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# Ford School of Public Policy 555: Microeconomics A <br> Fall 2011 

Exam 3 December 13, 2011
Professor Kevin Stange

This exam has 8 questions and spans the topics we have covered in the third part of the course. Please explain your answers when asked and always show your work. It is in your best interest to show each of your steps clearly in order to receive partial credit and so that you are not penalized in later parts for math mistakes in earlier ones. You will only receive partial credit if you do not show your work. You have 80 minutes to complete the exam. Each question indicates the points each question is worth - you should use this as a guide to the number of minutes you can spend on each question. The points sum to 90 , so 10 points are bonus. Good luck!

## Short Answer [22]

1. [5] Elephant Ears is a monopoly supplier of designer baby shoes in Ann Arbor. Suppose the demand function it faces is $Q=200-10 \mathrm{P}$ and its constant marginal cost is $\$ 15$ per unit. Currently, it is selling 50 units at a price of $\$ 15$ each. Is it maximizing profits? If not, should it be producing more or less? Explain briefly.
2. [4] There are two leading candidates for the Republican primary: Mitt Romney and Newt Gingrich. Suppose each is considering launching a negative ad campaign that attacks the personal character of the other. The payoffs (expected vote share) of this action to each candidate (given in the matrix below) depend, in part, on whether the other candidate also goes negative. Assume that the candidates decide simultaneously, act rationally to maximize vote share, and have perfect information about the payoffs. According to an MSNBC analyst, "It is inevitable that the Republican candidates go negative." Is she correct? Explain.

|  | Gingrich |  |  |
| :--- | :--- | :--- | :--- |
|  | Go negative |  | Don't |
| Romney | Go negative | 30,25 | 60,20 |
|  | Don't | 25,55 | 40,35 |

$\qquad$
3. [4] Suppose that a large hospital is a monopsonist employer in the nurse labor market in Anchorage, Alaska. If this labor market suddenly became competitive (say, because of a growth in opportunities to work as a travel nurse outside of Alaska) but the supply curve didn't change, what would you expect to happen to nurse wages and employment in Anchorage? Explain.
4. [5] True or false: "If firms in a duopoly industry compete on price, have the same cost function, and make pricing decisions simultaneously, then equilibrium price will always be pushed down to marginal cost." Explain briefly.
5. [4] The graph below plots the demand curve, marginal revenue curve, and marginal cost curve for a profit-maximizing monopolist. Fill in the blanks with numbers (nearest whole number).
a. The firm will charge a price of $\qquad$ .
b. To maximize total surplus, government should impose a price ceiling of $\qquad$ .
c. Any price ceiling greater than $\qquad$ will have no effect on output.
d. If a price ceiling of $\$ 30$ were implemented, the firm would set price equal to $\qquad$ .


Your Name: $\qquad$

## Question 6: Monopoly in Rail Travel [22]

Amtrak has a monopoly over inter-city rail service in most of the United States. Suppose the aggregate demand function is given by

$$
Q=100-P
$$

where $Q$ is the number of passenger-trips and $P$ is the price. Amtrak's total cost function is given by $T C=1000+20 Q$
a. [2] What is the marginal revenue function? What is the marginal cost function?
b. [5] If Amtrak maximized profits, what price would it charge per trip and how many trips would it sell?
c. [2] What is Amtrak's total profit at this price and quantity?

Amtrak is entirely owned by the U.S. government, so it is unlikely that it would ever be allowed to pursue this monopolist strategy.
d. [4] What would price and quantity be if Amtrak instead acted to maximize total surplus?
$\qquad$
e. [3] How much (if at all) would the government need to subsidize Amtrak so that it stayed in business at this price and quantity from part (d)?
f. [6] If the government wanted Amtrak to be entirely self-supporting (i.e. not need subsidies), what is the lowest possible price it could set and still have Amtrak stay in business? [Note: You will need to use the quadratic formula to solve this problem]

Your Name: $\qquad$

## Question 7: Oligopoly [20]

On some routes, Amtrak competes with another company (Southern Railroad) for rail travel. Suppose that the service provided Amtrak and Southern is identical and that the two companies compete by setting quantities. The market (inverse) demand curve in these cities is given by $P=1000-4 Q$, where $Q$ $=Q_{A}+Q_{s}$ is the total rides sold and $Q_{j}$ is the amount sold by each firm ( $j=A$ or $S$ ). Suppose each firm has a cost function given by: $\operatorname{Cost}_{j}=40 \mathrm{Q}_{\mathrm{j}}$ ( $\mathrm{j}=\mathrm{A}$ or S ). Amtrak sets its quantity first, then Southern follows, so this is the Stackelberg model. Ignore everything about questions 6 and 8 when solving this problem.
a. [6] What is Southern's optimal output as a function of Amtrak's output? (i.e. find Southern's reaction function.)
b. [10] What is Amtrak's quantity produced? What is Southern's quantity produced?
c. [2] What is the market price at this equilibrium?
d. [2] Would total output be closer to or further from the socially optimal level if the two firms made output decisions simultaneously? Explain briefly. [Note: no calculations needed]
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## Question 8: Amtrak Pricing [26]

Thinking that Amtrak's business model is not sustainable, the government decides to pursue three alternative pricing strategies. After some market research, Amtrak determines that it has two types of customers: business and leisure travelers. The demand function of individuals in each group is:

$$
\begin{aligned}
& q_{\text {Business }}=100-P \\
& q_{\text {Leisure }}=80-2 P
\end{aligned}
$$

Amtrak's total cost function is still given by $T C=1000+20 Q$

First assume that the monopolist cannot distinguish between business and leisure travelers.
a. [9] The first pricing option is to operate only on a membership basis, where riders pay \$T for an Amtrak membership each year and then pay \$P for each train ride they take. Could Amtrak implement such a policy that only attracts leisure travelers? If so, how how much should it charge for the membership and how much should it charge for each trip? If not, explain why not.

Your Name: $\qquad$

From now on assume that Amtrak has some way of perfectly distinguishing between business and leisure travelers.
b. [11] A second possibility is to price discriminate. If Amtrak were to practice $3^{\text {rd }}$ degree price discrimination (but not the two-part tariff described above), charging a different per-ride price to business and leisure travelers, what price will it charge business travelers? How many trips will it sell to business travelers? What price will it charge leisure travelers? How many trips will it sell to leisure travelers?
$\qquad$
c. [6] A third possibility is to combine the third-degree price discrimination with a two-part tariff, by charging a different membership fee ( T ) and per-ride price ( P ) to the two different groups. If Amtrak were to implement this policy, describe qualitatively how
a. The per-ride price $(P)$ charged to the two groups would or would not differ
b. The membership fee ( $T$ ) charged to the two groups would or would not differ
c. The profits under this scenario would compare to those with a two-part tariff or price discrimination in isolation
[Hint: this problem is easier and requires fewer calculations than it may look.]

Your Name: $\qquad$

## Useful Formulas

The partial derivative of a function $F(X, Y)=a X^{n} Y^{m}$ with respect to X is $\frac{\partial F(X, Y)}{\partial X}=a n X^{n-1} Y^{m}$ where $a, n$, and $m$ are numbers.

Formula for optimal mark-up for a profit-maximizing monopolist: $\frac{P-M C}{P}=-\frac{1}{\varepsilon_{d}}$
Formula for finding the root of quadratic function $a Q^{2}+b Q+c=0: Q^{*}=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
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## Short Answer [22]

1. [5] Elephant Ears is a monopoly supplier of designer baby shoes in Ann Arbor. Suppose the demand function it faces is $Q=200-10 \mathrm{P}$ and its constant marginal cost is $\$ 15$ per unit. Currently, it is selling 50 units at a price of $\$ 15$ each. Is it maximizing profits? If not, should it be producing more or less? Explain briefly.
2. 

Elephant Ears is not maximizing profits and should be producing less than it is. They should be producing where $M R=M C$. Invert the demand function to get $P=20-0.10 Q$
$\operatorname{Rev}=P^{*} Q=(20-0.10 Q) Q=20 Q-0.10 Q^{2}$
$M R=20-0.20 Q$ and $M C=15$
Set $M R=M C: 20-0.20 Q=15 \rightarrow 5=0.20 Q \rightarrow Q=25$
Therefore Elephant Earns should reduce its quantity from 50 to 25. You also could have just calculated that $M R=10$ when $Q=50$, so $M R<M C$. This implies that they should reduce quantity. Finally, you could have simply noted that if the monopolist is producing where $P=M C=15$, then this is the competitive quantity, which is more than the profit-maximizing quantity. Therefore quantity should be reduced.
3. [4] There are two leading candidates for the Republican primary: Mitt Romney and Newt Gingrich. Suppose each is considering launching a negative ad campaign that attacks the personal character of the other. The payoffs (expected vote share) of this action to each candidate (given in the matrix below) depend, in part, on whether the other candidate also goes negative. Assume that the candidates decide simultaneously, act rationally to maximize vote share, and have perfect information about the payoffs. According to an MSNBC analyst, "It is inevitable that the Republican candidates go negative." Is she correct? Explain.

|  | Gingrich |  |  |
| :--- | :--- | :--- | :--- |
|  | Go negative | Don't |  |
| Romney | Go negative | 30,25 | 60,20 |
|  | Don't | 25,55 | 40,35 |

She is correct. Both Gingrich and Romney have a dominant strategy of "Go negative," so this is going to be the Nash Equilibrium that the candidates end up at.
$\qquad$
4. [4] Suppose that a large hospital is a monopsonist employer in the nurse labor market in Anchorage, Alaska. If this labor market suddenly became competitive (say, because of a growth in opportunities to work as a travel nurse outside of Alaska) but the supply curve didn't change, what would you expect to happen to nurse wages and employment in Anchorage? Explain.

We would expect nurse wages and employment in Anchorage to increase. In a monopsonist market where there is a single buyer for a good or service (here nursing services), the monopsonist can maximize their surplus (analogous to profits in a monopolist case) by purchasing too little and lowering prices below the competitive level. Thus when the market becomes competitive, we expect both wages and employment to rise.
5. [5] True or false: "If firms in a duopoly industry compete on price, have the same cost function, and make pricing decisions simultaneously, then equilibrium price will always be pushed down to marginal cost." Explain briefly.

False. The statement is only true if the products are homogeneous (identical). If the products are differentiated, then the equilibrium prices will be above marginal cost.
6. [4] The graph below plots the demand curve, marginal revenue curve, and marginal cost curve for a profit-maximizing monopolist. Fill in the blanks with numbers (nearest whole number).
a. The firm will charge a price of $\qquad$ 40 _.
b. To maximize total surplus, government should impose a price ceiling of $\qquad$ .
c. Any price ceiling greater than $\qquad$ 40 will have no effect on output.
d. If a price ceiling of $\$ 30$ were implemented, the firm would set price equal to $\qquad$ 30 .


Your Name: $\qquad$

## Question 6: Monopoly in Rail Travel [22]

Amtrak has a monopoly over inter-city rail service in most of the United States. Suppose the aggregate demand function is given by

$$
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$$

where $Q$ is the number of passenger-trips and $P$ is the price. Amtrak's total cost function is given by

$$
T C=1000+20 Q
$$

a. [2] What is the marginal revenue function? What is the marginal cost function?
$P=100-Q$
$R e v=(100-Q) Q$
$M R=100-2 Q$
$M C=20$
b. [5] If Amtrak maximized profits, what price would it charge per trip and how many trips would it sell?
Set $M R=M C$ :
$100-2 Q=20$
$2 Q=80$
$Q=40 ; P=60$
c. [2] What is Amtrak's total profit at this price and quantity?

$$
\begin{aligned}
& \text { Profit }=P Q-\text { Cost } \\
& \text { Profit }=(60)^{*}(40)-1000-20(40) \\
& \text { Profit }=2400-1000-800 \\
& \text { Profit }=600
\end{aligned}
$$

Amtrak is entirely owned by the U.S. government, so it is unlikely that it would ever be allowed to pursue this monopolist strategy.
d. [4] What would price and quantity be if Amtrak instead acted to maximize total surplus?

The competitive equilibrium is the point that maximizes total surplus.
This is where $P=M C=20$.

$$
\begin{array}{|l|}
\hline \mathrm{P}=20 \\
\mathrm{Q}=80
\end{array}
$$

$\qquad$
e. [3] How much (if at all) would the government need to subsidize Amtrak so that it stayed in business at this price and quantity from part (d)?
Profit $=P Q-$ Cost
Profit $=(20)^{*}(80)-1000-20(80)$
Profit $=1600-1000-1600$
Profit $=-1000$
So the government would need to subsidize Amtrak by $\$ 1000$ if it were to stay in business at the social-maximizing quantity and price.
f. [6] If the government wanted Amtrak to be entirely self-supporting (i.e. not need subsidies), what is the lowest possible price it could set and still have Amtrak stay in business? [Note: You will need to use the quadratic formula to solve this problem]

The lowest possible price that would still have Amtrak stay in the business will be where $P=$ ATC, where $A T C=1000 / Q+20$
$100-Q=1000 / Q+20$
$0=Q-80+1000 / Q$
$0=Q^{2}-80 Q+1000$
From the quadratic formula:

$$
\begin{aligned}
& Q^{*}=\frac{-(-80) \pm \sqrt{(-80)^{2}-4(1)(1000)}}{2(1)} \\
& Q^{*}=\frac{80 \pm \sqrt{2400}}{2} \\
& Q^{*}=\frac{80 \pm 49}{2} \\
& Q^{\star}=64.5 \text { or } 15.5 \\
& P=35.5 \text { or } 84.5 . \text { Pick the lowest one. } \\
& P=35.5
\end{aligned}
$$

## Question 7: Oligopoly [20]

On some routes, Amtrak competes with another company (Southern Railroad) for rail travel. Suppose that the service provided Amtrak and Southern is identical and that the two companies compete by setting quantities. The market (inverse) demand curve in these cities is given by $P=1000-4 \mathrm{Q}$, where Q $=Q_{A}+Q_{S}$ is the total rides sold and $Q_{j}$ is the amount sold by each firm ( $j=A$ or $S$ ). Suppose each firm has a cost function given by: Cost $_{j}=40 \mathrm{Q}_{\mathrm{j}}(\mathrm{j}=\mathrm{A}$ or S$)$. Amtrak sets its quantity first, then Southern follows, so this is the Stackelberg model. Ignore everything about questions 6 and 8 when solving this problem.
a. [6] What is Southern's optimal output as a function of Amtrak's output? (i.e. find Southern's reaction function.)
$R e v_{s}=P Q_{s}=(1000-4 Q) Q_{s}$
$R e v_{s}=\left(1000-4 Q_{A}-4 Q_{S}\right) Q_{S}$
$R e v_{s}=1000 Q_{S}-4 Q_{A} Q_{S}-4 Q_{S}^{2}$
$M R=1000-4 Q_{A}-8 Q_{S}=M C$
$M R=1000-4 Q_{A}-8 Q_{S}=40$
$8 Q_{S}=960-4 Q_{A}$
$Q_{S}=120-0.5 Q_{A}$
b. [10] What is Amtrak's quantity produced? What is Southern's quantity produced?
$\operatorname{Re} v_{A}=P Q_{A}=(1000-4 Q) Q_{A}$
$\operatorname{Re} v_{A}=\left(1000-4 Q_{A}-4\left(120-0.5 Q_{A}\right)\right) Q_{A}$
$\operatorname{Re} v_{A}=\left(1000-4 Q_{A}-480+2 Q_{A}\right) Q_{A}$
$R e v_{A}=\left(520-2 Q_{A}\right) Q_{A}$
$R e v_{A}=520 Q_{A}-2 Q_{A}^{2}$
$M R=520-4 Q_{A}=M C$
$M R=520-4 Q_{A}=40$
$4 Q_{A}=480$
$Q_{S}=120 ; Q_{S}=60$
c. [2] What is the market price at this equilibrium?
$P=1000-4 Q$
$P=1000-4(120+60)$
$P=1000-720$
$\mathrm{P}=\$ 280$
d. [2] Would total output be closer to or further from the socially optimal level if the two firms made output decisions simultaneously? Explain briefly. [Note: no calculations needed] Further. If the two firms made output decisions simultaneously, then this would be the Cournot model. This situation would result in a lower total quantity than Stackelberg, so it is even further from the competitive equilibrium (socially optimal) quantity.

Your Name: $\qquad$

## Question 8: Amtrak Pricing [26]

Thinking that Amtrak's business model is not sustainable, the government decides to pursue three alternative pricing strategies. After some market research, Amtrak determines that it has two types of customers: business and leisure travelers. The demand function of individuals in each group is:

$$
\begin{aligned}
& q_{\text {Business }}=100-P \\
& q_{\text {Leisure }}=80-2 P
\end{aligned}
$$

Amtrak's total cost function is still given by $T C=1000+20 Q$
First assume that the monopolist cannot distinguish between business and leisure travelers.
a. [9] The first pricing option is to operate only on a membership basis, where riders pay \$T for an Amtrak membership each year and then pay \$P for each train ride they take. Could Amtrak implement such a policy that only attracts leisure travelers? If so, how how much should it charge for the membership and how much should it charge for each trip? If not, explain why not.

This is a two-part tariff scheme. The optimal twopart tariff for a single type of consumer is to charge a per-unit price equal to marginal cost and a
membership fee equal to the consumer surplus at that price. So for leisure travelers, this would mean charging a price of $P=20$ and a membership fee

Business
Leisure equal to the area denoted in the graph by the dotted triangle. This membership fee equals $0.5 *(40-20) * 40=\$ 400$.

P

$$
P=20
$$

However, since this membership fee is also less than the consumer surplus that business travelers would have at that price (denoted by the area of the dashed triangle, which equals $\left.0.5^{*}(100-20)^{*}(80)=\$ 3200\right)$, then all business travelers would also want to join. In fact, there is not a two-part tariff pricing scheme that would attract only leisure travelers and not business travelers. Any membership fee that is low enough to attract leisure travelers will also attract business travelers.

Your Name: $\qquad$

From now on assume that Amtrak has some way of perfectly distinguishing between business and leisure travelers.
b. [11] A second possibility is to price discriminate. If Amtrak were to practice $3^{\text {rd }}$ degree price discrimination (but not the two-part tariff described above), charging a different per-ride price to business and leisure travelers, what price will it charge business travelers? How many trips will it sell to business travelers? What price will it charge leisure travelers? How many trips will it sell to leisure travelers?
Amtrak will set MR = MC for each group separately. Since MC is constant, we don't actually have to figure out the total marginal cost and then apply that to each group.

Business
$\operatorname{Re} v=P q$
$\operatorname{Re} v=\left(100-q_{b}\right) q_{b}$
$\operatorname{Re} v=100 q_{b}-q_{b}^{2}$
$M R=100-2 q_{b}=M C$
$100-2 q_{b}=20$
$80=2 q_{b}$
$q_{b}=40 ; P_{b}=60$

Leisure
Rev $=P q$
$\operatorname{Re} v=\left(40-.5 q_{L}\right) q_{L}$
$\operatorname{Re} v=40 q_{L}-.5 q_{L}^{2}$
$M R=40-q_{L}=M C$
$40-q_{L}=20$
$20=q_{L}$
$q_{L}=20 ; P_{L}=30$
c. [6] A third possibility is to combine the third-degree price discrimination with a two-part tariff, by charging a different membership fee $(T)$ and per-ride price $(P)$ to the two different groups. If Amtrak were to implement this policy, describe qualitatively how
a. The per-ride price $(P)$ charged to the two groups would or would not differ
b. The membership fee $(T)$ charged to the two groups would or would not differ
c. The profits under this scenario would compare to those with a two-part tariff or price discrimination in isolation
[Hint: this problem is easier and requires fewer calculations than it may look.]

If Amtrak could identify which consumers were business travelers and which were leisure travelers, then it could potentially set a different two-part tariff for each group.

- The per-ride price would be equal to marginal cost, which is the same for both groups.
- The membership fee would be set equal to the consumer surplus of each group at the above perride price. In section (a) above we showed how this consumer surplus is greater for the business travelers than the leisure travelers, so the membership fee charged to business travelers will be higher than that charged to leisure travelers.
- The profits in this case are greater than either a single two-part tariff that would attract both consumer groups and are greater than the price discrimination scheme derive in part (b). In fact, this separate two-part tariff scheme described here is the best that the monopolist can do because it transfers all consumer surplus to the producer in the form of profits.

Your Name: $\qquad$

## Useful Formulas

The partial derivative of a function $F(X, Y)=a X^{n} Y^{m}$ with respect to $X$ is $\frac{\partial F(X, Y)}{\partial X}=a n X^{n-1} Y^{m}$ where $\mathrm{a}, \mathrm{n}$, and m are numbers.

Formula for optimal mark-up for a profit-maximizing monopolist: $\frac{P-M C}{P}=-\frac{1}{\varepsilon_{d}}$
Formula for finding the root of quadratic function $a Q^{2}+b Q+c=0: Q^{*}=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

