• Course webpage: */www-personal.umich.edu/~mmustata/632-2018.html*

• Office hours: Tuesdays 3-4 PM and by appointment

• This is the second part of a two-semester course giving an introduction to algebraic geometry. The goal in this semester is to introduce more advanced topics, such as a systematic use of sheaves and their cohomology, and discuss geometric applications. Time permitting, we might discuss other topics, such as transcendental methods in the study of complex algebraic varieties.

• Prerequisites: I will assume that everybody is familiar with the topics we covered in the first semester. Lecture notes for this first part of the course are available at

www-personal.umich.edu/~mmustata/ag-1213-2017.pdf

We will make use of slightly more advanced topics in commutative algebra than those we needed in the first semester. Some of these (associated primes, primary decomposition, completion) are usually covered in a first commutative algebra course, at the level of Atiyah-Macdonald. For this material I will prepare some handouts that should suffice for people who did not see these things before. We will discuss some more advanced material in class, after introducing cohomological techniques.

• I plan to run this semester as well a one-hour weekly problem session (day and time to be decided).

• There will be weekly homework assignments.

• Grading policy: the grade will be based on homework (70%) and on a final take-home exam (30%).

• Here is a rough outline of the course, though how much of this we will be able to cover will depend on the time available.

1) Coherent sheaves on algebraic varieties (definition and basic properties, operations with coherent sheaves, vector bundles, the global MaxSpec and MaxProj constructions, the cotangent sheaf)
2) Divisors on algebraic varieties (normal varieties, Weil and Cartier divisors, linear series, the class group and the Picard group, regular local rings are UFDs)
3) General facts about cohomology of sheaves (rudiments about the derived category, cohomological functors, singular cohomology as sheaf cohomology, cohomology of manifolds via differential forms: the de Rham theorem)
4) Cohomology of coherent sheaves (cohomology on affine varieties, Cech cohomology, coherent sheaves and cohomology on projective varieties)

5) Ample line bundles (morphisms to projective spaces, ample and very ample line bundles, Hilbert polynomials, numerical intersection theory)
6) An interlude: some local algebra (regular sequences, depth, Cohen-Macaulay rings, projective dimension, the local criterion for flatness). Exactly how much of this we will prove, will depend on the available time
7) Advanced topics in cohomology (Serre duality, the theorem on formal functions, the base-change theorems). Most likely, we will not go into details about all the proofs.
8) Curve theory (the Riemann-Roch theorem and applications, the Riemann-Hurwitz theorem and applications, embedding of curves in the projectve space)
9) Complex algebraic varieties (the classical topology, holomorphic functions, Serre's GAGA comparison theorem)
10) Introduction to Hodge theory