Problem session 6

Problem 1. Show that if X is an irreducible variety, that is smooth in codimension 1, then

$$\mathrm{Cl}(X \times \mathbb{P}^n) \simeq \mathrm{Cl}(X) \oplus \mathbb{Z}.$$

Problem 2. Let X be an algebraic variety and U an open subvariety. Prove the following assertions:

i) If $\mathcal{F}$ is a coherent sheaf on U, then there is a coherent sheaf $\mathcal{G}$ on X such that $\mathcal{G}|_U \simeq \mathcal{F}$.

ii) Furthermore, if $\mathcal{M}$ is a quasi-coherent sheaf on X such that $\mathcal{F}$ is a subsheaf of $\mathcal{M}|_U$, then we may take $\mathcal{G}$ to be a subsheaf of $\mathcal{M}$.

Hint: consider the following intermediate steps:

a) Show that if X is an *affine* variety and $\mathcal{P}$ is a quasicoherent sheaf on X, and if $(\mathcal{P}_i)_i$ is the family of coherent subsheaves of $\mathcal{P}$, then for every open subset U of X we have $\mathcal{P}(U) = \bigcup_i \mathcal{P}_i(U)$.

b) Prove i) and ii) above when X is affine.

c) Prove ii), and then i) above in the general case.