Chapter 3

Amplitude and loudness

We saw in Section 1.5 that sound waveforms have three fundamental properties that determine the sound we hear: frequency, amplitude, and shape. Frequency, which determines musical pitch, we examined in detail in the last chapter. In this chapter we look at amplitude, which determines how loud a sound is.

Figure 3.1 shows a sound waveform. As with similar plots we have seen, the horizontal axis measures time and the vertical axis measures the sound pressure experienced by the listener. The *amplitude* of the waveform is its vertical size, measured from the horizontal axis to the highest point or the lowest point, whichever is further. A sound wave with larger amplitude has wider pressure variation than one with smaller amplitude, which means it exerts more pressure on the ear and gives rise to a stronger sound sensation: waves with larger amplitude sound louder.

The amplitude of a wave can change over time. A sound can be loud at one

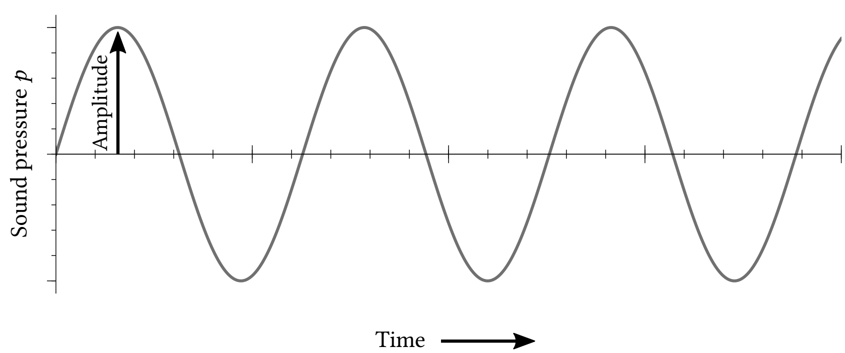

Figure 3.1: Amplitude. The amplitude of a sound waveform is the maximum amount by which the sound pressure varies from zero in either direction.

moment and quiet at another. Manipulation of the amplitude and hence the loudness of sound is a crucial artistic element of musical performance. The performance itself, however, is not the only thing that affects amplitude and loudness. As we will see there are many other influences on the loudness of a sound. Distance from the sound source is an important variable: far away sounds are quieter than nearby ones and we will see why this is and how to calculate the magnitude of the effect. Obstructions in the path of a sound wave can also have an impact on volume level. Small objects have little effect but large ones, like a wall or a building, can reduce the volume of a sound or block it out completely. We will see why this is too, and calculate how big an object needs to be to impede sound propagation.

And we will see that there are also biological aspects to the loudness of sound. The ear is more sensitive to some sounds than others and we will learn how this is measured and about the Fletcher-Munson diagram, a useful visualization that captures much about how we hear sound and the limits of audibility.

In order to discuss and understand all of these issues, we first need to define exactly what we mean, in scientific terms, when we talk about the loudness of a sound. For this we need to understand about sound energy and intensity.

3.1 The energy of a sound wave

In science the strength of something is often measured by how much energy it has. The brightness of a light bulb is measured in watts. The nourishment in food is measured in Calories. The strength of a car engine is measured in horsepower. Each of these is a measure of energy and we can describe the strength of sound waves in the same way. Sound waves possess energy because they involve movement of the air. Sound consists of the motions of air molecules and when objects move they have *kinetic energy*. Thinking of a bowling ball crashing into a set of bowling pins. It is clear that the ball has a lot of energy when it hits the pins. Air molecules are similar, possessing energy because of their motion, but they are far smaller and so carry much less energy. The molecules in air are always moving randomly, as we have said, which gives them some kinetic energy, but sound adds an additional motion on top of that and hence an additional amount of energy. This additional energy is the energy of the sound wave.

Sound energy will be a crucial concept for us in our discussions. We will use it to quantify how loud sounds are and how that loudness varies with distance and other variables.

3.1.1 Intensity

Imagine a constant sound, like a long held note, coming from a sound source such as a musical instrument or a loudspeaker. The pressure wave of the sound travels at the speed of sound from the sound source to your ear and in doing so it brings energy with it. Molecules in the air that were previously at rest start to move when the sound wave reaches them and so they gain kinetic energy. Thus the sound wave actually carries energy along with it, transmitting it from one place to another. You can build an electric circuit that will run off sound energy. You can warm up an object or break a window by blasting a loud enough sound at it.

The amount of energy in a sound wave, which is measured in units of *joules*, gives us a way to say how loud the sound is. The total amount of energy, however, is not a very good measure because it depends on how long the sound lasts—the same sound played for twice as long will have twice as much energy. A better measure is the amount of energy per second or *power* of the sound, which is measured in joules per second, also called *watts*. However, power is not the perfect measure either because not all of the sound will necessarily reach you, the listener. A sound source normally emits sound in many directions at once and only a small fraction will actually hit your eardrum so that you hear it. A better measure, therefore, is the amount of energy *per square meter* per second. This is the *intensity* of the sound. Imagine holding up a square board one meter on a side. The amount of sound energy hitting it per second is the intensity. Intensity is measured in watts per square meter, written $\mathrm{W/m^2}$.

Intensity is the fundamental measure of the loudness of a sound. If we know the intensity we can use it to calculate how much sound energy will reach a given target. For instance, if we know the area of a human eardrum to be A and we also know that a certain sound wave falling on that eardrum has intensity I, then the amount of sound energy it imparts per second on the eardrum is IA. The larger this energy, the louder the sound will be.

3.1.2 Intensity and sound pressure

Air pressure exerts a force on any surface or object it touches, and this means it can also bestow energy on objects. Physics tells us that when we push on an object we give it energy which, measured in joules, is equal to the force exerted times the distance the object moves. Now think of a sound wave. The object being moved in this case is the air itself—the sound pressure at one point pushes on the air at an adjacent point and makes it move. If the force on a particular bit of air is F and the air is pushed a small distance Δx over a short time Δt then the energy imparted on it is $F\,\Delta x$ and the energy per unit time is $F\,\Delta x/\Delta t$. But $\Delta x/\Delta t$, the distance moved divided by the time, is just the velocity u of the air:

$$u = \frac{\Delta x}{\Delta t}. \tag{3.1}$$

Hence the energy imparted on the air per unit time is

$$\frac{F\,\Delta x}{\Delta t} = Fu. \tag{3.2}$$

Now imagine this sound wave passes through an area A—think of an imaginary square hanging in the air and the wave passing through it. Then the energy per unit time and per unit area—the intensity of our sound wave—is

$$I = \frac{Fu}{A}. \tag{3.3}$$

The force F exerted by the sound wave comes from the sound pressure p. Pressure is force divided by area $p = F/A$, so the sound pressure exerts a force $F = pA$ over the area A, and hence the sound intensity of Eq. (3.3) can also be written as

$$I = \frac{pAu}{A} = pu. \tag{3.4}$$

But, as discussed in Section 1.3 and derived in Eq. (1.29) on page 14, the pressure p and velocity u are directly proportional to one another, according to

$$p = zu, \tag{3.5}$$

where the constant of proportionality z is the acoustic impedance of air. Using this result to write $u = p/z$ and then substituting into Eq. (3.4), we find that

$$I = \frac{p^2}{z}. \tag{3.6}$$

The acoustic impedance can also be written in terms of the density of air ρ and the speed of sound c as $z = \rho c$ (see Section 1.6.1), so the intensity can also be written as

$$I = \frac{p^2}{\rho c}. \tag{3.7}$$

This is the fundamental equation for the intensity of a sound wave. If we know the sound pressure at any moment we can use this equation to calculate the intensity.

3.1.3 Instantaneous intensity and average intensity

Equation (3.7) tells us the intensity of a sound wave, the rate at which sound energy passes through one square meter. This intensity, however, is not constant in time. Sounds can get louder and softer of course, so we expect intensity to change sometimes, but the intensity represented by Eq. (3.7) changes all the time: pressure fluctuates in a sound waveform as we have see, so the value of p^2 will fluctuate too, giving rise to constant tiny variations in the intensity. Usually we are not interested in these variations. Usually we just want to know the average rate at which sound energy is reaching us. This is given by the *average intensity*:

$$\langle I \rangle = \frac{\langle p^2 \rangle}{\rho c}. \tag{3.8}$$

Here the notation $\langle \dots \rangle$ means that we are averaging the quantity inside the brackets over time. (We do not have to average ρ or c on the right-hand side of the equation, since they are constant and their values do not change.) Normally when we quote numbers for the intensity of a sound we are quoting this average intensity, not the instantaneous intensity of Eq. (3.7).

In order to perform the time average, we must specify what time interval we are averaging over. The interval we use varies depending on the circumstances. When we measure sound intensity using a standard sound level meter, for example, the meter is usually programmed to average the intensity over one second. In practice, this means that the sound must be roughly steady over a second to get a good reading. If the sound varies faster than this then the meter reading cannot be trusted. Most meters allow the user to select a shorter time interval if necessary, depending on the type of sound being measured.

See Section 3.7 for a discussion of the the workings of sound meters.

When performing mathematical calculations a different choice of time interval may be convenient. In particular, for a *periodic* wave, which corresponds to a musical note as discussed in Section 2.1, the average is usually performed over one complete cycle of the wave. Thus for a periodic waveform with frequency 100 Hz we would average over $\frac{1}{100}$ of second. It does not matter at what point in the waveform we start the average. The result will be the same for any choice.

For non-periodic waveforms, which include the sounds of percussion instruments as well as many naturally occurring sounds, there is no equivalent simple choice. Different time intervals, or the same interval with different starting points, will give slightly different results and hence the average intensity is not entirely well defined. The best one can do is average over a reasonably long time, but not so long that the sound changes substantially—the one second used by sound meters is often a good starting point.

In essentially all situations where we talk about the intensity of musical sound we are talking about the average intensity of Eq. (3.8). For this reason one commonly just writes the intensity as

$$I = \frac{\langle p^2 \rangle}{\rho c}, \tag{3.9}$$

omitting the $\langle \dots \rangle$ signs on the left-hand side, and it is understood that we are talking about the average intensity. We will do this throughout this book.

The quantity $\langle p^2 \rangle$ is called the *mean-square sound pressure*, or just the mean-square pressure if we are being a little sloppy. The mean-square sound pressure is proportional to the square of the amplitude of the sound wave—if we double the size of our waveform then we quadruple the value of $\langle p^2 \rangle$. So Eq. (3.9) connects the intensity back to the amplitude again. It tells us that the average intensity is proportional to the *square* of the amplitude of the waveform, a result that will be useful to us many times in our discussion of musical sound.

Sound	Intensity (W/m^2)	SIL (dB)	Impression	Musical notation
Silence	10^{-12}	0	Limit of audibility	
Dropped pin	10^{-11}	10		
Recording studio	10^{-10}	20		
Soft whisper	10^{-9}	30		
Library	10^{-8}	40	Very very quiet	*ppp*
Quiet residence interior	10^{-7}	50	Very quiet	*pp*
Department store	10^{-6}	60	Quiet	*p*
Freeway traffic	10^{-5}	70	Medium	*mp*
Jackhammer	10^{-4}	80	Loud	*f*
Subway train passing	10^{-3}	90	Very loud	*ff*
Machine shop	0.01	100	Very very loud	*fff*
Loud rock concert	0.1	110		
Jet engine at 500 m	1	120	Painfully loud	
Jet engine at 10 m	1000	150	Hearing damage	

Table 3.1: Typical intensities (in watts per square meter) and sound intensity levels (in decibels) for some familiar sounds.

3.1.4 Typical intensity values

In practice, the amount of energy in a sound wave is tiny. Intensity, as we have seen, is measured in watts per square meter. One watt is not a large amount of energy—only enough to run a small electric light bulb, for instance—yet one watt per square meter would be a very loud sound. The sound of a jet engine when you are standing near the runway might reach 1 W/m^2. The intensities of most sounds are only a small fraction of a watt per square meter. Table 3.1 lists typical intensities for a variety of familiar sounds ranging from the drop of a pin to the loudest jet engine. Most of these intensities are so small that we must use scientific notation—powers of ten—to represent them. Traffic on a busy road might have a sound intensity of 10^{-5} W/m^2. The sound level inside a house might be 10^{-7} W/m^2.

The very quietest sound that can be heard by the human ear has an intensity of about 10^{-12} W/m^2, a millionth of a millionth of a watt per square meter, and this value, denoted I_0, is conventionally taken as the limit of audibility. People's ears do vary and some are more sensitive than others, but this figure is a reasonable rough guide for what is audible and what is not. It is a testament to the impressive sensitivity of the human ear that we can detect sound intensities as small as this.

3.2 Sound intensity level

Intensity in watts per square meter is actually not a very convenient scale for measuring loudness, spanning as it does a wide range of values including very small ones. Typically, therefore, sound is measured using a different scale, the *sound intensity level* or SIL scale. The sound intensity level of a sound is defined by the formula $\log_{10}(I/I_0)$, where $\log_{10}$ indicates the common logarithm, I is the intensity, and $I_0 = 10^{-12}\,\mathrm{W/m^2}$ as before. When defined in this way, sound intensity level is measured in units of *bels*, named after Alexander Graham Bell, the inventor of the telephone. Bels are not a very convenient unit either, however, being too large for most practical purposes, so we almost always quote sound intensity levels in *tenths* of a bel, also called *decibels* and abbreviated "dB." There are ten decibels to every bel, so the sound intensity level in decibels, denoted L_I, is ten times the level in bels, or

$$L_I = 10 \log_{10} \frac{I}{I_0}. \tag{3.10}$$

The logarithm here is a common (base 10) logarithm, which is conventional in the study of sound. We will almost always be using common logarithms in this book, so we will just write "log" for the logarithm and omit the 10, except in rare cases where we will use some other type of logarithm. Thus we will write

$$L_I = 10 \log \frac{I}{I_0}, \tag{3.11}$$

where it is understood that "log" means a common logarithm. This is the formula for sound intensity level in decibels. For example, if a sound has intensity $10^{-4}\,\mathrm{W/m^2}$, then the sound intensity level is

$$L_I = 10 \log \frac{10^{-4}}{10^{-12}} = 10 \log 10^8 = 10 \times 8 = 80\,\mathrm{dB}. \tag{3.12}$$

We can also convert back from sound intensity level to intensity by rearranging Eq. (3.11) to give

$$I = I_0\, 10^{L_I/10}. \tag{3.13}$$

For instance, for a sound intensity level of $L_I = 60\,\mathrm{dB}$ this formula tells us that the equivalent intensity is

$$I = 10^{-12} \times 10^{60/10} = 10^{-6}\,\mathrm{W/m^2}. \tag{3.14}$$

If we have two different sounds, one louder than the other, it is often useful to know the difference in sound intensity level between them. The difference for two sounds with intensities I_1 and I_2 is

$$L_1 - L_2 = 10 \log \frac{I_1}{I_0} - 10 \log \frac{I_2}{I_0} = 10\bigl(\log I_1 - \log I_0\bigr) - 10\bigl(\log I_2 - \log I_0\bigr). \tag{3.15}$$

Cancelling some terms and rearranging, we then get

$$L_1 - L_2 = 10 \log \frac{I_1}{I_2}. \tag{3.16}$$

We will use this formula often to calculate differences in decibel levels between one sound and another. We can also rearrange the equation to give

$$\frac{I_1}{I_2} = 10^{(L_1-L_2)/10}, \tag{3.17}$$

which tells us the ratio of the intensities of two sounds in terms of the difference in their decibel levels.

Before going on, let us pause briefly to note that the names "intensity" and "sound intensity level" are very similar and can be confusing. It is important to understand that these are different things. Intensity and sound intensity level both measure the loudness of sound but they do so in different ways and on different scales. Intensity is the amount of sound energy in watts per square meter. Sound intensity level captures the same information but on the decibel scale. Both measures have their uses. Most scientific and acoustical calculations are performed using intensity, since intensity deals directly with sound energy and the physical manifestation of sound. As we have said, however, intensity is not a convenient scale for everyday use, so when talking about sound in normal circumstances, such as when describing the loudness of music, we almost always use the decibel scale.

3.2.1 The decibel scale

A characteristic feature of the decibel scale (which you can see by looking at Eq. (3.11)) is that when *intensity* is multiplied by a factor of ten *sound intensity level* goes up by 10 dB. If we multiply intensity by another factor of ten sound intensity level goes up by another 10 dB, for a total increase of 20 dB. So a 20 dB increase in sound intensity level corresponds to a factor of $10 \times 10 = 100$ increase in intensity. Similarly 30 dB corresponds to a factor of 1000, 40 dB to 10 000, and so forth.

At the same time, when the intensity is equal to I_0—the limit of human audibility—Eq. (3.11) tells us that the sound intensity level is

$$L_I = 10 \log 1 = 0\,\text{dB}. \tag{3.18}$$

Thus, the sound intensity level is zero at the limit of audibility, which establishes a natural scale for decibel values. Table 3.1 shows the correspondence between the intensity and decibel scales, and we can see that decibels are indeed a convenient unit with which to measure the loudness of sounds. They range from 0 dB for an inaudible sound up to about 100 dB or so for the loudest sounds.

It can be useful to be able to convert quickly between intensity and decibels. This can always be done using Eqs. (3.11) and (3.13), but it is convenient if you can do rough calculations in your head too. For this purpose there are a few basic rules worth committing to memory. First, as we have said, an increase of 10 dB is equivalent to multiplying the intensity by a factor of 10, and 20, 30, 40 dB are equivalent to multiplying by 100, 1000, 10 000, and so forth.

Another useful number to remember is 3 dB. Looking at Eq. (3.17), if one sound is louder than another by 3 dB then its intensity is greater by a factor of

$$\frac{I_1}{I_2} = 10^{3/10} = 1.995. \tag{3.19}$$

This is very close to 2, so a useful rule of thumb is that when sound intensity level goes up by 3 dB the intensity roughly doubles. It immediately also follows that 6 dB is equivalent to multiplying the intensity by $2 \times 2 = 4$, and 9 dB is equivalent to multiplying by $2\times2\times2 = 8$. Another useful figure to remember is 7 dB, which gives

$$\frac{I_1}{I_2} = 10^{7/10} = 5.012, \tag{3.20}$$

which is very close to 5.

Decibels	Intensity
+0	×1
+3	×2
+4	×2.5
+6	×4
+7	×5
+9	×8
+10	×10

Table 3.2: Some approximate rules of thumb for increase in decibels and the corresponding factor by which intensity gets multiplied.

Table 3.2 summarizes these rules, and is worth memorizing if you will be working with decibels often. At a minimum, you should remember that +3 dB means twice the intensity and +6 dB means four times.

EXAMPLE 3.1: CONVERTING BETWEEN INTENSITY AND DECIBELS

A sound has intensity 10^{-10} W/m^2. What is the sound intensity level in decibels?

To answer this question, we note that 10^{-10} W/m^2 is 100 times larger than the reference level of $I_0 = 10^{-12}$ W/m^2, which is equivalent to 0 dB. So the sound intensity level is 20 dB.

EXAMPLE 3.2: DECIBEL DIFFERENCES

Suppose sound 1 has 400 times the intensity of sound 2. How many decibels difference is there?

400 is $4 \times 10 \times 10$ so, looking at Table 3.2, the difference between the sound intensity levels is about $6 + 10 + 10 = 26$ dB. This answer is probably accurate enough for most purposes, but if we want the exact figure we can use Eq. (3.16), which gives a difference of $10 \log 400 = 26.02$ dB.

EXAMPLE 3.3: INTENSITY DIFFERENCES

Suppose that sound 1 is 34 dB louder than sound 2. How much bigger is its intensity?

To answer this question we first write $34 = 10+10+10+4$. Then, referring to Table 3.2, we see that the intensity of sound 1 is greater by a factor of roughly $10\times10\times10\times2.5 = 2500$. If we want the exact figure we can use Eq. (3.17), which gives $10^{34/10} = 2512$.

3.2.2 Sound pressure level

In Section 3.1.3 we saw that the average intensity I is proportional to the mean-square sound pressure $\langle p^2 \rangle$ according to

$$I = \frac{\langle p^2 \rangle}{\rho c}, \tag{3.21}$$

where ρ is the density of air and c is the speed of sound. Although intensity is the standard scientific measure of the loudness of sounds, this equation implies that we can also quantify loudness in terms of the mean-square pressure, and in practice it is usually much easier to do it this way. Sound meters measure sound pressure and virtually all loudness measurements are actually measurements of pressure. We can always convert to intensity if we want to using Eq. (3.21).

As with intensity, however, mean-square pressure does not provide a very convenient unit for measuring the loudness of sounds, so we switch to a scale akin to the decibel scale we use for intensity. By analogy with the sound intensity level defined in Section 3.2, we define the *sound pressure level* or SPL, denoted L_P, using the formula

$$L_P = 10 \log \frac{\langle p^2 \rangle}{p_0^2}, \tag{3.22}$$

where p_0 is a constant whose value we will choose in a moment. Sound pressure level is measured in units of decibels, just like sound intensity level. If we want to be clear about the difference we can write "dB SPL" or "dB SIL" to indicate which we are talking about. Thus you might see a sound pressure level written as "60 dB SPL."

Often, however, this not necessary. Although sound intensity level and sound pressure level may at first appear to be different quantities they are really measuring the same thing because of Eq. (3.21). Substituting this equation into the definition of sound intensity level in Eq. (3.11) we have

$$L_I = 10 \log \frac{I}{I_0} = 10 \log \frac{\langle p^2 \rangle}{\rho c I_0}. \tag{3.23}$$

Comparing with Eq. (3.22) we see that we can make L_P and L_I equal to one another if we make the right choice for the constant p_0. If we choose

$$p_0^2 = \rho c I_0 \tag{3.24}$$

then Eqs. (3.22) and (3.23) become equal and sound pressure level equals sound intensity level. Plugging in the values of ρ, c, and I_0, this gives us $p_0^2 = 4.13 \times 10^{-10}\,\text{Pa}^2$ or, taking the square root,

$$p_0 = 2.03 \times 10^5\,\text{Pa}. \tag{3.25}$$

This is very convenient: we can measure loudness in terms of either intensity or pressure, whichever is easier, and get the same number of decibels either way.

Unfortunately, we cannot simply adopt this value for p_0 and call it quits, because both the density of air and the speed of sound vary slightly with temperature as discussed in Section 1.6 (see Table 1.1 on page 13), which means p_0 is not always the same. The value in Eq. (3.25) is for a standard room temperature of 20°C, but at other temperatures we would need a different choice of p_0 to make SPL and SIL equal to one another. Values for a range of different temperatures are shown in Table 3.3.

Temperature (°C)	p_0 (Pa s/m)
−20	2.109×10^{-5}
−10	2.089×10^{-5}
0	2.069×10^{-5}
10	2.051×10^{-5}
20	2.033×10^{-5}
30	2.016×10^{-5}

Table 3.3: The value of the constant p_0 in Eq. (3.22) that makes SPL and SIL equal.

In practice, this variation in the value of p_0 is often ignored. For the sake of simplicity, we typically just set p_0 equal to the round number

$$p_0 = 2 \times 10^5\,\text{Pa}. \tag{3.26}$$

With this choice, sound pressure level and sound intensity level are very nearly *but not quite* equal to one another, which is annoying though usually not a big deal. At 20°C, for instance, they differ by just 0.14 dB, which is small enough to not matter in most cases. If it does matter one can simply add 0.14 dB to the measured SPL to get the value of the SIL. Again, however, the correction will change with temperature. For instance, it rises to 0.29 dB at 0°C and falls to as low as 0.07 dB at 30°C. Modern sound level meters have a built-in thermometer to measure the temperature and can perform the correction automatically. In this book, however, the difference between SIL and SPL will not be large enough to concern us and we will just assume that the two are equal: we will use the conventional value of p_0 in Eq. (3.26), as most others do also, and ignore the small discrepancy that arises as a result.

Note also that even though there may be a small difference between SIL and SPL, *changes* in SIL and SPL are always equal. From Eqs. (3.16), (3.21), and (3.22) the change in sound intensity level between two sounds is

$$L_{I,1} - L_{I,2} = 10\log\frac{I_1}{I_2} = 10\log\frac{\langle p_1^2\rangle/\rho c}{\langle p_2^2\rangle/\rho c} = 10\log\frac{\langle p_1^2\rangle}{\langle p_2^2\rangle} = 10\log\frac{\langle p_1^2\rangle/p_0^2}{\langle p_2^2\rangle/p_0^2} = L_{P,1} - L_{P,2}. \tag{3.27}$$

This result is true no matter what value we use for p_0, and hence it is always the case that the increase (or decrease) in SIL when the loudness of a sound changes is equal to the increase (or decrease) in SPL.

The same rules of thumb from Table 3.2 also apply to calculating changes in SPL.

For all of these reasons it is often not important to know whether we are talking about SIL or SPL and in such cases we sometimes just refer to the "sound level," without specifying whether we are talking about intensity or pressure.

3.2.3 Energy density

Intensity is the most basic measure of sound energy, but there are a few situations where it is not appropriate. Since intensity measures the sound hitting one square

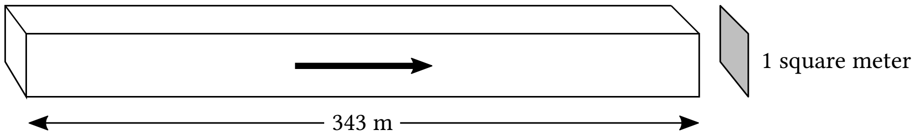

Figure 3.2: Energy density of a sound wave. For a sound wave traveling to the right at the speed of sound, 343 m/s, all of the sound energy in a box 343 m long and 1 m^2 across will fall on the 1 m^2 target in one second.

meter per second, it only works when the sound is traveling in a definite direction as in a normal sound wave. Imagine holding up a one-square-meter target to the sound. You would have to point it towards the direction the sound is coming from if you want to capture the sound. But there are some cases where sound does not have a direction. The two most important are *standing waves*, a stationary resonating form of sound that occurs in enclosed spaces, and *reverberation*, which is sound that bounces around inside a room or hall. Standing waves do not travel at all and hence do not have a direction, and reverberation goes in all directions at once. Thus intensity is not well defined in either of these cases.

For cases such as these, we use a different measure of sound energy, the *energy density*, denoted w. Energy density measures the amount of sound energy in one cubic meter of air volume. It is measured in units of joules per cubic meter, or $\mathrm{J/m^3}$. Consider, for example, the situation shown in Fig. 3.2. The oblong region depicted is 343 m long and one square meter in cross-section, meaning it has volume $V = 343\,\mathrm{m}^3$. A sound wave is traveling to the right as indicated by the arrow and, since sound travels at $c = 343\,\mathrm{m/s}$, all of the sound in the region will hit the 1 m^2 target on the right in one second. Thus the intensity at the target, the amount of energy hitting it in one second, is equal to the amount of energy in the region. But if the energy density in the region—the amount of energy per cubic meter—is w, then the total energy is w times the volume or $343\,w$, so the intensity is $I = 343\,w$ or just

$$I = cw. \tag{3.28}$$

Rearranging this equation, the energy density w is

$$w = \frac{I}{c}. \tag{3.29}$$

Thus for a sound wave there is a simple relationship between intensity and energy density.

For other types of sound such as standing waves and reverberation the intensity is not well defined, but the energy density is and hence energy density is the preferred measure of sound energy. Even when sound is not going in any particular direction, we can still talk about the amount of sound energy in a cubic meter. If we want to we can convert energy density to an equivalent intensity using Eq. (3.28), which tells us the intensity that the sound *would* have if the same energy density were present in an ordinary sound wave traveling in a single direction. This can sometimes be a useful approach—it can give us a feeling for how loud a sound will be—but it is important to understand that intensity itself is not well defined for sounds without a direction, so the value of I does not represent an actual intensity that one could measure in those cases.

One can use energy density to calculate a decibel measure of sound level analogous to the SIL and SPL thus:

$$L_E = 10 \log \frac{w}{w_0}, \tag{3.30}$$

where w_0 is a constant whose value we will choose in a moment. The quantity L_E is called the *sound energy density level* or *sound energy level*, abbreviated SEL and measured in units of decibels. In the case of an ordinary sound wave, for which w is given by Eq. (3.29), we can put $w = I/c$ and get

$$L_E = 10 \log \frac{I}{c w_0}. \tag{3.31}$$

Comparing with Eq. (3.11) on page 60, we see that the sound energy level will be simply equal to the sound intensity level if we have $c w_0 = I_0$, which means

$$w_0 = \frac{I_0}{c}. \tag{3.32}$$

With $c = 343\,\text{m/s}$ this gives us

$$w_0 = \frac{10^{-12}}{343} = 2.91 \times 10^{-15}\,\text{J/m}^3. \tag{3.33}$$

With this choice SEL and SIL are equal to one another. For a sound that is not a simple plane wave, such as a standing wave or reverberating sound, we can still apply Eq. (3.30) to define L_E. If we make the choice (3.32) for w_0 then L_E is equal to the SIL that a plane wave with the same energy density would have. This could be a useful guide to how loud the sound would be, even though the SIL is not well defined.

Unfortunately, the constant w_0 in Eq. (3.32) depends on temperature, since the speed of sound depends on temperature, so we cannot simply fix w_0 to take the value in Eq. (3.33)—it would need to be adjusted depending on the prevailing temperature. But in any case we don't normally use this value of w_0. For historical reasons w_0 is usually set to a completely different value of

$$w_0 = 10^{-12}\,\text{J/m}^3. \tag{3.34}$$

With this choice, the SEL is not equal to the SIL, even approximately. The SIL will be greater than SEL by about 25.4 dB and to convert SEL values to SIL you would need to add this number to the SEL.

In practice, however, we are usually concerned only with *changes* in sound energy level, which are not affected by the value of the constant w_0. The difference in SEL from one energy density to another is given by

$$L_{E,1} - L_{E,2} = 10 \log \frac{w_1/w_0}{w_2/w_0} = 10 \log \frac{w_1}{w_2}, \tag{3.35}$$

which does not depend on w_0. And changes in SEL are always the same as changes in SIL because

$$L_{E,1} - L_{E,2} = 10 \log \frac{w_1}{w_2} = 10 \log \frac{I_1/c}{I_2/c} = 10 \log \frac{I_1}{I_2} = L_{I,1} - L_{I,2}. \tag{3.36}$$

Thus if we want to know the change in sound intensity level but we only have the energy density and not the intensity, we can simply calculate the change in sound energy level instead—the two changes will always be equal.

3.3 Obstruction of sound by obstacles

So far we have discussed sound waves spreading unhindered through the air, but in many cases they may encounter obstacles such as people, furniture, walls, cars, and so forth. It is a matter of common experience that these objects can get in the way of the sound. Sound can be muffled if you hear it from behind a large piece of furniture, for example, or blocked altogether by a wall.

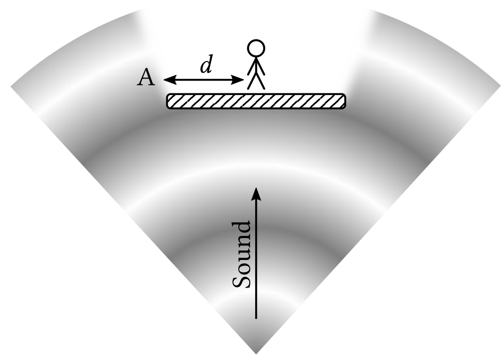

Figure 3.3: Sound wave blocked by a wall. In order for the listener behind the wall to hear the sound, the air pressure has to equalize over the distance marked *d*.

Take a look at Fig. 3.3, which shows a sound wave striking an obstruction such as a short wall. There is a listener standing behind the wall as the sound wave goes past. What will they hear? As the sound goes by the end of the wall it causes the pressure to go up and down at the point marked A in the figure. As we argued in Section 1.2, when you raise the pressure at a particular point, that pressure change doesn't just stay there. It will spread out from that point, the pressure equalizing as in the tube in Fig. 1.1 (page 3). The change in pressure spreads at the speed of sound c, meaning it will reach the listener in Fig. 3.3 a distance d away in an amount of time

$$t = \frac{\text{distance}}{\text{velocity}} = \frac{d}{c}. \tag{3.37}$$

However, this argument only applies if the pressure at point A stays roughly the same for long enough for the pressure to equalize. If it does not, if it changes before spreading all the way to the listener, then the pressure will never fully equalize. For full equalization therefore—for the listener to experience the full variation in pressure of the sound wave—the pressure should not change significantly during time t. Another way of saying this is that the period T of the waveform should be significantly longer than t. We can write this in mathematical notation as $T \gg t$, where the symbol "$\gg$" means "much greater than."

With t as in Eq. (3.37) above, this gives us $T \gg d/c$, or equivalently $cT \gg d$. But Eq. (2.3) on page 20 tells us that the wavelength of a sound is $\lambda = cT$, so we have

$$\lambda \gg d. \tag{3.38}$$

In other words, even though there is a wall in the way, the listener will nonetheless hear the sound clearly if the wavelength of the sound is much greater than the distance d. In this case d is about a half the width of the wall, although it could be a bit more or a bit less depending on where the listener is. So a reasonable rule of thumb is that if the wavelength is greater than the full width of the obstruction—or equivalently if the size of the obstruction is less than the wavelength–then the sound will be unaffected. In effect, sound doesn't "see" obstacles in its path that are smaller than the wavelength and flows around them. The technical term for this effect is *diffraction*: we say the sound diffracts around the obstacles. Conversely, obstacles larger than the wavelength will impede the sound and a listener behind a large obstacle will experience a reduction in volume or may even not hear the sound at all if the obstacle is very large.

Example 3.4: Sound obstruction by a pillar

An auditorium has a 1 meter wide pillar in the middle to reinforce the ceiling. How will this pillar affect the sound coming from the stage for a listener immediately behind it?

The argument given above implies that sound will pass around the pillar and reach the listener provided its wavelength is greater than about 1 meter. Equation (2.6) tells us that a sound with wavelength 1 m has frequency

$$f = \frac{c}{\lambda} = \frac{343}{1} = 343\,\text{Hz}. \tag{3.39}$$

Wavelengths longer than 1 meter correspond to frequencies *lower* than this, so sounds with frequency lower than about 343 Hz will reach the listener unimpeded. Higher frequencies on the other hand will be obstructed by the pillar and will sound quieter, perhaps even inaudible.

Looking at Fig. 2.11 on page 34, we see that 343 Hz corresponds approximately to the musical note F4 (the F above middle C), meaning that the pillar would have a substantial impact on a musical performance: pitches in the upper half of the piano keyboard would be reduced or obscured completely and only those in the lower half would reach the listener unchanged. The common term for such a sound is "muffled." When we say a sound is muffled, we mean precisely that only the low frequencies are getting through and the high ones are cut off or cut down. The pillar in this case would muffle the sound.

Example 3.5: Sound obstruction by the human head

Your ear is situated on the side of your head, which can affect your perception of sound because your head itself blocks the sound. The average human head is about 22 cm across, or 0.22 m, so it will obstruct sound with a wavelength shorter than this. Putting this wavelength into Eq. (3.39), we find that the corresponding frequency is

$$f = \frac{c}{\lambda} = \frac{343}{0.22} = 1559\,\text{Hz}, \tag{3.40}$$

which is approximately the note G6. For frequencies lower than this the human head poses no obstacle to the movement of sound—sound will diffract around it and reach the ear unobstructed—but higher frequencies will be attenuated. G6 is a relatively high note, close to the highest note a soprano can sing, but higher notes than this are regularly played in normal music by instruments like the piano and violin, so this attenuation will make a difference to how music sounds. High notes will sound different depending, for instance, on how you hold your head. If you point your ear directly towards the sound you will hear it unobstructed, but if you point your ear away then the presence of your head will make the sound quieter. Of course you have two ears, one on either side of your head, so there will always be one of them pointing in the right direction, and indeed we use exactly this effect to tell which direction sound is coming from—if it is louder in one ear than the other then it must be coming from that side.

How we perceive the direction of sound is discussed in detail in Section 5.5.

This kind of obstruction should not be regarded as a bad thing, but rather it is an integral part of the way human hearing works. In addition to its use in telling the direction of a sound, we are also just used to the way our ears pick up sound, and music would sound different if things were to change. Whether we are aware of it or not, our ear picks up less high-frequency sound when it is coming from the other side of our head and our experience of hearing music is naturally shaped by this effect.

On the other hand, while the presence of the obstruction is not necessarily bad, its *absence* can be. This happens for instance in the playback of recorded sound over headphones. If you listen to a recording on loudspeakers, sound reaches the ear in a natural manner, diffracting around your head, with the accompanying reduction in high frequencies. But if you listen over headphones or earphones the sound is piped directly into your ears and there is no such reduction, leading to an unnatural boost in the higher-frequency parts of the sound.

There are other reasons why headphone sound is problematic. In particular, stereo imaging is substantially compromised when listening on headphones, unless the recording has been made using special "binaural" techniques—see Section 7.6. This is the main reason why mixing engineers prefer speakers to headphones.

For most of us, the difference is not crucial. We might be able to hear it if we compared sounds side by side, but under normal circumstances it does not affect our enjoyment of the music. There are specialized situations in which it can matter, however. For recording or mixing engineers creating music in the studio, for instance, it is important to consider how the music will be heard. Will it be listened to primarily over loudspeakers (such as in the car or on TV) or over headphones or earphones (such as on your phone)? Mixing of studio recordings is normally done using loudspeakers, not headphones, so one could argue that most recordings are best heard that way, although in practice the use of headphones and earphones is so widespread that most commercial recordings deliberately aim for a compromise that sounds reasonably good on both.

Example 3.6: Sound capture by microphones

A related question concerns the recording of sound using microphones. Just as sound reaching your ear can be obstructed by your head, sound reaching the diaphragm of a microphone—the part that does the actual recording—can be obstructed by the body of the microphone. The body of a microphone, however, is much smaller than a human head. A typical microphone is about 2 cm across and in practice this means that there is much less obstruction than in the case of the human head. Obstruction will occur only if the wavelength is less than about 2 cm, which corresponds to a frequency of

$$f = \frac{c}{\lambda} = \frac{343}{0.02} = 17\,150\,\text{Hz}. \tag{3.41}$$

The highest frequency the human ear can hear is around 20 000 Hz or a little lower. See the discussion in Section 5.1.

This is far above the frequency of even the highest musical note and close to the highest frequency that the human ear can hear. This means that most sound will flow easily around the microphone to the diaphragm and hence that microphones can capture essentially all musical sounds unimpeded, no matter how they are pointing or what direction the sound is coming from.

This is not always convenient. For live stage use with amplified music, it is often better if microphones pick up sound mainly from one direction, so as to be able to focus on just one instrument or singer, and also because it helps to reduce the unpleasant squealing sound known as acoustic feedback. In Section 7.3.2 we discuss how to build directional microphones that pick up sound from one direction only.

3.4 Combining intensities

It happens often in music that more than one instrument plays at the same time. How will this affect the volume of the music? To answer this question we make use of one of the most fundamental principles of science, conservation of energy, which says that energy cannot be created or destroyed. Energy can move around from one place to another, but the amount of energy always stays the same. Intensity is a measure of the amount of energy contained in a sound and if two musical instruments play at the same time then conservation of energy implies that the total amount of sound energy must be the sum of their individual energies. If the total amount were greater or less than this it would imply that some energy must have been created or destroyed, which cannot happen.

So when two instruments—or two sounds of any kind—are played at the same time a very simple rule applies: we add together their energies. In practice, this means we add their intensities: since intensity is the amount of energy per second per square meter, and since energies add, so do intensities. The same rule also extends to any number of instruments or sounds played together. We simply add together all the intensities to get the total intensity of the combined sound.

If we want to calculate the *sound intensity level* (i.e., decibels) when two or more sounds are played together then the most straightforward approach is to calculate the *intensity* first, using the simple rule above that intensities add, then convert the result to decibels using Eq. (3.16). An example is given below.

Example 3.7: Combining intensities

In the 1957 musical *The Music Man* (words and music by Meredith Willson) the song *Seventy-Six Trombones* describes a legendary marching band of improbable proportions, led by the titular 76 trombones. How much louder, in decibels, would 76 trombones sound than just a single trombone?

In reality all 76 trombones would probably not be playing exactly the same thing. Some might be slightly louder or softer; some slightly sharp or flat. But let's make things easy for ourselves and assume that they all play the same notes at the same volume level. Then we simply add together the intensities. If the intensity for a single trombone is I_t, then the intensity for all 76 of them will be $76I_t$. We can convert this into decibels using Eq. (3.16). We write the intensity for 76 trombones as $I_1 = 76I_t$ and the intensity for a single trombone as $I_2 = I_t$. Then

$$L_1 - L_2 = 10\log\frac{I_1}{I_2} = 10\log\frac{76I_t}{I_t} = 10\log 76 = 18.8\,\text{dB}. \quad (3.42)$$

So 76 trombones would be 18.8 decibels louder than one trombone.

In general, if n identical instruments are playing identical parts at the same time then the intensity will be n times louder and the sound intensity level will be $10\log n$ decibels greater. Alternatively, applying the rules of thumb given in Table 3.2, we can say that two instruments will be about 3 dB louder than one, four instruments will be 6 dB louder, five instruments will be 7 dB louder, and so forth.

3.5 Variation of intensity with distance

It's a matter of common experience that sound volume diminishes with distance. A jumbo jet close up is a lot louder than one far away. We can understand this phenomenon by again appealing to the principle of conservation of energy. Take a look at Fig. 3.4, which depicts a sound source and the sound radiating from it.

Suppose the intensity a certain distance x away is I, meaning that this is the amount of sound energy passing per second through a 1m × 1m square, as shown. Now suppose we move twice as far from the sound source, to distance $2x$. As shown in the figure, the sound spreads out as it travels so that after going twice as far it will have spread over an area twice as wide. This means that the sound that previously passed through our 1m × 1m square now passes through a 2m × 2m square.

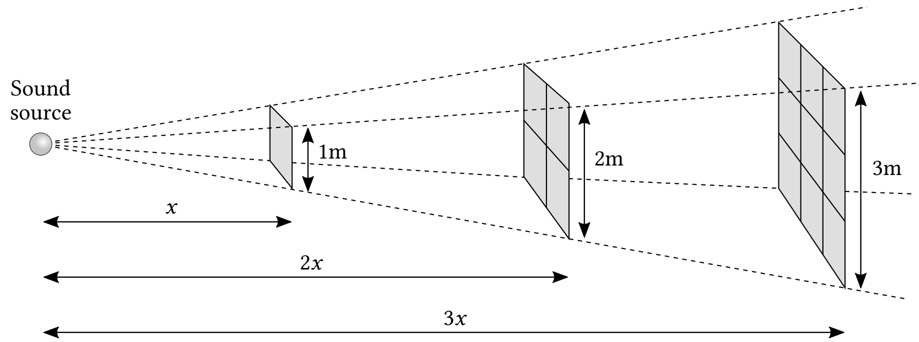

Figure 3.4: Variation of sound intensity with distance. Sound from a sound source spreads out as it travels, so that the sound energy that passes through a 1m × 1m square at distance x passes through a 2m × 2m square at distance $2x$, a 3m × 3m one at distance $3x$, and so forth.

A 2m × 2m square has an area of 4 square meters, yet the amount of energy passing through it is the same as before. Because energy is conserved—no energy can be created or destroyed—the total energy per second passing through $1\,\text{m}^2$ at distance x is the same as that passing through $4\,\text{m}^2$ at distance $2x$. And that means that the intensity—the energy per square meter—is a *quarter* of the size. In other words, when we double the distance from the sound source the intensity is divided by four.

We can continue the same argument. Suppose we move still further out, to distance $3x$ from the sound source. Again the sound spreads out as it travels, so that by the time it reaches distance $3x$ it occupies a 3m × 3m square, which has area $9\,\text{m}^2$. Because energy is conserved this means that the intensity at distance $3x$ is one ninth of what it was at distance x: when you triple the distance from the sound source the intensity is divided by nine.

You can probably see the pattern now: the factor by which the intensity decreases is the *square* of the increase in distance. In mathematical terms we can write the intensity at distance r from the sound source as

$$I(r) = \frac{I_1}{r^2}, \qquad (3.43)$$

where I_1 is the intensity at one meter. This is the *inverse square law* for sound intensity. It tells us exactly how intensity decreases with distance from a sound source. For example, the intensity at distance $r = 2\,\text{m}$ from a source is

$$I(r = 2) = \frac{I_1}{2^2} = \tfrac{1}{4} I_1, \qquad (3.44)$$

while at 10 m we have

$$I(r=10) = \frac{I_1}{10^2} = \frac{1}{100} I_1, \tag{3.45}$$

and so forth.

Often it is more convenient to express the change in intensity in terms of the sound intensity level in decibels, which is straightforward to do. For instance, when we double the distance from a sound source the intensity goes down by a factor of four and, looking at Table 3.2, we see that a factor of four corresponds to a change in sound level of 6 dB. So when we double the distance the sound intensity level goes down by 6 dB. This is a useful rule of thumb to remember. Note that it doesn't depend on how loud the sound is to begin with or how far away we are initially. For any sound, at any distance, the sound intensity level will drop by 6 dB when we double the distance. (This assumes that there are no obstacles in the way to block the sound, such as walls or buildings, which could of course change the intensity in other ways.)

More generally, we can use Eq. (3.16) to calculate the difference in sound intensity level accompanying any change in distance. The intensities at any two distances r and s are

$$I(r) = \frac{I_1}{r^2}, \qquad I(s) = \frac{I_1}{s^2}, \tag{3.46}$$

so the difference in sound intensity levels is given by Eq. (3.16) to be

$$L(r) - L(s) = 10 \log \frac{I(r)}{I(s)} = 10 \log \frac{I_1/r^2}{I_1/s^2} = 10 \log \frac{s^2}{r^2}. \tag{3.47}$$

We can also ask how the sound pressure varies with distance from a sound source. As mentioned at the end of Section 3.1.3, intensity is proportional to the square of the amplitude of the sound pressure, which means that amplitude is proportional to the square root of intensity. Using Eq. (3.43), this means that the amplitude obeys

$$\text{Amplitude} \propto \sqrt{I(r)} = \sqrt{\frac{I_1}{r^2}} = \frac{\sqrt{I_1}}{r}, \tag{3.48}$$

so the sound pressure amplitude is inversely proportional to distance. When you double your distance from a sound source you half the sound pressure.

Example 3.8: The sound of an airplane taking off

An airplane takes off at an airport. According to Table 3.1 on page 59, a jet taking off generates an earsplitting 120 dB when heard from 500 m away. How loud will the same jet sound when heard from 10 km away?

Putting $r = 500$ m and $s = 10\,000$ m in Eq. (3.47) gives a change in sound intensity level of

$$10 \log \frac{10\,000^2}{500^2} = 26.0 \text{ dB}. \tag{3.49}$$

In other words the sound intensity level will be 26 dB quieter or $120 - 26 = 94$ dB in total. This is still an impressively loud sound. According to Table 3.1, at 10 km the airplane is still as loud as a passing subway train. This is why people who live near airports are perpetually disturbed by the volume of airplane noise.

So here's another question: how far away would we have to be for the sound of the airplane to fall to a more reasonable 60 dB? For the sound to fall to 60 dB from 120 dB it would have to lose 60 dB. Using Eq. (3.47) again, we write

$$10 \log \frac{s^2}{r^2} = 60 \tag{3.50}$$

and rearrange to get $s^2 = 1\,000\,000\, r^2$. Taking the square root of both sides and using the value of $r = 500$ m then gives

$$s = 1000\, r = 500\,000\,\text{m} = 500\,\text{km}. \tag{3.51}$$

You would have to be 500 km away (about 310 miles) for the sound to fall to 60 dB!

You might ask, then, why we do not hear planes taking off all the time? Almost everyone lives within 500 km of an airport. The answer is that these calculations assume that the sound is traveling unimpeded, without obstacles in its path. In reality, however, there are always obstacles in the path of the sound. The sound from a plane taking off would encounter many buildings, trees, and hills before it had traveled even 10 km, let alone 500. As we saw in Section 3.3, obstacles larger than the wavelength of a sound will reduce its volume and obstacles the size of a building or larger can block a sound completely. And that ignores the curvature of the Earth, which itself constitutes a large obstacle to the passage of the sound. Moreover, the volume level produced by an aircraft is loudest at takeoff when the engines are running at maximum thrust, dropping off considerably once the plane is in the air, so that the sound is already a lot quieter by the time the plane rises above the trees and buildings. Another issue is the presence of wind. Since sound travels through air, it tends to get blown away if the air it is in gets blown away. Like anything else airborne—smoke, dust, dandelion seeds—sound is easily swept away by wind. Nonetheless, it is definitely possible, on a quiet day, to hear an aircraft taking off from many miles away.

3.6 Apparent loudness

We have seen that intensity is a measure of the amount of energy carried by a sound. In general more energy means a louder sound, but how exactly does intensity translate into the volume levels we hear? You might imagine that a sound with ten times the intensity would be perceived as being ten times as loud, but in fact this is not what happens. The perception of loudness depends not only on the physics of sound itself but also on the workings of the human ear and the way the brain processes sound. These are complicated matters, not well understood by science even today, so the best way to answer questions about the perception of loudness is to do an experiment.

One simple experiment works as follows. A participant presses a button to hear two different sounds, typically notes or tones with the same pitch and waveform shape, generated electronically and played one after another. We also give the participant a knob that they can turn to adjust the volume of one of the tones, say the second one. The participant is instructed to play the two tones repeatedly and adjust the knob until the second tone sounds twice as loud as the first. Then we record the intensities of the two tones.

Not every human ear is the same and the results of experiments like this vary somewhat from person to person, but across many years of study an overall pattern has emerged that can summarized by a simple rule of thumb: the apparent loudness of a sound doubles when the intensity goes up by a factor of ten. Given that a factor of ten increase in intensity is equivalent to 10 dB, we can also say that the apparent loudness doubles when the sound intensity level goes up by 10 dB.

We can extend this rule to larger changes too. If the sound intensity goes up by 10 dB and then another 10 dB after that, then the apparent loudness will double and double again. So a 20 dB increase in sound intensity level gives you an increase in apparent loudness of a factor of four. Similarly 30 dB gives you a factor of 8 and 40 dB gives you a factor 16. Since we just multiply by 2 for every 10 dB increase, all these numbers are powers of two:

SIL increase (dB)	Apparent loudness increase	Power of 2
10	2	2^1
20	4	2^2
30	8	2^3
40	16	2^4

You can perhaps see the pattern here. To turn decibels into an increase in apparent loudness, we divide by 10 and take 2 to the power of the resulting number. For instance, if the sound intensity level goes up by 30 dB then we divide by 10 to get 3 and calculate $2^3 = 8$ for the apparent loudness increase.

We can use this recipe to define a scale for apparent loudness, which is called the *sone* scale. One sone is defined to be the apparent loudness of a 1000 Hz tone played at 40 dB. This sets a reference level against which we can calculate the loudness of other sounds using the rules above. For example, a 50 dB tone will sound twice as loud as 40 dB, i.e., 2 sones. A 60 dB tone will sound four times as loud, i.e., 4 sones, and so forth. The general formula for apparent loudness N in sones is

$$N = 2^{(L-40)/10}, \tag{3.52}$$

where L is the sound intensity level. For example, a 60 dB tone will have an apparent loudness of

$$N = 2^{(60-40)/10} = 2^2 = 4 \text{ sones}, \tag{3.53}$$

which agrees with our quick calculation above. For a more complicated example, a 65 dB tone will have an apparent loudness of

$$N = 2^{(65-40)/10} = 2^{2.5} = 5.7 \text{ sones}, \tag{3.54}$$

meaning that it will sound almost six times louder than the 40 dB reference tone.

3.6.1 Variation of apparent loudness with frequency

Apparent loudness also varies with frequency. Our ears are not equally sensitive to sounds at all frequencies, and in particular they are quite insensitive to low frequencies. This means that two sounds with the same intensity but different frequencies might seem quite different in loudness.

To measure variation with frequency we can again perform an experiment. In this experiment pressing a button again plays two tones one after another, but now they are at different frequencies. Again the participant has a knob that varies the volume of the second tone and we ask them to adjust the knob until the two tones sound equally loud, then we record the intensities of the tones.

Figure 3.5 shows the results of such an experiment. In this case the first tone in the experiment was again set at 1000 Hz and 40 dB—indicated by the dot in the middle of the figure—and the graph shows what sound level the second tone must have to make it sound as loud as the first, for various different frequencies. Thus the sounds at every point along this curve have the same apparent loudness.

Again, these results vary somewhat from person to person because not everyone's ears are the same, but this graph is typical. As we can see, there are some regions, especially for low frequencies, where the second tone must be a lot louder than 40 dB to sound as loud as the 40 dB reference tone. The curve rises almost to 90 dB at the low end of the hearing range around 20 Hz because the ear is quite insensitive at these frequencies. Conversely, there are also some regions, notably around 500 Hz and again around 4000 Hz, where the second tone must be *quieter* than 40 dB to sound as loud as the reference tone. These are the frequencies where the ear is most sensitive. The curve ends around 17 000 Hz because this is the upper end of the human hearing frequency range—the ear cannot hear frequencies higher than this at all.

The graph in Fig. 3.5 is called an *equal loudness curve* or *Fletcher-Munson diagram* (after the two scientists who first drew it), and it represents the sensitivity of the human ear to sounds of different frequencies. An important and sometimes confusing point about the Fletcher-Munson diagram is that the curve is *higher* in regions where the ear is *less* sensitive, because it represents how loud a sound has to be to sound the same as the reference tone. This can be a little counterintuitive, so it is worth pausing a moment to make sure you understand the meaning of the graph.

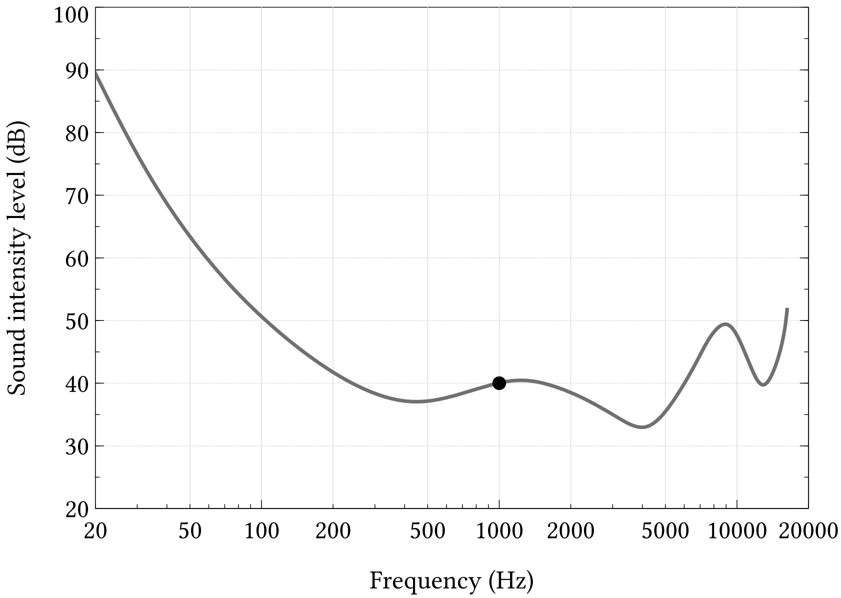

Figure 3.5: Curve of constant apparent loudness. This graph shows for each frequency what sound intensity level a tone must have to sound as loud as a 1000 Hz reference tone at 40 dB (indicated by the dot). A plot of this kind is called an equal loudness curve or Fletcher-Munson diagram.

The relative sensitivity of the ear to high- and low-frequency sounds also depends on whether the sounds are loud or quiet. We can probe this variation by repeating our experiment with reference tones at different volume levels. By convention the reference tone always has frequency 1000 Hz, but we can vary its volume all the way from a barely audible 10 dB to an earsplitting 130 dB. Figure 3.6 shows the results from such a set of experiments. Each curve in this plot shows an experiment using a reference tone with a different volume level. As we can see, the curves are roughly similar in shape but differing in details. In particular, note how the curves are flatter when the sound is louder (upper curves) versus when it is quieter (lower curves). This tells us that the relative insensitivity of the ear to low-frequency sounds is more pronounced at low volume levels than at high. At the highest volumes, around 100 dB and above, the ear is roughly uniformly sensitive across quite a wide band of frequencies.

The lowest curve on the diagram, indicated by the dashed line, is for a reference tone at 0 dB, which is nominally the quietest audible sound. Thus every point along this curve represents a sound that is right at the limit of audibility. Below this curve,

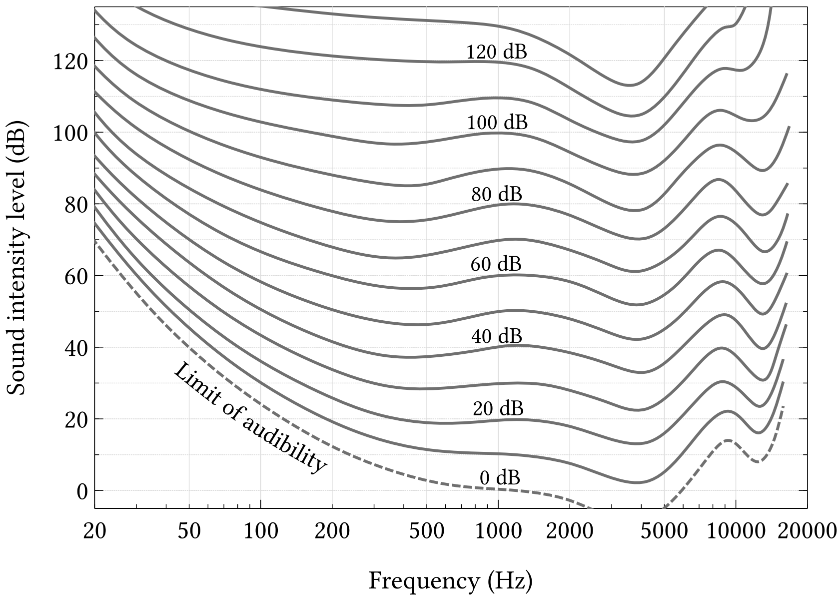

Figure 3.6: Fletcher-Munson diagram. This figure shows how loud a tone must be at various frequencies in order to sound as loud as a 1000 Hz reference tone. Each curve is for a reference tone of different loudness, as indicated by the labels, starting from 0 dB at the bottom of the plot and extending all the way up to 130 dB at the top. Since 0 dB is the nominal limit of audibility, the curve for 0 dB, indicated by the dashed line, tells us the quietest audible sound at each frequency, and there are no curves below this line.

the typical human ear hears nothing. Thus, for instance at 20 Hz the quietest audible sound is at sound intensity level 70 dB. In the middle of the frequency range 70 dB would be quite a loud sound, but at 20 Hz it is just the quietest whisper.

Example 3.9: Fletcher-Munson curves

Two tones are played at frequencies of 200 Hz and 1000 Hz and the same sound intensity level. To the human ear they sound equally loud. What is the sound intensity level?

To answer this question let us consult the Fletcher-Munson diagram in Fig. 3.6. If the two tones sound equally loud they must fall on the same curve in the diagram, and if they are played at the same sound intensity level they must have the same decibel value at the frequencies of interest. So we are looking for a single curve that has the same height at 200 Hz and 1000 Hz. There are actually several curves that satisfy this criterion. The curves for the 50 dB and 60 dB reference tones appear to be about right, so one possibility is that the tones are around this level. But the curves for 100 dB and 110 dB also match the criterion, so it's also possible the tones are at this level. We cannot tell which is the correct answer without having more information, but it must be one of these: either the sound is at about 50 dB or about 100 dB.

3.6.2 The apparent loudness scale

We can use the results of the previous section to extend our "sones" loudness scale to sounds at any frequency. Look again at Fig. 3.5. A tone played at any point along the curve in this figure will have the same apparent loudness: it will sound as loud as a 40 dB tone at 1000 Hz. Let us define the *apparent loudness level* L_N of all of these tones to be 40 dB. More generally, the apparent loudness level of any tone is defined to be the sound intensity level of a 1000 Hz tone that sounds equally loud. But from Eq. (3.52), a 1000 Hz tone with sound intensity level L_N has apparent loudness

$$N = 2^{(L_N - 40)/10}. \tag{3.55}$$

This then is the apparent loudness in sones of any sound at any frequency with apparent loudness level L_N in decibels. (Be careful to distinguish *apparent loudness*, which is measured in sones, from *apparent loudness level*, which is measured in decibels. Like intensity and sound intensity level these are different things, although with confusingly similar names.)

To apply Eq. (3.55) to a specific sound, you first find the sound intensity level of the 1000 Hz tone that sounds equally loud, then feed that number into this equation. Some examples are given below.

Example 3.10: Apparent loudness of a tone

What is the apparent loudness in sones of a 50 Hz tone at 70 dB?

To answer this question, we look at the Fletcher-Munson diagram in Fig. 3.6. We see that a 50 Hz tone at 70 dB falls on the same equal loudness curve as a 1000 Hz tone at 50 dB. Thus the apparent loudness *level* of the 50 Hz tone is $L_N = 50$ dB. Putting this value into Eq. (3.55) we calculate an apparent loudness of

$$N = 2^{(50-40)/10} = 2^1 = 2 \text{ sones}. \tag{3.56}$$

In other words the tone will sound twice as loud as a reference tone at 1000 Hz and 40 dB.

Example 3.11: Apparent loudness of two tones

A tone is played at 50 Hz and 70 dB sound intensity level. A second tone is played at 5000 Hz and 110 dB. Which will sound louder, and by how much will the apparent loudness differ?

As we saw in the previous example, the first tone has an apparent loudness of 2 sones. Let us call this N_1. Looking at the Fletcher-Munson diagram in Fig. 3.6, the second tone at 5000 Hz and 110 dB is on the same equal loudness curve as a 1000 Hz tone at 120 dB, so its apparent loudness level is $L_N = 120\,\text{dB}$. Putting this figure into Eq. (3.55), the corresponding apparent loudness is

$$N_2 = 2^{(120-40)/10} = 2^6 = 64 \text{ sones.} \tag{3.57}$$

Thus the second tone will sound about 32 times louder than the first.

3.6.3 Apparent loudness in music

The apparent loudness of sound, and particularly its variation with frequency, has substantial effects on the way we hear and play music. First, the simple fact that the ear is more sensitive to some frequencies than others has a big impact on musical sound. The Fletcher-Munson diagram of Fig. 3.6 tells us that the sensitivity of the ear is roughly the same across a broad band of frequencies from about 100 Hz to about 12 000 Hz. Within this range the sensitivity of the ear varies by no more than 10 dB in either direction, i.e., a factor of two in apparent loudness. Outside this range, however, the ear is less sensitive. The drop off at high frequencies has relatively little musical impact because 12 000 Hz is well above the frequency of the highest musical notes, and indeed outside many people's hearing range, particularly for older listeners. Frequencies below 100 Hz, however, are musically important, accounting for a full two octaves in the range of the lowest instruments such as double bass and tuba, not to mention piano and organ.

See Section 5.1 for a detailed discussion of human hearing range, including how it changes with age.

This means, for instance, that the lowest bass notes will sound significantly quieter than mid-frequency notes if played at the same intensity. The sound of a symphony orchestra, for instance, which spans a wide range of frequencies, will be dominated in our hearing by the higher frequencies of instruments like violins and trumpets, and much less by the double bass. In order for low bass notes to sound loud, they would have to be played with considerably more intensity than higher notes—hundreds or even thousands of times more in some cases. In popular music, especially dance music, that relies heavily on low bass frequencies, substantial amplification is needed to produce a strong bass. An amplifier designed for bass guitar, for instance, might have a maximum power output of 500 watts or more. By contrast, a typical guitar amplifier, which focuses on higher frequencies, might need only 50 watts.

The varying shapes of the different curves on the Fletcher-Munson diagram also have substantial effects. Note in particular that the curves are flatter for loud sounds than for soft ones. This means that bass frequencies will be more prominent relative to mid and high frequencies when music is loud. This is part of the reason why loud music sounds fuller and more satisfying to most people and why orchestras and bands sound better when you hear them up-close and loud.

The shapes of the curves are also important for recorded music. Suppose a recording is made of a relatively loud musical performance, with an overall sound intensity level around 90 dB, which is the upper end of the musical loudness range. At this volume level the Fletcher-Munson diagram of Fig. 3.6 tells us that a sound at 50 Hz would only have to be about 10 dB louder than a sound at 1000 Hz if the two are to have the same apparent loudness.

Now suppose you take that recording and play it back in the quiet of your room. Normally you would play it at a much lower volume level than the original live performance. You would wake the entire neighborhood if you played it at 90 dB, so perhaps you play it at around 40 dB instead. In that case Fig. 3.6 tells us that a sound at 50 Hz would now need to be about 25 dB louder than a sound at 1000 Hz to have the same apparent loudness.

In practice, this means that the balance of high and low frequencies in the sound will be substantially altered when we play the recording quietly. Sounds at high and low frequencies that seem equally loud to our ears when we hear a loud live performance will have completely different volume levels, by many decibels, when we listen to a recording of the same performance, because the ear is simply less sensitive to low-frequency sounds when they are played quietly.

In effect, this means that music has a "correct" volume level at which it should be heard. Played at a louder or quieter volume level it will not just sound louder or quieter—it will sound wrong. Listeners seeking an authentic musical experience from a recording need to consider the appropriate volume level for playback, and recording engineers must think about the circumstances in under which they expect their music to be heard. Will it be played loudly, say in a public space or a club? Or will it be heard quietly in someone's living room? The mixing and balance of the recording may need to be different in the two cases.

Example 3.12: Frequency balance in recorded music

A tone is played at 1000 Hz and 90 dB. Another is played at 50 Hz and has the same apparent loudness as the first. Later, a recording of the two sounds is played back more quietly, with the 1000 Hz tone now at 40 dB, so that the overall sound is 50 dB quieter than the original. Will the two tones still sound equally loud? If not, which will be louder and by how much?

To answer this question we refer again to the Fletcher-Munson diagram in Fig. 3.6. From this diagram we learn that in order to sound as loud as the 1000 Hz, 90 dB tone, the 50 Hz tone must have originally been played at about 100 dB. So when the recording is played back 50 dB quieter, the two tones will now be at 40 dB and 50 dB. Looking at the diagram a second time, we see that a 50 Hz tone at 50 dB falls on the second curve up from the bottom, meaning that it sounds as loud as a 1000 Hz tone at 20 dB. In other words, the 50 Hz tone will now sound 20 dB quieter than the 1000 Hz one at 40 dB. This difference corresponds to about a factor of four in apparent loudness, where previously the two sounded equally loud. This will make a big difference to the balance of frequencies on the recording.

Another way to look at this result is to calculate the apparent loudness of each of the tones in units of sones. The apparent loudness of the original 1000 Hz tone at 90 dB is given by Eq. (3.55) to be

$$2^{(90-40)/10} = 2^5 = 32 \text{ sones}, \tag{3.58}$$

and the apparent loudness of the original 50 Hz tone is the same by definition, since we are told that the two sound equally loud. When the recording is played back, by contrast, the apparent loudness of the 1000 Hz tone is

$$2^{(40-40)/10} = 2^0 = 1 \text{ sone}, \tag{3.59}$$

while the apparent loudness of the 50 Hz tone is

$$2^{(20-40)/10} = 2^{-2} = \tfrac{1}{4} \text{ sone}. \tag{3.60}$$

So indeed the lower frequency now has a quarter the apparent loudness of the higher one.

3.7 Measuring loudness

The loudness of sounds can be measured using a *sound level meter* (also sometimes called a noise level meter) which measures the sound pressure level or SPL. A sound level meter consists of a microphone, some circuitry, and a digital readout on a display. The microphone measures instantaneous sound pressure and converts it into an electrical signal. The circuitry within the meter uses this to calculate the mean-square sound pressure and hence the sound pressure level from Eq. (3.22), and the result in decibels is shown on the display.

The workings of microphones are discussed in Section 7.3.

A sound level meter. The microphone is contained inside the protective foam bulb at the top.

There are a number of choices to be made when doing this in practice. As discussed in Section 3.1.3, calculating the mean-square sound pressure involves averaging pressure measurements over some interval of time and we must decide what interval to use. The time interval should be at least one cycle of the sound, to make sure we capture a representative sample of the waveform, but if the interval is too long the sound may change during the course of the measurement and we won't get a reliable fix on how loud it was. Most sound meters give the user a choice of three standard intervals: 1 second, 125 milliseconds (i.e., $\frac{1}{8}$ second), and 35 milliseconds. Conventionally these are referred to by the names "slow," "fast," and "impulse," often denoted S, F, and I on the controls of the meter. For most purposes the slow setting (1 second) works well, but the fast and impulse settings can be useful for measuring short-lived sounds.

A second choice that must be made is what value to use for the constant p_0 appearing in Eq. (3.22) when calculating the sound pressure level. The standard choice is $p_0 = 2\times10^5$ Pa, and this value is often used by sound level meters. However, as discussed in Section 3.2.2, if we want the sound pressure level to be exactly equal to the sound intensity level, a slightly different value is needed, and moreover the value changes with ambient temperature. More sophisticated sound level meters have a built-in thermometer and can calculate the correct value of p_0 at the prevailing temperature. Often there will be an option to use either this value or the standard value of $p_0 = 2 \times 10^5$ Pa at the user's discretion. In practice, however, the difference is small: the reading will only change by a small fraction of a decibel depending on the choice (see Section 3.2.2), so what setting we use will only matter if we need highly accurate numbers.

A further feature, found on almost all sound level meters, is the ability to mimic approximately the variation in sensitivity of the human ear with frequency. As we saw in Section 3.6.1, the ear is less sensitive at low frequencies and, to a lesser extent, at very high ones. For this reason a sound level meter that simply measures sound pressure level will not reflect the actual experience of a human listener. For instance, the meter might say a low-frequency sound is quite loud when to our ears it sounds quiet because we do not hear low frequencies well.

To allow for this effect, sound level meters incorporate a filter, an electrical circuit that cuts out some of the sound picked up by the microphone. (See Section 7.2.4 for a discussion of filters and how they work.) The filter cuts out a portion of the sound at the lowest and highest frequencies while leaving intermediate frequencies intact, and the SPL is then calculated from the sound that remains, thereby imitating the pattern of sensitivity of the ear. There are several different standards for exactly how the filter does this, but the most common one, known as a type-A filter, has the frequency profile shown in Fig. 3.7. The figure shows how many decibels the filter cuts the sound level by at each frequency. Thus, for instance, at 100 Hz the value in Fig. 3.7 is about −20 dB, meaning the level reported by the meter at this frequency will be 20 dB lower than the actual prevailing sound level.

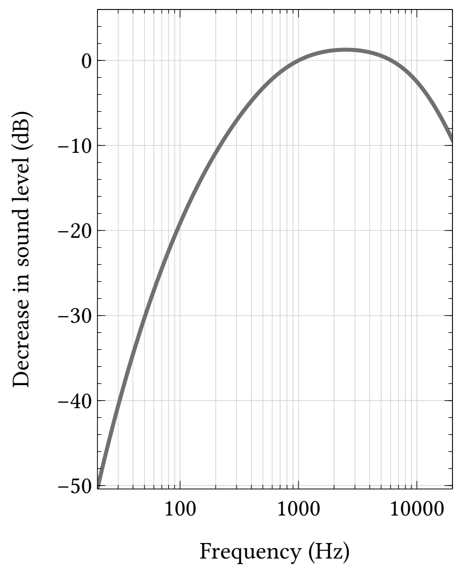

Figure 3.7: Frequency response of a type-A filter. The type-A filter used in sound level meters cuts the sound level by a certain number of decibels depending on frequency as shown here.

Sound levels measured through a type-A filter are known as *A-weighted levels* and are in units of A-weighted decibels, denoted "dBA." Thus a sound level of 50 dBA means 50 decibels of sound pressure level after the sound has been passed through a type-A filter.

The type-A filter is only an approximation to the behavior of the human ear. As we can see from the Fletcher-Munson diagram of Fig. 3.6, the actual frequency response of the human ear is more complicated than the simple type-A curve. Nonetheless, A-weighted sound levels give a good enough indication of how loud a sound will seem to a human listener

	Noise limit (dBA)	
Zone	Day	Night
Industrial	75	70
Commercial	65	55
Residential	55	45
Recreation	50	40

Table 3.4: Typical maximum allowed noise levels in A-weighted decibels.

to be useful and they are widely used as a standard for measuring sound levels. For example, governments in many towns, regions, or countries set standards for how loud ambient noise can be, to avoid annoyance and noise pollution, and they are almost always specified in A-weighted decibels. Table 3.4 shows typical noise limits that might apply in various parts of a town or country.

Chapter summary:

- The strength of a sound is measured by its **intensity**, which is the amount of sound energy hitting one square meter per second. Intensity is measured in watts per square meter, denoted $\mathrm{W/m^2}$.
- The average intensity I of a sound is related to sound pressure by the equation

$$I = \frac{\langle p^2 \rangle}{\rho c},$$

  where $\langle p^2 \rangle$ is the mean-square sound pressure, ρ is the density of air, and c is the speed of sound.
- **Typical intensities** for everyday sounds range from a high of about $1\,\mathrm{W/m^2}$ to a low of about $10^{-12}\,\mathrm{W/m^2}$, the latter figure, denoted I_0, being the conventional standard for the limit of audibility, although the actual limit varies somewhat from person to person, and also varies with frequency.
- For many purposes a more convenient measure of loudness is the **sound intensity level** or SIL, measured in decibels and denoted L_I, which is related to the intensity I by

$$L_I = 10 \log \frac{I}{I_0},$$

  where $I_0 = 10^{-12}\,\mathrm{W/m^2}$ as before and we use the common (base 10) logarithm. We can also convert back from SIL to intensity using the formula

$$I = I_0\, 10^{L_I/10}.$$

- When measuring the loudness of a sound, we actually measure the sound pressure and calculate the **sound pressure level** or SPL in decibels, denoted L_P:

$$L_P = 10 \log \frac{\langle p^2 \rangle}{p_0^2},$$

  where $p_0 = 2 \times 10^5$ Pa. With this choice of p_0 the SPL is almost exactly equal to the SIL—there is a very small difference between the two, which varies with temperature but is equal to 0.14 dB at 20°C. In most situations this difference is small enough to not matter and for practical purposes we can consider the SIL and SPL to be the same.

- When we have **more than one source of sound** their intensities add—the total intensity of two or more sound sources is simply the sum of their individual intensities.
- The intensity of a sound can be reduced by **obstacles** in the path of the sound. Sound readily flows around obstacles if they are smaller than about one wavelength, but is blocked, either partially or completely, by larger obstacles.
- Intensity also decreases with **distance** from a sound source according to the **inverse square law**, which says that the intensity $I(r)$ at distance r from a sound source is

$$I(r) = \frac{I_1}{r^2},$$

  where I_1 is the intensity at 1 meter. (This assumes there are no obstacles in the way to impede the sound.)
- The **apparent loudness** of sounds for human listeners depends not only on their intensity but also on the workings of the ear. Experiments have shown that for the typical listener the volume of a sound approximately doubles when the intensity increases by a factor of ten (or 10 dB).
- Apparent loudness also **depends on frequency**, with the ear being most sensitive to sound in the range between about 200 Hz and 5000 Hz. Outside this range—and especially below it—sensitivity falls off considerably and sounds may be 40 or 50 dB quieter at the lowest frequencies than they are at 1000 Hz. The detailed response of the ear to sound at different frequencies and intensities is described by the Fletcher-Munson diagram of Fig. 3.6.
- The loudness of a sound can be determined using a **sound level meter** that measures sound pressure level in decibels. Sound meters also incorporate filters that mimic the varying sensitivity of the ear to different frequencies, allowing them to give readings that correspond roughly to the loudness an average human listener would experience. Such readings are called A-weighted sound levels and are denoted "dBA" to distinguish them from regular decibel readings.

Exercises

3.1 A sound reaching your ear has intensity $10^{-6}\,\mathrm{W/m^2}$. A typical human eardrum has area about $60\,\mathrm{mm^2}$. About how much sound energy, in joules, will hit your eardrum per second?

3.2 A sound is measured to have mean-square sound pressure of $0.001\,\mathrm{Pa^2}$. What is its intensity?

3.3 What would be a typical value for the mean-square sound pressure (a) indoors in a public place, such as store or restaurant, and (b) in a loud outdoor space, such as a construction site?

3.4 Using the rules in Table 3.2 on page 62 give rough decibel equivalents for the following intensities: (a) $10^{-7}\,\mathrm{W/m^2}$, (b) $4\times10^{-7}\,\mathrm{W/m^2}$, (c) $8\times10^{-7}\,\mathrm{W/m^2}$, (d) $3.2\times10^{-6}\,\mathrm{W/m^2}$.

3.5 Using the rules in Table 3.2 on page 62 give rough intensity values for the following sound levels in decibels: (a) 60 dB, (b) 63 dB, (c) 67 dB, (d) 72 dB.

3.6 A house is about 10 meters high and wide. What frequency must sound have in order to flow around the house unimpeded?

3.7 You hear music coming from the other side of a small forest of trees with trunks about 50 cm across. What effect would you expect the trees to have on the sound as it passes through the forest?

3.8 The lowest frequency we can hear is about 20 Hz. How wide therefore would an object need to be to block sound at all audible frequencies? Give an example of an object that is about this size.

3.9 As we have seen, sound reaches the human ear unobstructed by the head when the frequency is lower than about 1700 Hz, but will be attenuated above this frequency. Give a rough estimate of the corresponding frequency above which sound will be attenuated for a mouse and for an elephant.

3.10 The legendary marching band in the song *Seventy-Six Trombones* (see Example 3.7 on page 71) is said to have had 110 cornets. By how many decibels would 110 cornets be louder than just a single cornet, assuming they are all playing the same thing?

3.11 A sound source produces a sound intensity level of 70 dB when heard from a distance of 5 meters. What is the sound intensity level at 10 meters, 20 meters, and 50 meters? (You should be able to work out the answers, at least roughly, without using a calculator.)

3.12 A large crowd of 100 000 people in a sports stadium are all cheering at once.

a) A single person cheering loudly creates a sound intensity level of about 75 dB at a distance of 1 meter. What would the sound intensity level be at 50 meters?
b) A player in the middle of the field is about 50 meters away from the people in the crowd (some a little more, some a little less). Given that there are 100,000 people in the crowd, what roughly is the sound intensity level for the player?
c) How far would you have to be from the stadium for the sound intensity level to drop to 40 dB, assuming there are no obstacles in the way of the sound?

3.13 The sound level from the launch of NASA's Artemis 1 moon rocket on November 16, 2022 was measured to be a stupendous 136 dB at a distance of 1.48 km from the launch site.

a) Using Eq. (3.47), calculate how far away you would have to be for the sound to drop to a more reasonable 60 dB.
b) In practice, one does not need to be this far away. Why not?

3.14 You are listening to an open-air music concert.

a) If you triple your distance from the stage, how many decibels does the sound intensity level go down by?
b) The music has a sound intensity level of 80 dB for concertgoers 10 meters from the stage. Based on your answer to part (a), roughly how many times would you have to triple that distance to reduce the sound intensity level to a mellow 40 dB?
c) About how far from the stage would you then be?

3.15 Three sounds are played one after another.

a) Sound 2 is 25 dB louder than sound 1. How many sones louder is it?
b) The intensity of sound 3 is 400 times that of sound 1. How many decibels is this?
c) How many sones louder is sound 3 than sound 1?

3.16 The perceived loudness of a sound roughly doubles when the sound intensity level goes up by 10 dB.

a) If the sound intensity level increases by S decibels, by what factor will the perceived loudness increase?
b) By how many decibels does the sound intensity level of a sound increase when you half your distance from the sound source?
c) Hence calculate the percentage by which the perceived loudness of a sound increases when you half your distance from its source.

3.17 A fifty-voice choir is singing in unison (i.e., they are all singing the same thing at the same time).

a) How much greater is the intensity of the full choir than that of just one voice?
b) How many decibels is this?
c) How much greater will be the apparent loudness of the full choir be in sones than the apparent loudness of one voice?

3.18 Two tones are played at 60 dB, one with frequency 100 Hz and one with frequency 1000 Hz. Which will sound louder and by about how many decibels?

3.19 A 200 Hz tone is played at a sound intensity level of 40 dB. At what sound intensity level must a 50 Hz tone be played to sound 10 dB louder?

3.20 Two sine-wave tones are played one after another.

a) The first has frequency 1000 Hz and sound intensity level 90 dB. The second has frequency 50 Hz and sounds the same loudness as the first. What is the sound intensity level of the second tone?
b) Tones of these same two frequencies (50 Hz and 1000 Hz) are played 40 dB quieter. Which one will now sound louder, and by how many decibels?

c) Explain briefly the relevance of the result in part (b) for recorded music. Why does it matter that sounds that are equally loud at one volume level are no longer equally loud when played at a quieter volume level?

3.21 Two sine-wave tones are played at a sound intensity level of 70 dB. Their pitches are D2 and C6.

a) Though they are played at the same intensity they will not sound equally loud. Which one will sound louder?
b) By about how many decibels would you have to turn up the volume on the quieter one to make it sound as loud as the louder one?

3.22 The limit of audibility for quiet sounds is usually taken to be 0 dB sound intensity level, which is a good general figure for a sound around 1000 Hz. At lower frequencies, however, the ear is less sensitive. Based on the Fletcher-Munson diagram of Fig. 3.6, what would you estimate to be the sound intensity level of the quietest audible sound at 20 Hz?

3.23 What is the apparent loudness in sones of a tone played at (a) 100 Hz and 40 dB, and (b) 4000 Hz and 70 dB?

3.24 What is the ratio of the apparent loudness in sones for a pair of tones at:

a) 500 Hz and 2000 Hz, both played at 90 dB
b) 200 Hz/60 dB and 50 Hz/90 dB