Chapter 1

The Neomoneatarist KE-LM Model

The Basic Neomoneatarist Model is an attempt to find a counterpart in modern, fully dynamic theory to the old-fashioned static IS-LM model. For readers familiar with the traditional static IS-LM model, the Basic Neomoneatarist model offers some surprises. In many ways it is similar to what Tobin and Sargent called the Dynamic Aggregative Model, except that it has sticky prices instead of sticky wages.

The distinction between long-run and short-run responses is a familiar one. Mathematically, the language of short-run/long-run distinctions corresponds to approximating different speeds of adjustment for various dimensions of a model by a hierarchy of different time scales. In understanding the Basic Neomoneatarist it is helpful to have in mind a hierarchy of different time scales with even more levels. From fastest to slowest (according to our assumptions) these are

1. almost instantaneous adjustment of asset markets (putting the economy on the LM curve),
2. ultra-short-run adjustment of output (toward short-run equilibrium at the intersection of KE and LM)
3. short-run adjustment of prices (toward medium-run equilibrium at full employment),
4. medium-run entry and exit of firms and adjustment of the capital stock (toward long-run equilibrium on the steady state growth path of the
economy), and

5. long-run growth.

In this chapter we will formally model only the third level of “short-run” dynamics, discussing the other levels in an informal way.

1.1 The LM Curve

1.1.1 Money Demand

Let \( P \) be the aggregate price level, \( Y \) the aggregate level of output, \( M \) the nominal money supply and let \( V \) the autonomous component of velocity (the part of velocity not determined by output or the interest rate). For these four variables, use the lower-case letters \( p, y, m \) and \( v \) to represent the natural logarithms of upper-case letters. Based on the underlying equation \( MV/P = Y h L(r + \pi) \), which allows for any constant elasticity \( h \) of money demand with respect to aggregate output and a general dependence of money demand on the nominal interest rate \( r + \pi \) generated by the monotonically decreasing function \( L(\cdot) \), money demand can be written in logarithmic form as

\[
m + v - p = hy + \ln(L(r + \pi)).
\] (1.1)

As long as the nominal interest rate is not changing too dramatically, (1.1) can be approximated by

\[
m + v - p = hy - \ell \cdot (r + \pi)
\] (1.2)

where

\[
\ell = \frac{-L'(r^* + \pi^*)}{L(r^* + \pi^*)} > 0
\]

is the interest semi-elasticity of money demand at an average or typical level for the nominal interest rate and \( V \) has been appropriately normalized to absorb the constant term. If the interest elasticity of money demand

\[
\frac{-(r + \pi)L'(r + \pi)}{L(r + \pi)}
\]

were a constant, as it is, for example, in the Baumol-Tobin case, then the interest semi-elasticity of money demand at \( r^* + \pi^* \) would be
1.1. THE LM CURVE

\[ \ell = \frac{\text{constant}}{r^* + \pi^*}. \]

To make a less strong and therefore safer statement, in general, it is likely that the interest semi-elasticity of money demand \( \ell \) is fairly low at high levels of the nominal interest rate—which would be relevant in countries facing high levels of inflation. On the other hand, the interest semi-elasticity of money demand \( \ell \) is likely to be fairly high at low levels of the nominal interest rate—which would be relevant in countries facing low levels of inflation or even deflation. For simplicity, we can log-linearize around \( r^* + \pi^* \), but it will be important at a later stage to consider the effects of different average or typical levels of inflation on the interest semi-elasticity of money demand \( \ell \).

1.1.2 The Dynamics of the Velocity-Adjusted Real Money Supply

Let us give the autonomous-velocity-adjusted log real money supply its own letter, \( x \):

\[ x = m + v - p. \]

Also, define \( \mu \) as the mean growth rate of the autonomous-velocity-adjusted nominal money supply:

\[ \mu = \frac{d}{dt} [m + v] = \frac{dm}{dt} + \frac{dv}{dt} = \dot{m} + \dot{v}, \]

where a dot is used to represent a time derivative. For brevity, we will sometimes drop the adjective “autonomous-velocity-adjusted,” and refer to \( x \) as “real money balances” or as the “real money supply” and to \( \mu \) as the “growth rate of the (nominal) money supply.”

By definition, the inflation rate \( \pi \) is

\[ \pi = \dot{p}. \]

Given these definitions,

\[ \dot{x} = \mu - \pi. \quad (1.3) \]

A phase diagram is a useful tool for understanding dynamics because it shows how rates of change in key variables relate to the current levels of those
variables at any time. The phase diagram for the Basic Neomonetarist model can be drawn with the autonomous-velocity-adjusted log real money supply \( m + v - p \) on the horizontal axis and inflation \( \pi \) on the vertical axis. On the phase diagram, the \( \dot{x} = 0 \) locus is the horizontal line \( \pi = \mu \). Above the line, \( \dot{x} < 0 \). Below the line, \( \dot{x} > 0 \). (See the leftward and rightward arrows in Figure 1, indicating these dynamics.)

1.1.3 The LM Curve and the Phase Diagram

The LM curve itself belongs on a graph with output \( y \) on the horizontal axis and the real interest rate \( r \) on the vertical axis as in Figure 2. Rearranging (1.2),

\[
r = \frac{h y - x}{\ell} - \pi.
\]

The slope of the LM curve is the coefficient of \( y \), \( \frac{h}{\ell} \), which is always positive. Increases in the real autonomous-velocity-adjusted money supply \( x \) shift the LM curve outward and down. For example, a .01 increase in \( x \), which represents a 1% increase in the autonomous-velocity-adjusted real money supply, shifts the LM curve outward by \( \frac{1}{h}(.01) \), which represents a \( \frac{1}{h}\% \) higher level of output for a given real interest rate. Finally, an increase in inflation \( \pi \) shifts the LM curve down in a one-for-one fashion: an increase in inflation by one percentage point (say from 4% per year to 5% per year) forces the real interest rate at a given level of output to be one percentage point lower (say 1% per year instead of 2% per year).

Notice that the two variables that shift the LM curve—\( x \) and \( \pi \) are the two variables on the phase diagram. Thus, a point on the phase diagram indicates a specific position for the LM curve. To be specific, the location of the LM curve in \( y-r \) space is determined by the value of \( \frac{x}{\ell} + \pi \) in \( x-\pi \) space. The curves \( \frac{x}{\ell} + \pi = \text{constant} \) on the phase diagram (that is, curves with slope \(-\frac{1}{h}\)) represent a fixed position of the LM curve. (See Figure 3.) Rightward and upward movement in the phase diagram represents an outward shift of the LM curve, while leftward and downward movement in the phase diagram represents an inward shift of the LM curve.

1.1.4 The LM Curve and Other Monetary Rules

The LM curve is the right tool of analysis when the default policy for the central bank (the Federal Reserve in the U.S.) is to keep the autonomous-velocity-adjusted nominal money supply growing at a constant rate, with some
1.1. THE LM CURVE

tinkering. But if the central bank follows some other policy on a day-to-day basis, it is important to build that rule into the analysis in a more fundamental way.

Real-life central bankers often think of themselves as setting the nominal interest rate. A reasonable, fairly general specification for a monetary rule is for the central bank to set the nominal interest rate \( r + \pi \) by

\[
r + \pi = a + by + g\pi
\]

where \( b \) and \( g \) are constant coefficients and \( a \) evolves gradually in a way that tightens monetary policy if inflation is above its target rate and loosens monetary policy if inflation is below its target rate:

\[
\frac{da}{dt} > 0 \quad \text{when } \pi > \pi^*
\]

and

\[
\frac{da}{dt} < 0 \quad \text{when } \pi < \pi^*.
\]

An LM curve generated by a nominal money supply growing at a constant rate fits this description of monetary policy with \( g = 0 \), \( a = -\frac{\pi}{\ell} \), \( \pi^* = \mu \) and \( b = h/\ell \). (For reasonable values of the income and interest rate elasticities of money demand, \( h = 1 \) and \( \ell = 1 \) year, \( b = h/\ell \) might be on the order of 1/year.) As an alternative, the Taylor rule fits this description of monetary policy reasonably well with \( g = 1.5 \) and \( b \) (also) on the order of 1/year. The key difference between an LM curve and the Taylor rule becomes clear when one focuses on the determination of the real interest rate. The LM curve implies that

\[
r = a + by - \pi
\]

while the Taylor rule implies

\[
r = a + by + .5\pi.
\]

Thus, when inflation falls temporarily as a result of a technological improvement, an LM curve would shift up, while the Taylor rule curve would shift down. In between these two cases is a real interest rate rule given by

\[
r = a + by.
\]

Although the analysis in the rest of this chapter will continue to use an LM curve as the monetary rule, it is good to keep in mind the alternatives.
1.2 Aggregate Supply and the Net Rental Rate (NRR or KE) Curve

In the absence of investment adjustment costs, the sticky-price version of Tobin’s KE is the same thing as the Net Rental Rate curve which gives the rate, net of depreciation, at which capital services can be rented for a given level of aggregate output. To emphasize this point, I will call the KE curve the Net Rental Rate (NRR) curve in this chapter.

One of the key features of the Basic Neomonetarist Model is that the location and shape of the Net Rental Rate (NRR) curve—which is the NRR curve in the absence of investment adjustment costs—is intimately related to the details of aggregate supply. Thus, the traditional approach of combining NRR and LM to get aggregate demand and only then bringing in aggregate supply does not work in the Basic Neomonetarist model. Instead, the NRR (=NRR) curve and aggregate supply are joined from the start, almost like two sides of the same coin. It is the combination of NRR and aggregate supply which is then brought together with the LM curve.

1.2.1 Short-Run Aggregate Supply

Both aggregate supply and the net rental curve are based on the technology, preferences and industrial organization of the economy. Medium and long-run aggregate supply are determined as in Real Business Cycle models with some degree of imperfect competition and increasing returns to scale. The actual short-run aggregate supply curve is just the current level of the slowly moving aggregate price—a horizontal line in $y-p$ space. (See Figure 4.) However, the actual (log) aggregate price $p$ need not be the same as the (log) desired price $p^\#$ of a typical firm. The instantaneously optimal desired price of a typical firm is defined as the price that a firm would choose if it never had any costs or barriers to price adjustment, while all the other firms in the economy remained subject to those limitations in their price-setting. The notional short-run aggregate supply curve shown in Figure 4 gives the (log) desired price $p^\#$ as a function of (log) aggregate output $y$:

$$p^\# = p + \beta(y - y^f). \tag{1.4}$$

The quantity $y^f$ is log full-employment output, where full-employment output is defined as what output will be after the price adjustment process is complete. Full employment can also be defined as a situation where marginal revenue equals marginal cost for the typical firm—or equivalently, where
1.2. AGGREGATE SUPPLY AND THE NET RENTAL RATE (NRR OR KE) CURVE

The desired markup (price/[marginal revenue]) is equal to the actual markup (price/[marginal cost]). The slope $\beta$ is the elasticity of the desired price with respect to aggregate output.\(^1\)

### 1.2.2 The Rental Rate of Capital

The net rental rate is simply the gross rental rate of capital $R$ net of the depreciation rate $\delta$. (Note that, contrary to the convention above, $r$, the real interest rate, is not the natural logarithm of $R$, the gross rental rate. The need to subtract the depreciation rate from $R$ makes it awkward to use natural logarithms indiscriminately at this juncture.) Investment demand depends critically on the gap $R - \delta - r$ between the net rental rate and the real interest rate. Indeed, in the absence of investment adjustment costs the elasticity of investment demand with respect to the gap $R - \delta - r$ is infinite, as illustrated by the horizontal investment demand curve in Figure 5. If $R - \delta > r$, investment demand is infinite. If $R - \delta < r$, demand for gross investment is zero. Clearly, infinite investment is impossible in investment market equilibrium (that is, equilibrium in the market for loanable funds). Zero gross investment only occurs in circumstances like those in the Great Depression (discussed later on). In non-depression circumstances, an absence of investment adjustment costs forces the real interest rate to equal the net rental rate in equilibrium:

$$r = R - \delta.$$

Since the depreciation rate $\delta$ is a constant, the rental rate $R$ is the key to the determination of the real interest rate $r$.

If prices were not sticky, the rental rate $R$ could be determined in turn by the marginal revenue product of capital. But with prices sticky, the firm cannot necessarily sell additional output at the temporarily fixed price. Therefore, the rental rate of capital is best seen as determined by the marginal cost product of capital. The marginal cost product of capital is the reduction in the cost of other inputs needed to produce a given level of output resulting

\(^1\)In “The Quantitative Analytics of the Basic Neomonetarist Model,” Kimball (1995) shows that

$$\beta = \frac{\Omega}{\epsilon \omega}$$

where $\epsilon$ is the price elasticity of demand for a typical firm, $\Omega$ is the elasticity of (marginal cost)/(marginal revenue) with respect to expansions of aggregate output and firm output, $\omega$ is the elasticity of (marginal cost)/(marginal revenue) with respect to increases in firm output alone.
from an increase in capital services. In the simplified model here, all of the inputs other than capital are represented by the variable input labor, denoted \( N \). Although the assumption is not necessary, nothing important is lost by assuming a generalized Cobb-Douglas production function for each firm \( i \): 
\[
Y_i = F\left(K_i^{\alpha}(ZN_i)^{1-\alpha}\right),
\]
where \( K_i \) is the amount of capital used by firm \( i \), \( Z \) is the level of labor-augmenting technology (common to all firms), \( \alpha \) is capital’s share in costs and \( 1 - \alpha \) is labor’s share in costs (both common to all firms). Given the assumption of a generalized Cobb-Douglas production function, cost minimization for firm \( i \) to produce output \( \bar{Y}_i \) requires the solution of

\[
\min_{K_i, N_i} RK_i + WN_i
\]

s.t.

\[
K_i^{\alpha}(Z_iN_i)^{1-\alpha} = F^{-1}(\bar{Y}_i)
\]

where \( W \) is the real wage at which labor is available on a competitive spot market.

A standard result for Cobb-Douglas technology, which can be re-derived here using the Lagrange multiplier technique, is that

\[
\frac{RK_i}{WN_i} = \frac{\alpha}{1 - \alpha}.
\]

Rearranging and adding up \( K_i \) and \( N_i \) over all firms to put the equation in terms of the aggregate quantities of capital \( K \) and labor \( N \) leads to

\[
RK = R \sum_i K_i = \frac{\alpha}{1 - \alpha} W \sum_i N_i = \frac{\alpha}{1 - \alpha} WN.
\]

Solving for the rental rate \( R \),

\[
R = \frac{\alpha}{1 - \alpha} \frac{WN}{K}. \tag{1.5}
\]

This means that the real interest rate, equated to the net rental rate, is given by

\[
r = R - \delta = \frac{\alpha}{1 - \alpha} \frac{WN}{K} - \delta.
\]
1.2. AGGREGATE SUPPLY AND THE NET RENTAL RATE (NRR OR KE) CURVE

1.2.3 The Net Rental Rate (NRR) Curve

Since an increase in output raises the demand for capital services, while the supply of capital remains essentially the same (given how much slower capital accumulation is than typical business cycle movements), one would expect an increase in output to lead to an increase in the rental rate $R$. (Indeed, a tendency toward a procyclical rental rate of capital is an almost universal feature of serious business cycle models.) Equation (1.6) confirms the idea that the rental rate will increase with output. Given how slowly the quantity of capital moves, in the short run, in the absence of any change in technology, an increase in output requires an increase in the quantity of labor $N$. In the absence of any change in the representative household’s medium-to-long-run situation, getting the standard representative Neoclassical household to supply a larger quantity of labor requires a higher real wage $W$. (Constancy of the household’s medium-to-long-run situation makes this only a temporary increase in the real wage, and so blocks any substantial wealth or income effect. To be specific, the marginal utility of consumption $\lambda$ is little affected by the short-run movements of the economy. Therefore, the Frisch or marginal-utility-of-consumption-held-constant labor supply curve, which is definitely upward sloping, is the relevant labor supply curve.) With both $N$ and $W$ increasing with output, the rental rate must increase with output.

Changes in technology or in the medium-to-long-run situation of the representative household are real shocks, which will be discussed later. For a given level of technology and a given medium-to-long-run situation of the representative household, the level of output determines the rental rate $R$ by way of $N$ and $W$. Thus, write

$$r = R - \delta = r^f + \phi(y - y^f),$$

(1.6)

where $r$ is the real interest rate, $r^f$ is the full-employment net rental rate, $y - y^f$ is the logarithmic gap between output and full-employment output, and the slope $\phi$ is the semi-elasticity of the real interest rate with respect to the log output gap. Equation (1.6) is depicted as the NRR curve in Figure 6. As noted above, the slope $\phi$ is positive because an increase in output raises the demand for capital services, a fact confirmed by the increased price and quantity of the alternative input, labor. The upward-slope of the NRR curve makes the NRR curve different from the traditional NRR curve as it is usually drawn. (However, even traditional treatments sometimes admit the possibility that the “investment accelerator” might be so strong that the NRR curve would be upward-sloping.)
The size of the slope $\phi$ is given by

$$\phi = R \frac{1 + \zeta^{-1}}{\Gamma(1 - \alpha)},$$

where $\zeta$ is the Frisch labor supply elasticity and $\Gamma$ is aggregate degree of returns to scale. In words, the slope of the NRR curve depends on:

(a) the base level of the gross rental rate $R$, which indicates how proportional changes in the gross rental rate translates into percentage point changes in the real interest rate; (b) the elasticity of labor requirements with respect to output, equal to $\frac{1}{\Gamma(1 - \alpha)}$; (c) the reciprocal of the Frisch labor supply elasticity, $\zeta^{-1}$, which indicates how the increase in labor $N$ translates into an increase in the real wage $W$. To explain further, (a) a 1% increase in $R$ might be an increase, from say 15% per year to 15.15% per year, which is not the same as a one percentage point increase in $R$, say from 15% per year to 16 per year; (b) $\Gamma$ is the elasticity of output with respect to a proportional increase in all inputs, but when only labor is increasing, with capital held constant, output does not move as much, so labor has to increase more to get a given increase in output; (c) since $\zeta^{-1}$ is the elasticity of $W$ with respect to $N$, $1 + \zeta^{-1}$ is the elasticity of $WN$ with respect to $N$.

The NRR curve also differs from the traditional NRR curve in being unaffected by short-run movements in any variable other than technology (since short-run movements in any variable cannot change the medium-to-long-run situation of the representative household). This is discussed more below in connection with the effects of real shocks.

### 1.3 Dynamic Aggregate Supply

Price-setting is an ongoing process, with some firms changing their prices at the same time other firms are leaving their prices fixed. This pattern of staggered price setting and overlapping fixed prices is something worth capturing in a model of sticky prices. However, because introducing sticky prices into a model creates many complications, it is a good idea to begin by introducing them in a simple way. Calvo (1982) pioneered a particularly convenient way of dealing with overlapping fixed prices set on a staggered schedule. Calvo’s simplified model of price setting gets at the idea of time-dependent staggered price setting in a way that leaves the frequency of price adjustment by firms—which we will denote $\theta$—as a flexible parameter of the model.
1.3. DYNAMIC AGGREGATE SUPPLY

The simplifying trick in Calvo’s model is to think of firms as getting the chance to consider and adjust their prices after random intervals of time, determined by a Poisson process. This insures that the small subset of firms changing their price at any point in time are a random sample of all firms—with an average old price equal to the aggregate price. Assume also that all of the firms are essentially identical other than each firm’s old price carried over from the past. Then, in setting a new price, each firm is in essentially the same situation, and there is a single optimal reset price $B$ for all firms that get the opportunity to change their prices at a given time.

1.3.1 The Evolution of the Aggregate Price Level

In order to keep the model of dynamic aggregate supply simple, we will also make some fairly innocuous approximations. To begin with, regardless of the elasticities of substitution between the prices of the varieties of goods, the logarithmic (percentage) change in the aggregate price level is equal, to a first order approximation, to a weighted average of the logarithmic (percentage) changes in the individual prices—weighted by the share of each variety in total spending. The share of each variety in total spending will remain close enough to the average share that changes in this weighting create only second-order adjustments in the behavior of the aggregate price level. Thus, given underlying symmetry between all the firms, to a first-order approximation, the logarithmic change in the aggregate price level is equal to a simple average of the logarithmic changes in individual prices. Since, on average, firms are changing their log prices from the log aggregate price $p$ to the log reset price $b$ at the Poisson rate of price adjustment $\theta$, the rate of inflation $\pi$ is

$$\pi = \dot{p} \approx \theta (b - p).$$

(1.7)

1.3.2 The Determination of the Optimal Reset Price

The next key issue is the determination of the optimal reset price. Intuitively, the optimal reset price should be a weighted average of the (instantaneously optimal) desired prices over the period of time for which the newly reset price will be fixed. As long as the microeconomic rate of price adjustment $\theta$ is large relative to the difference between the real interest rate and the growth rate of the economy, such a weighted average of future desired prices is very close to correct. In the present context, the small gap having to do with discounting of future considerations at the real interest rate can be appropriately neglected.
A first-order approximation also makes it possible to state the relationship in similar form in terms of log prices:

\[ b_t = \int_t^\infty [\theta e^{-\theta (\tau-t)}] p^{#}_\tau \, d\tau. \]  

(1.8)

Since \( \int_0^\infty [\theta e^{-\theta (\tau-t)}] \, d\tau = 1 \), (1.8) states that the log reset price \( b_t \) is a weighted average of future log desired prices \( p^{#}_\tau \). Desired prices in the more distant future are discounted at rate \( \theta \) simply because it is unlikely that the firm will still be stuck with the price it is currently resetting for that long. A desired price beyond the time of the next opportunity to adjust prices is irrelevant to the decision of how to set the price now.

1.3.3 Digression on Leibniz’ Rule

It is sometimes desirable to convert integral equations, such as (1.8), into differential equations. For this purpose, Leibniz’ rule is often useful. Leibniz’ rule states that if

\[ g(t) = \int_{A(t)}^{Z(t)} f(\tau, t) \, d\tau, \]

then

\[ \frac{dg(t)}{dt} = \int_{A(t)}^{Z(t)} \frac{\partial f(\tau, t)}{\partial t} \, d\tau \]

\[ + \frac{dZ(t)}{dt} f(Z(t), t) \]

\[ - \frac{dA(t)}{dt} f(A(t), t). \]  

(1.9)

Note that \( t \) acts as a parameter in the integration while \( \tau \) is the dummy variable over which integration is performed. The function \( g(t) \) is the area under the curve of \( f(\tau, t) \) as \( \tau \) goes from \( A(t) \) to \( Z(t) \). To get some intuition for Leibniz’ rule, consider what the picture looks like if \( A(t), Z(t) \) and \( f(\tau, t) \) all increase with the parameter \( t \). Multiplying the right-hand side of (1.9) by \( dt \) yields the additional area when \( t \) is increased to \( t + dt \). On the right-hand side of (1.9), the first line indicates the additional ribbon of area added on top. (See Figure 7.) The second line indicates the additional sliver of area added at the right as \( Z(t) \) increases. The third line indicates the sliver of area sliced off at the left as \( A(t) \) increases. The area of each sliver is equal to base times
1.3. DYNAMIC AGGREGATE SUPPLY

height. The base is proportional to \( \frac{dZ(t)}{dt} \) on the right and proportional to \( \frac{dA(t)}{dt} \) on the left. The height is equal to \( f(Z(t), t) \) on the right and to \( f(A(t), t) \) on the left. The small squarish shapes in the corners can be ignored because they are small, being proportional to \( dt^2 \). Similar pictures can be drawn when one or more of \( A(t), Z(t) \) and \( f(\tau, t) \) decreases with the parameter \( t \).

1.3.4 Differential Equations for the Optimal Reset Price and for Inflation

Applying Leibniz’ rule to (1.8),

\[
\dot{b}_t = \frac{d}{dt} \int_{\tau}^{\infty} \left[ \theta e^{-\theta(\tau - t)} \right] p^\#_\tau d\tau = \int_{\tau}^{\infty} \left[ \theta^2 e^{-\theta(\tau - t)} \right] p^\#_\tau d\tau - \theta p^\#_t = \theta(b_t - p^\#_t).
\]

With the dummy variable \( \tau \) eliminated from the equation, the time subscript can once again be safely omitted: \( \dot{b} = \theta(b - p^\#) \).

Equation (1.10) leads to a differential equation for inflation. Since \( \pi = \theta[b - p] \)

\[
\dot{\pi} = \theta[b - \dot{p}] = \theta[\theta(b - p^\#) - \theta(b - p)] = \theta^2(p - p^\#).
\]

Using the notional short run aggregate supply curve, (1.4), to relate the “inflationary gap” \( p^\# - p \) to the “output gap” \( y - y^f \),

\[
\dot{\pi} = -\theta^2 \beta(y - y^f).
\]

Equation (1.11) is the dynamic aggregate supply equation. Note that \( \dot{\pi} = 0 \) when \( y = y^f \); the dynamic aggregate supply equation is consistent with any constant level of inflation when the economy is at full employment. In Phillips curve space with log output \( y \) on the horizontal axis and inflation on the vertical axis, the dynamic aggregate supply curve is vertical at \( y^f \). (See Figure 8.) In the absence of new information, when output is above full employment, inflation must be falling; below full employment, inflation must be rising.
This may sound strange, but it actually makes sense in a model with rational, forward-looking agents. As shown below, in dynamic general equilibrium, output above full employment tends to be associated with high and falling inflation, while output below full employment tends to be associated with low and rising inflation. The mantra “high and falling or low and rising” is invaluable in making sense of the negative sign in the dynamic aggregate supply equation (1.11).

1.4 Short-Run Equilibrium

Figure 9 illustrates short-run equilibrium at the intersection of the LM and NRR curves. To solve for this equilibrium algebraically, substitute from the equation for the NRR curve (1.6) into the equation for the LM curve (1.2):

\[
m + p - v = x = hy - \ell [y^f + \phi (y - y^f)] - \ell \pi
\]
\[
= (h - \ell \phi) (y - y^f) - \ell \pi + [hy^f - \ell r^f].
\]

Solving for the output gap \(y - y^f\),

\[
y - y^f = \frac{x + \ell \pi - [hy^f - \ell r^f]}{h - \ell \phi}.
\]

Thus, assuming the denominator \(h - \ell \phi\) is positive, output depends positively on the autonomous-velocity-adjusted real money supply \(x\) and on the rate of inflation \(\pi\), which tends to lead to economizing of transaction money balances, making a given amount of money go further. (Other than noting that they are not independent, we will defer discussion of changes in full-employment output and the full-employment real interest rate until later, when we discuss real shocks.)

The denominator \(h - \ell \phi\) in (1.13) is closely related to the slopes of the NRR and NRR curves. With log output \(y\) on the horizontal and the real interest rate \(r\) on the vertical axis, the slope of the NRR curve is \(\phi\), while the slope of the LM curve is \(h/\ell\). Thus

\[
h - \ell \phi = \ell [\frac{h}{\ell} - \phi] = \ell [(\text{slope of LM}) - (\text{slope of NRR})].
\]

With the NRR curve upward-sloping, stability of the short-run equilibrium requires the LM curve to have a steeper upward slope than the NRR curve. Stability is easier to guarantee the lower is \(\ell\). If \(\ell\) becomes too great, as it may
if inflation is very low, it calls the stability of the short-run equilibrium into question as the LM curve becomes relatively flat. In effect, the upward-sloping NRR curve makes a kind of liquidity trap possible even for finite values of $\ell$ that might occur for strictly positive values of the nominal interest rate. This is very relevant in thinking about the Great Depression. But for now, assume that $h - \ell \phi > 0$, ensuring stability.

1.5 The Phase Diagram for the Basic Neomonetarist Model

1.5.1 The Dynamics of Inflation

Equation (1.13) makes it possible to express $\dot{\pi}$, the rate of change in inflation, as a function of $x = m + v - p$ and $\pi$:

$$\dot{\pi} = -\theta^2 \beta (y - y^f)$$

$$= -\frac{\theta^2 \beta}{h - \ell \phi} [x + \ell \pi - (hy^f - \ell r^f)]. \quad (1.14)$$

On the phase diagram, the $\dot{\pi} = 0$ locus is downward-sloping, with slope $-\frac{1}{\ell}$. It intercepts the $x$-axis at $x = hy^f - \ell r^f$. To the right of the $\dot{\pi} = 0$ locus, $\pi$ is falling. To the left of the $\dot{\pi} = 0$ locus, $\pi$ is rising. (See Figure 10.)

Note that since it has the slope $-\frac{1}{\ell}$, $x + \ell \pi$ is constant along the $\dot{\pi} = 0$ locus and the LM curve is in the same position everywhere along the $\dot{\pi} = 0$ locus—the position of the LM curve that puts the short-run equilibrium at full employment output $y^f$.

1.5.2 The Convergence Rate

Figure 11 combines the dynamics for $x$ and $\pi$. The intersection of the $\dot{x} = 0$ and $\dot{\pi} = 0$ loci is the phase diagram’s steady state. We will call this point “full employment,” the “medium-run steady-state,” or “medium-run equilibrium” in contrast to the “long-run steady state,” which is where the economy ends up after both prices and the capital stock have adjusted.

The short-run dynamics shown in Figure 11 imply an upward-sloping saddle path. (The rule guaranteeing a downward-sloping saddle path applies only when the two axes are the state and costate variables of an optimizing model.)

In matrix form, the dynamic equations are
\[
\begin{bmatrix}
\dot{x} \\
\dot{\pi}
\end{bmatrix} = \begin{bmatrix}
0 & -1/	heta \beta h \\
-\theta \beta (hy - f - r f)/h - \ell \phi & h - \ell \phi
\end{bmatrix} \begin{bmatrix}
x \\
\pi
\end{bmatrix} + \begin{bmatrix}
\mu \\
\theta \beta (hy - f - r f)/h - \ell \phi
\end{bmatrix}.
\]

The characteristic equation for an eigenvalue \( \Lambda \) is
\[
\Lambda^2 - \text{trace} \Lambda + \text{determinant} = \Lambda^2 + \left( \frac{\ell \theta^2 \beta}{h - \ell \phi} \right) \cdot \Lambda - \frac{\theta^2 \beta}{h - \ell \phi} = 0.
\]

By the quadratic formula,
\[
\Lambda = -\frac{\ell \theta^2 \beta}{2(h - \ell \phi)} \pm \sqrt{\left( \frac{\ell \theta^2 \beta}{2(h - \ell \phi)} \right)^2 + \frac{\theta^2 \beta}{h - \ell \phi}}.
\]

Subtracting the square root yields the negative root. The convergence rate \( \kappa \) is the absolute value of this negative root. Simplifying,
\[
\kappa = \frac{\ell \theta^2 \beta}{2(h - \ell \phi)} \left[ 1 + \sqrt{1 + 4 \frac{h - \ell \phi}{\ell^2 \theta^2 \beta}} \right]. \quad (1.15)
\]

### 1.5.3 The Slope of the Saddle Path

The eigenvector associated with the convergence rate shows the slope of the saddle path. To find the convergent eigenvector, it is necessary to solve the equation
\[
\begin{bmatrix}
\dot{x} \\
\dot{\pi}
\end{bmatrix} = \begin{bmatrix}
0 & -1/	heta \beta h \\
-\theta \beta (hy - f - r f)/h - \ell \phi & h - \ell \phi
\end{bmatrix} \begin{bmatrix}
x \\
\pi
\end{bmatrix} = -\kappa \begin{bmatrix}
1 \\
1
\end{bmatrix}.
\]

for \( \sqcap \). Focusing on the top scalar equation indicates that
\[
\begin{bmatrix}
0 & -1
\end{bmatrix} \begin{bmatrix}
1 \\
\sqcap
\end{bmatrix} = -\sqcap = -\kappa.
\]

Thus, \( \sqcap = \kappa \) and the convergent eigenvector is
\[
\begin{bmatrix}
1 \\
\kappa
\end{bmatrix}.
\]

It is straightforward but tedious to show that this eigenvector solves the bottom equation as well. Therefore, the slope of the saddle path is \( \kappa \).

In words, the argument proving that the slope of the saddle path is \( \kappa \) is equivalent to the following argument. If autonomous velocity-adjusted real money balances \( x \) are 1% above their full employment level, they must decline by \( \kappa \) percentage points per year. To accomplish this, inflation must be \( \kappa \) percentage points per year above normal.
1.6. CHANGES IN THE GROWTH RATE OF MONEY

1.5.4 Determinants of the Convergence Rate, the Slope of the Saddle Path and the Contract Multiplier

Knowing the equality of the convergence rate and the slope of the saddle path, we can kill at least two birds with one stone in analyzing the determination of $\kappa$ in (1.15). The slope of the saddle path is important because it shows the size of the response of inflation to a one-shot increase in the autonomous-velocity-adjusted money supply $m + v$—or an increase in $m + v$ coming out of an i.i.d. time series of changes. The convergence rate is important because it is the macroeconomic rate of price adjustment. It is also of interest to look at the contract multiplier $\theta \frac{\kappa}{\kappa}$—the extent to which the macroeconomic rate of price adjustment is slower than the microeconomic rate of price adjustment.

In dynamic general equilibrium the sensitivity of output to a one-shot (or i.i.d.) increase in the autonomous-velocity-adjusted real money supply is given by

$$\frac{dy}{dx} = \frac{\partial y}{\partial x} + \kappa \frac{\partial y}{\partial \pi} = \frac{1 + \kappa \ell}{h - \ell \phi} = \frac{\kappa^2}{\theta^2 \beta}.$$  

The last line is an implication of the characteristic equation satisfied by $-\kappa$. It implies

$$\kappa = \theta \sqrt{\beta \frac{dy}{dx}} = \theta \sqrt{\beta \left(\frac{1 + \kappa \ell}{h - \ell \phi}\right)}.$$  

The contract multiplier, $\frac{\theta}{\kappa}$, is then

$$\frac{\theta}{\kappa} = \beta^{-0.5} \left(\frac{dy}{dx}\right)^{-0.5}.$$  

In the quantity theory case with $\ell = 0$ and $h = 1$, $\frac{dy}{dx} = 1$ and the contract multiplier is $\frac{\theta}{\kappa} = \beta^{-0.5}$, making the question of how to get a larger contract multiplier the same as the question of how to get a low value of $\beta$. Ball and Romer (1990) give the name “real rigidity” to having a low value of $\beta$. Therefore, real rigidity is a key to getting a large contract multiplier. More generally, the contract multiplier is increased by real rigidity and by a small dynamic general equilibrium sensitivity of output to money $\frac{dy}{dx}$.

1.6 Changes in the Growth Rate of Money

Since i.i.d. monetary shocks move the economy around on the saddle path, they can create the illusion of a stable Phillips curve, with inflation and the
level of output covarying. But attempting to exploit this “Phillips curve,” with
the idea of accepting higher inflation in exchange for higher output, would lead
to an increase in the expected rate of autonomous-velocity-adjusted money
growth \( \mu \). As can be seen in Figure 12, this shifts the medium-run steady
state to a higher level of inflation \( \pi \) and a lower level of real money balances
\( x \). The new saddle path, leading to the new steady state, shifts up to a higher
level of inflation for any value of real money balances \( x \). The shift in the saddle
path also implies a higher level of inflation for any value of output \( y \)—in effect
an upward shift of the Phillips curve, as shown more directly in Figure 13.
The advantage of Figure 12 as compared to Figure 13 is the knowledge that
the initial jump must be vertical in Figure 12. In Figure 13, the initial jump
is on a slant.

The dynamics of the short-run adjustment process go as follows. Since
there has been no sudden change in \( m + v \), but only a change in the growth
rate, and the price level \( p \) cannot jump, \( x = m + v - p \) does not jump. Therefore,
without any jump in \( x \), inflation \( \pi \) must jump up to the new saddle path. After
that, the economy follows the new saddle path down to the medium-run steady
state.

During this whole process, the NRR curve does not move. Every point on
the phase diagram corresponds to a position for the LM curve. Lines parallel to
the \( \dot{\pi} = 0 \) locus (\( y = y^f \) locus) are sets of points with the same position for the
LM curve. The upward jump in \( \pi \) causes the LM curve to shift to the right,
pushing the economy above full employment. (See Figure 14.) The phase
diagram treats the adjustment to the short-run equilibrium at the intersection
of NRR and the new LM as already completed. As the economy follows the
saddle path down to the left, the LM curve gradually shifts back to the left
until the economy returns to full employment.

In the very short-run, output has not yet adjusted to the NRR curve.
Before output has had any chance to adjust, the economy jumps down to the
new LM curve at the old level of output. Since the LM curve shifts down by
exactly the change in inflation \( \Delta \pi \), this initial very short-run equilibrium has
a real interest rate \( r \) that is lower by \( \Delta \pi \) but an unchanged nominal interest
rate \( r + \pi \). Then the economy follows the new LM curve up to the right until
it meets the NRR curve.

A one-shot or i.i.d. increase in the money supply, which moves the economy
up along the saddle path, induces a similar pattern of short-run adjustment in
the LM-NRR graph. The LM curve initially shifts to the right, then gradually
moves back to the left as prices adjust. In this case, the LM curve moves down
by more than \( \Delta \pi \), so that both the real and the nominal interest rates fall in
the very short run, then rise again through the very short-run dynamics as the economy moves toward the NRR curve. Because of the upward slope of the NRR curve, the real interest rate (and \textit{a fortiori}, the nominal interest rate) is higher in short-run equilibrium at the intersection of NRR and LM than it was before the shock.

1.7 Real Shocks

So far we have discussed \textit{monetary} shocks which have little or no effect on the real variables in the economy. By contrast a “real shock” is a shock that causes a significant change in the level of full-employment output $y^f$ or the full-employment real interest rate $r^f$—typically both in tandem. By convention, a \textit{positive} real shock is one that increases full employment output. (In the unlikely case in which only $r^f$ changes, a positive real shock would be one that increased $r^f$.) Where monetary shocks shift the $\dot{x} = 0$ locus, real shocks shift the $\pi = 0$ locus.

A set of four types of real shocks gives a good sense of the range of possible reactions of the economy to real shocks: labor supply shocks, technology shocks, government purchase shocks and industrial organization shocks.

1.7.1 Labor Supply Shocks

The phrase “labor supply shocks” refers to a wide variety of different shocks that shift the labor supply curve. Regardless of the cause of the shift of the labor supply curve, it results in similar short-run dynamics as the economy adjusts to a new level of full-employment output and of the full-employment real interest rate. Labor supply as seen by firms may shift out for any of the following reasons:

1. an increase in patience—that is, an increase in households’ willingness to defer gratification and work for the future;

2. fears of hard times (or the certainty of hard times) in the future that households want to prepare for by saving more;

3. an increase in the long-term real interest rate, making saving look more attractive;

4. an influx of new potential workers into the economy, either by immigration or by the maturation of a large cohort of young people;
5. a reduction in the effective marginal tax rate on labor;

6. a reduction in the market power of unions;

7. an increase in the desire for consumption goods relative to leisure, making households willing to work harder (to be able to purchase those goods);

8. a reduction in the need to spend time at home, whether because of household labor-saving devices or because of having fewer children at home.

The first three sources of labor supply shocks all operate through the marginal utility of consumption $\lambda$ for a fixed felicity (instantaneous utility) function. These movements in $\lambda$ follow the logic of the Basic Real Business Cycle Model with essentially no change to that model, as long as we assume that the desired markup of price over costs at full employment is small. Studying the other sources of labor supply shocks formally would require additional machinery.

**The Effect on $y_f$**

The Basic Real Business Cycle Model, with or without peripheral add-ons, implies that in medium-run (full-employment) equilibrium, a shift outward in labor supply leads to an increase in full-employment output $y_f$. To see why, remember the characterization of full employment as a situation in which the typical firm has marginal revenue equal to marginal cost. The shift outward and therefore downward in the labor supply curve means that at the old level of full employment output, the real wage is lower than before the shock. The lower real wage results in lower marginal cost at the old level of full-employment output, while marginal revenue at that level of output remains unchanged. Since marginal revenue is above marginal cost at the old level of full-employment output, the new level of full employment output must be higher.

**The Effect on $r_f$**

Because it means more output for each unit of capital, this increase in full-employment output leads to an increase in the full-employment rental rate unless the full-employment profit rate rises very fast with the level of full em-
1.7. REAL SHOCKS

Because the rental rate is likely to increase, it is likely that a positive labor supply shock will increase the full-employment real interest rate \( r^f \).

The Shift of the NRR Curve

A shift outward and downward in the labor supply curve means that for any given quantity of labor, the real wage is lower. With no change in technology or in the capital stock, there is no change in the amount of labor needed to produce a given amount of output. (See Figure 15.) Therefore, an increase in labor supply implies a lower real wage for any given amount of output, and a lower value of

\[
R = \frac{\alpha WN}{1 - \alpha K}
\]

for a given amount of output. As a consequence, the NRR curve, which gives the mapping from output \( y \) to the net rental rate \( R - \delta \), shifts downward. (See Figure 16.)

The Shift of the \( \dot{\pi} = 0 \) locus

On the LM-NRR diagram, the \( y^f - r^f \) point indicates full employment. Define the full-employment LM curve as the LM curve that exists for some values of \( x \) and \( \pi \) which passes through the \( y^f - r^f \) point. The \( \dot{\pi} = 0 \) locus shows the set of points on the \( x - \pi \) plane that yield a full-employment LM curve. Therefore (given the positive relationship between \( x \) and the LM curve), a rightward shift of the full-employment LM curve corresponds to a rightward shift of the \( \dot{\pi} = 0 \) locus. Conversely, a leftward shift of the full-employment LM curve corresponds to a leftward shift of the \( \dot{\pi} = 0 \) locus.

In response to a positive labor supply curve, both the full-employment LM curve and the \( \dot{\pi} = 0 \) locus shift to the right. Here is why. The new \( y^f - r^f \) point must be on the new NRR curve. The increase in full-employment output \( y^f \), combined with the downward shift in the NRR curve puts the new \( y^f - r^f \) point unambiguously to the right of the old full-employment LM. Therefore, the new full-employment LM curve after a positive labor supply shock must be to the right of the old full-employment LM curve. (See Figure 17.)

---

2 Given a Cobb-Douglas production function, \( R = \alpha(1 - \Pi)(Y/K) \), where \( \Pi \) is the actual shadow profit rate at any point in time. Since \( 1 - \Pi = \text{[degree of returns to scale]/[desired markup]} \), for the full-employment profit rate to rise that fast, either the degree of returns to scale must fall very fast or the desired markup must rise very fast, or both.
The Short Run Dynamics on the $x-\pi$ Phase Diagram

On the $x-\pi$ phase diagram, the medium-run (full-employment) equilibrium is at the intersection of the $\dot{\pi} = 0$ locus with the $\dot{x} = 0$ locus. The $\dot{x} = 0$ locus is unchanged at $\pi = \mu$. Thus, the rightward shift of the $\dot{\pi} = 0$ locus in reaction to an increase in labor supply implies a higher value of $x^f$, the autonomous-velocity-adjusted real money supply at full employment. The full-employment level of inflation, $\pi^f$ is unchanged. From the standpoint of the money demand equation, the higher level of full-employment output $y^f$, with no change in $r^f + \pi^f$ requires a higher level of real money balances $x^f$.

Figure 18 shows the short-run dynamic general equilibrium effects of a permanent rightward shift in the $\dot{\pi} = 0$ locus. On impact, inflation jumps down to the new saddle path. Then the economy follows the new saddle path up to the new medium-run steady state. The moment the point on the phase diagram crosses the old $\dot{\pi} = 0$ locus is distinguished as the moment the shifting LM curve passes over the original LM curve. Beyond that point, the LM curve continues shifting toward the $y^f - r^f$ point on the new NRR curve. (See Figure 19.)

The Contractionary Effects of an Increase in Labor Supply in the Short Run

The position of the LM curve is fully determined by the real money supply $x$ and the inflation rate $\pi$; the mapping from the phase diagram to the position of the LM curve is unaffected by real shocks. In dynamic general equilibrium, in the absence of any change in autonomous-velocity-adjusted money $m + v$, $x = m + v - p$ will not jump on impact. In this case, the initial shift of the LM curve in reaction to any real shock will be determined by the jump in the inflation rate, which is forward-looking.

Figure 18 shows the initial fall in the inflation rate in response to an increase in labor supply. This disinflationary response is typical of positive real shocks. A fall in inflation causes an upward shift in the LM curve, as a given nominal interest rate gets mapped into a higher real interest rate.

Figure 19 shows this initial upward jump in the LM curve. The upward shift in LM, combined with the downward shift in NRR, leads to an unambiguous short-run reduction in output $y$. Thus, in response to an increase in

\footnote{For $\pi^f$ to be unchanged, the medium-run dynamics of the capital stock must be much slower than the macroeconomic rate of price adjustment. When this is not a good approximation, one must allow for possible changes in $\pi^f$. Analyzing changes in $\pi^f$ requires a good grasp of the medium-run effects of a given real shock.}
labor supply, which increases full employment output $y^f$, output falls in the short run! This perverse short-run effect on output is typical of real shocks.

**Other Monetary Policy Rules**

What if the central bank follows a policy that automatically changes the money supply in reaction to a shock? First, if the central bank fully understood the nature of the shock, it could increase the money supply enough to immediately reach the new equilibrium level of the real money supply $(x^f)'$ instead of getting to $(x^f)'$ through an extended disinflationary period. However, this would require a lot of knowledge on the part of the central bank. Suppose the central bank does not have that much knowledge, but follows a Taylor rule. Then the Monetary Policy (MP) curve, which otherwise looks like an LM curve, shifts in the opposite direction from an LM curve in response to a drop in $\pi$. (See Figure 20.) The $x - \pi$ phase diagram is now replaced by an $a - \pi$ phase diagram. The $\dot{\pi} = 0$ locus still shifts to the right, along with the saddle path. But now on impact, the economy moves to a point on the new saddle path to the right of the old $\dot{\pi} = 0$ locus, since the MP curve jumps down and to the right in response to a fall in $\pi$. (See Figure 21. Note that $\pi$ cannot jump up, since then the economy would have to jump to a point on the saddle point to the left of the old $\dot{\pi} = 0$ locus, which is impossible if $\pi$ jumps up.) From that point, the economy moves along the saddle path toward medium-run equilibrium in a familiar way. The initial jump downward in the monetary policy curve combined with the downward shift in the NRR curve makes it ambiguous what happens to output in the short run. Finally, suppose the central bank follows a real interest rate rule, so that the MP curve does not jump in response to inflation. This case is left as an exercise for the reader.

**1.7.2 Technology Shocks**

Because it increases their permanent income, an improvement in technology can shift the labor supply curve back, but because it raises the long-run real interest rate, an improvement in technology can shift the labor supply curve out. A sufficient statistic for both of these effects is the movement in the marginal utility of consumption $\lambda$ induced by the improvement in technology.\footnote{The perverse short-run effect of this and other positive real shocks on output depends crucially on a non-zero interest elasticity of money demand; if the LM curve were vertical, real shocks would initially have no effect on output.}
For simplicity, we will assume in this subsection that the wealth effect of the increase in permanent income is cancelled out by the long-run real interest rate effect. That is, we will assume that the improvement in technology has no impact effect on $\lambda$ and therefore no impact effect on labor supply. To the extent that a technology shock does have an impact effect on labor supply, the responses discussed in the previous subsection about labor supply shocks can be added to the direct effects of the technology shock discussed in this subsection.

The effects of technology shocks are quite similar to the effects of labor supply shocks.

One of the most unambiguous results from the Basic Real Business Cycle model is that a permanent improvement in technology results in an increase in output on impact. Since the Basic Real Business Cycle model has perfectly flexible prices, this increase in output corresponds to an increase in medium-run equilibrium (full-employment) output $y_f$.

For any given capital stock $K$ an increase in the level of labor-augmenting technology $Z$ reduces the amount of labor $N$ needed to produce any given amount of output. With an unchanging labor supply curve, this reduction in the labor requirements for given $Y$ reduces the real wage $W$ associated with that value of $Y$. (See Figure 22.) Since both $N$ and $W$ for given output fall, the rental rate

$$R = \frac{\alpha}{1 - \alpha} \frac{WN}{K}$$

for given $Y$ must fall. As a consequence, the NRR curve, which gives the mapping from (log) output $y$ to the net rental rate $R - \delta$, shifts downward.

The downward shift in the NRR curve, combined with the increase in full-employment output $y_f$, guarantees that the full-employment LM curve must shift to the right. As a consequence, the new $\pi = 0$ locus must be to right of the old one. The dynamics on the short-run $x-\pi$ phase diagram are very similar to those induced by an increase in labor supply. Inflation $\pi$ jumps down on impact, resulting in an upward jump in the LM curve. In concert with the downward shift of the NRR curve, the upward jump in the LM curve leads to a short-run reduction in output $y$, even though the improvement in technology increases full-employment output $y_f$.

1.7.3 Government Purchase Shocks

Government spending is divided into transfers (such as social security payments) which can be though of as negative taxes, and government purchases
of goods and services. Turning to government purchases of goods and services, some types of government purchases are close substitutes for private purchases, and can be lumped in with private consumption or investment. Other types of government purchases, such as most military spending, have little interaction with the private economy except for the use of resources. This is the type of government purchases we focus on here.

Long-lasting increases in government purchases, whether financed by current taxes or by borrowing backed by future taxes, makes households significantly poorer. This impoverishment of households makes them eager to earn more money by working more. Thus, the impoverishment effect of government purchases tends to increase labor supply.

If the additional government purchases are financed by current lump-sum taxes and by borrowing backed by future taxes, the impoverishment effect is the key effect on labor supply. However, if the additional government purchases are financed in part by increases in the current labor income tax rate, this increase in the marginal labor income tax rate has an adverse effect on labor supply that acts in the opposite direction.

Given households obeying the permanent income hypothesis, relatively brief increases in government purchases—lasting, say, four to five years time—might have only a trivial impoverishment effect on labor supply.

In this subsection, we will analyze the effects of an increase in government purchases is either brief enough to have little effect on labor supply, or is financed by an increase in labor income taxes that cancels out its effect on labor supply. Depending on how long it takes prices to adjust, it may be possible to have an increase in government purchases that lasts long enough for prices to adjust to a new medium-run (full-employment) equilibrium, but not long enough to have a significant effect on labor supply. To the extent an increase in government purchases affects labor supply, the effects of the previous subsection on labor supply shocks can be added to the direct effects of an increase in government purchases discussed here.

In the absence of investment adjustment costs, the direct effect of an increase in government purchases is simply to crowd out investment one-for-one, with no effect on medium-run equilibrium output \( y^f \). Moreover, in the absence of any change in labor supply, the increase in government purchases has no effect on the level of \( N \) or \( W \) for a given level of output and therefore no effect on the NRR curve. The full-employment LM curve does not move, and so the \( \dot{\pi} = 0 \) locus remains where it started. Other than the increase in government purchases \( G \) and the corresponding reduction in investment \( I \), nothing happens, either in the medium-run equilibrium or in the short-run! Of course, the
qualifications we have made, point to some of the places one might look to restore the idea that government purchases should have an effect on output. But restoring the traditional Keynesian story is much more difficult than one might think.

1.7.4 Industrial Organization Shocks

Suppose the desired markup falls because of an increase in consumers’ readiness to switch between brands. This results in lower desired prices in an otherwise similar economic situation. Ignoring any effect on labor supply, this results in an outward shift in the $\dot{\pi} = 0$, with consequent dynamics as shown on the phase diagram. The outward shift in the $\dot{\pi} = 0$ locus guarantees an increase in $x^f$ and an outward shift in the full-employment LM curve. With no shift in labor supply or in technology, the NRR curve stays put. Thus, the outward shift in the full-employment LM curve implies a shift in the $y^f - r^f$ point up along the NRR curve. Other than the absence of a shift in the NRR curve, the analysis is similar to that for an increase in labor supply or an improvement in technology.

1.7.5 Open Economy Shocks

Events abroad can have an important effect on an economy. There is no way to deal with all of the important open economy issues here, but it is useful to relate open economy shocks to the shocks we have discussed so far while tacitly assuming a closed economy.

1. The Balance of Trade. An increase in the trade surplus acts much like an increase in government purchases, since, like government purchases, it widens the gap between output and $C + I$. Since exports may not generate as much money demand as other forms of output, a change in the balance of trade can also cause a change in what we have been calling autonomous velocity $v$.

2. The Terms of Trade. An improvement in the terms of trade—the quantity of imports than can be obtained from a given amount of exports—

5Ultimately, the increased competitive pressure will cause a shakeout in which many firms will exit, leaving fewer, larger firms that can take advantage of greater scale economies to avoid losses in the more competitive environment. But by the hierarchy of time-scales we have assumed, this occurs after the economy has already reached the new medium-run equilibrium.
acts much like an improvement in technology—which increases the quantity of output that can be obtained from a given amount of input. Changes in the terms of trade often also cause changes in the balance of trade.

3. Trade Barriers. A reduction in trade barriers is like an improvement in the terms of trade. Like other improvements in the terms of trade, a reduction in trade barriers can also result in an increase in the effective level of competition—yielding a positive industrial organization shock.

4. The Exchange Rate. An increase in the exchange rate causes an improvement in the terms of trade, and often causes a reduction in the trade surplus. If the exchange rate will go back to its previous value over time, the fall in the exchange rate from its high value tends to push down the long-run real interest rate. (A reduction in the long-run real interest rate tends to reduce labor supply.)

5. International Oil and Raw Materials Prices. Increases in international oil and raw materials prices represent a worsening of the terms of trade—often with the associated effects on the balance of trade and on autonomous velocity. To complicate matters further, increases in oil and raw materials prices are sometimes caused by a worldwide increase in long-term real interest rates.

The main message to be drawn from this itemization is that international shocks—including oil shocks—can have quite complex effects. Open economy macroeconomics is inherently more difficult than closed economy macroeconomics and is correspondingly less well understood.

In its effects (though not of course in its origins), a terms of trade shock is like a technology shock, since trade is like a production function that converts exports (the inputs) into imports (the outputs). To treat a change in the terms of trade as a technology shock, one must add trade to the list of sectors whose production function one considers. In its effects, an increase in the trade surplus is similar to an increase in autonomous government purchases, since both affect the material balance condition in the same way.

1.8 The Composition of Output

So far, we have used the Basic Neomonetarist Model to determine output and other variables without regard to the composition of output as between
consumption and investment. The key to the analysis of the composition of output is to realize that with no investment adjustment costs, investment is passively determined as a residual after consumption is determined. (Government purchases are exogenous, and net exports are determined in a complex way in an open economy model.)

The short-run movements in consumption are entirely governed by interaction between consumption and labor in the utility function. With additively separable utility between consumption and labor, consumption is always very close to its medium-run equilibrium value $C^f$, since the marginal utility of consumption $\lambda$ is determined by medium-run considerations, not by short-run considerations.

In response to a monetary expansion, consumption only increases to the extent that consumption is complementary to labor in the utility function. With additively separable utility, consumption would move very little.

The effect on full-employment consumption of an increase in labor supply depends on the source of the increase in labor supply:

1. an increase in patience increases labor supply but reduces $C^f$;
2. fears of hard times (or the certainty of hard times) in the future increase labor supply but reduce $C^f$;
3. an increase in the long-term real interest rate increases labor supply, but reduces $C^f$;
4. an influx of new potential workers into the economy, either by immigration or by the maturation of a large cohort of young people raises both labor supply and $C^f$;
5. a reduction in the effective marginal tax rate on labor raises both labor supply and $C^f$;
6. a reduction in the market power of unions or in other labor market distortions raises both labor supply and $C^f$;
7. an increase in the desire for consumption goods relative to leisure, raises both labor supply and $C^f$;
8. a reduction in the need to spend time at home raises labor supply and increases $C^f$ if it is because of household labor-saving devices, but reduces $C^f$ if it is because of having fewer children at home.
Other than any effects operating through changes in labor supply from sources 2 and 3, an improvement in technology causes $C^f$ to change only insofar as consumption is complementary with labor. But even then, the change depends on the sign of the change in $N^f$, which is ambiguous. If utility is additively separable between consumption and labor, consumption is determined by $\lambda$ alone.

Other than its effects on $\lambda$, an increase in government purchases has no effect on consumption.

Finally, a reduction in the desired markup tends to increase $C^f$.

Even when medium-run equilibrium consumption $C^f$ increases, consumption can fall in the short-run if consumption is complementary with labor.

Further analysis of consumption and investment in medium-run equilibrium requires a deeper investigation of medium-run dynamics and long-run equilibrium.

**Exercises**

1. If $Y = F(K, ZN)$, show that the marginal product of capital $F_K(K, ZN)$ can be expressed as a function of $Y$ and $K$ that is invariant to changes in $Z$.

2. Analyze the effects of a temporary decrease in the growth rate of autonomous-velocity-adjusted money $\mu$.

3. Analyze the effects of an anticipated decrease in the growth rate of money $\mu$. 
Real Rigidity

\[ \frac{p^*}{p} \]

\[ p^*/p \]

\[ 1 \]

\[ D(y^f) \]

\[ D(y) \]

\[ MC(y, Y) \]

\[ MR(y, Y) \]

\[ MC(y, Y^f) \]

\[ MR(y, Y^f) \]

\[ y^f \]

\[ y^# \]

\[ Y \]
\[ \dot{x} = \mu - \Pi \]

Figure 1

\[ \pi = \mu \quad \pi = m + v - p \]

Figure 2

LM (slope = \( \frac{h}{k} \))
Figure 3

\[ x + \pi = \text{constant; LM fixed} \]

\[ \text{(slope } = -\frac{1}{2} \) \]

\[ x = m + v - p \]

Figure 4

\[ p, p^\# \]

slope = \beta

\[ p^\# : \text{Notional SR} \]

SRAS
Figure 5

Figure 6

slope = \phi

NRR (15)