Homework #2

MEAM 501 Matrix/Analytical Methods in Engineering

Kikuchi 97F

Suppose that the following data have been obtained in a laboratory:

Time (t)	Response (x)	Standard Deviation (σ)
0	7	0.02
0.1	4	0.02
0.2	-2	0.02
0.3	3	0.05
0.4	-1	0.05
0.5	-2	0.07
0.6	-1	0.05
0.7	0	0.02
0.8	1	0.02
0.9	3	0.02
1.0	4	0.03
1.1	3	0.03
1.2	2	0.03
1.3	2	0.03
1.4	3	0.02
1.5	8	0.02
1.6	12	0.02
1.7	18	0.01
1.8	7	0.02
1.9	2	0.02
2.0	1	0.01

Using the following basis functions:

$$f_1(t) = 1$$
$$f_2(t) = t$$
$$f_3(t) = t^2$$
$$f_4(t) = t^3$$
$$f_5(t) = t^4$$
$$f_6(t) = \exp(-t)$$
$$f_7(t) = \exp(-(t-1.7)^2)$$

$$f_8(t) \;=\; \cos(1.7 - t)$$

we would like to make a curve fit of the data obtained as follows:

$$x(t) = \sum_{k=1}^{8} c_k f_k(t)$$

where c_k *,* $k = 1, ..., 8$*, are the coefficients which must be determined. If a merit function is defined by*

$$\Xi^2 = \sum_{i=1}^{21} \left[\frac{x_i - \sum_{k=1}^{8} c_k f_k(t_i)}{\sigma_i} \right]^2$$

where (t_i , x_i , σ_i)*,* $i = 1, ..., 21$*, are the data set, then the coefficient* c_k *,* $k = 1, ..., 8$*, are determined so as to minimize the merit function* Ξ^2 *. That is, the coefficients* c_k *,* $k = 1, ..., 8$*, of the curve fit is the solution of the least squares problem . If a matrix* $\boldsymbol{A} = [a_{ij}]$ *and a vector* $\boldsymbol{b} = \{b_i\}$ *are defined by*

$$a_{ij} = \frac{f_j(t_i)}{\sigma_i} \quad \text{and} \quad b_i = \frac{x_i}{\sigma_i}$$

the merit function X^2 *can be written as*

$$\Xi^2 = \frac{1}{2}(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{c})^T(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{c}).$$

(a) Determine the coefficient matrix $\boldsymbol{A}$ *and the vector* $\boldsymbol{b}$ *using the data given.*

(b) Obtain the range $R(\boldsymbol{A})$ *of the matrix* $\boldsymbol{A}$*. What is the dimension of* $R(\boldsymbol{A})$*?*

(c) Obtain the null space $N(\boldsymbol{A})$*. What is the dimension of* $N(\boldsymbol{A})$*?*

(d) Using the Householder transformation, modify the matrix $\boldsymbol{A}^T\boldsymbol{A}$ *to the form of the matrix* $\left(\boldsymbol{A}^T\boldsymbol{A}\right)^*$ *such that its lower triangular portion becomes zero, that is,*

$$\left(\boldsymbol{A}^T\boldsymbol{A}\right)^*_{ij} = 0 \quad , \quad i > j \, .$$

(e) Make a LU decomposition of $\boldsymbol{A}^T\boldsymbol{A}$.

(f) Compute det $\boldsymbol{A}^T\boldsymbol{A}$ *and rank* $\boldsymbol{A}^T\boldsymbol{A}$.

(g) Compute det $\boldsymbol{A}\boldsymbol{A}^T$ *and rank* $\boldsymbol{A}\boldsymbol{A}^T$.

(h) Tridiagonalize the matrix $\boldsymbol{A}^T\boldsymbol{A}$.

(i) Compute the 2 norm $|| \, \boldsymbol{A}^T\boldsymbol{A} \, ||_2$ *of the matrix* $\boldsymbol{A}^T\boldsymbol{A}$.

(j) Find the minimizer $\boldsymbol{c}$ *of the merit function* Ξ^2 .

(k) Using $\boldsymbol{c}$ *obtained in (j), plot the curve x(t) obtained.*

(l) If the singular value decomposition of the matrix $\boldsymbol{A}$ *is obtained as* $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$, *the covariance of the coefficient* c_k , *k=1, ..., 8, is defined by*

$$Cov\left(c_j, c_k\right) = \left(\boldsymbol{V}\boldsymbol{\Sigma}^+\boldsymbol{\Sigma}^+\boldsymbol{V}^T\right)_{jk} .$$

Compute the covariance.

(m) Solve the eigenvalue problem

$$\left(\boldsymbol{A}^T\boldsymbol{A}\right)\boldsymbol{y} = \lambda\boldsymbol{B}\boldsymbol{y}$$

where

Case (1)

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Case (2)

$$\boldsymbol{B} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Obtain all the eigenvalues and the corresponding eigenvectors.

MATLAB Program

```
% Homework #2, Fall 1997
%
_____________________________________________________________
_____________
%
% setup data
%
t=(0:0.1:2)';
x=[7,4,-2,3,-1,-2,-1,0,1,3,4,3,2,2,3,8,12,18,7,2,1]';
s=[0.02,0.02,0.02,0.05,0.05,0.07,0.05,0.02,0.02,0.02,0.03,0.0
3,0.03,0.03,...
0.02,0.02,0.02,0.01,0.02,0.02,0.01]';
% define coefficient matrix and righthand side
```

```
A=[ones(21,1)./s,t./s,(t.^2)./s,(t.^3)./s,(t.^4)./s,exp(-
t)./s,exp(-(t-1.7).^2)./s,cos(1.7-t)./s]
b=x./s
%
% exercises of the Householder transformation
%
ATA=A'*A;
B=ATA;
n=8;
% upper triangulation of the matrix A'*A
for i=1:n-1
y=zeros(n,1);
v=zeros(n,1);
y(i:n)=B(i:n,i);
v(i)=y(i)-norm(y);
v(i+1:n)=y(i+1:n);
P=eye(n)-2*v*v'/(v'*v);
B=P*B;
end
B
% tridiagonalization of the matrix A'*A
B=ATA;
for i=1:n-2
y=zeros(n,1);
v=zeros(n,1);
y(i+1:n)=B(i+1:n,i);
v(i+1)=y(i+1)-norm(y);
v(i+2:n)=y(i+2:n);
P=eye(n)-2*v*v'/(v'*v);
B=P*B*P;
end
B
%
% exercises related to the matrix A
%
[U,S,V]=svd(A)
ATA=A'*A
[L,U,P]=lu(ATA)
P*ATA-L*U
det(ATA)
rank(ATA)
det(A*A')
rank(A*A')
```

```
norm(ATA)
%
% least squares solution and curve fitting
%
c=pinv(A)*b
F=[ones(21,1),t,(t.^2),(t.^3),(t.^4),exp(-t),exp(-(t-
1.7).^2),cos(1.7-t)];
xc=F*c
plot(t,x,'+',t,xc)
xlabel('t')
ylabel('x')
title('Curve Fitting with 8 Functions')
% covariant matrix
cov=V*pinv(S)*pinv(S)'*V'
%
% exercises of eigenvalue problems
%
b=[];
B=eye(8);
B(3,3)=0;
B
[X,R]=eig(ATA,B)
for i=1:7
B(i,i)=2;
B(i+1,i)=1;
B(i,i+1)=1;
end
B(1,1)=1;
B(8,8)=1;
B
[X,R]=eig(ATA,B)
```

```
Columns 1 through 7

   0.0500         0         0         0         0    0.0500    0.0028
   0.0500    0.0050    0.0005    0.0001    0.0000    0.0452    0.0039
   0.0500    0.0100    0.0020    0.0004    0.0001    0.0409    0.0053
   0.0200    0.0060    0.0018    0.0005    0.0002    0.0148    0.0028
   0.0200    0.0080    0.0032    0.0013    0.0005    0.0134    0.0037
   0.0143    0.0071    0.0036    0.0018    0.0009    0.0087    0.0034
   0.0200    0.0120    0.0072    0.0043    0.0026    0.0110    0.0060
   0.0500    0.0350    0.0245    0.0172    0.0120    0.0248    0.0184
   0.0500    0.0400    0.0320    0.0256    0.0205    0.0225    0.0222
   0.0500    0.0450    0.0405    0.0365    0.0328    0.0203    0.0264
   0.0333    0.0333    0.0333    0.0333    0.0333    0.0123    0.0204
```

```
    0.0333    0.0367    0.0403    0.0444    0.0488    0.0111    0.0233
    0.0333    0.0400    0.0480    0.0576    0.0691    0.0100    0.0260
    0.0333    0.0433    0.0563    0.0732    0.0952    0.0091    0.0284
    0.0500    0.0700    0.0980    0.1372    0.1921    0.0123    0.0457
    0.0500    0.0750    0.1125    0.1688    0.2531    0.0112    0.0480
    0.0500    0.0800    0.1280    0.2048    0.3277    0.0101    0.0495
    0.1000    0.1700    0.2890    0.4913    0.8352    0.0183    0.1000
    0.0500    0.0900    0.1620    0.2916    0.5249    0.0083    0.0495
    0.0500    0.0950    0.1805    0.3430    0.6516    0.0075    0.0480
    0.1000    0.2000    0.4000    0.8000    1.6000    0.0135    0.0914

  Column 8

   -0.0064
   -0.0015
    0.0035
    0.0034
    0.0053
    0.0052
    0.0091
    0.0270
    0.0311
    0.0348
    0.0255
    0.0275
    0.0293
    0.0307
    0.0478
    0.0490
    0.0498
    0.1000
    0.0498
    0.0490
    0.0955


b =

   1.0e+03 *

    0.3500
    0.2000
   -0.1000
    0.0600
   -0.0200
   -0.0286
   -0.0200
         0
    0.0500
    0.1500
    0.1333
    0.1000
    0.0667
    0.0667
    0.1500
    0.4000
    0.6000
    1.8000
    0.3500
    0.1000
    0.1000


B =
```

```
   1.0e+06 *

  Columns 1 through 7

    0.4330    0.7750    1.4189    2.6345    4.9434    0.0757    0.4038
   -0.0000    0.0370    0.0925    0.2000    0.4124   -0.0120    0.0147
   -0.0000    0.0000    0.0111    0.0373    0.0952    0.0020   -0.0031
   -0.0000    0.0000    0.0000    0.0013    0.0058   -0.0001   -0.0005
   -0.0000    0.0000   -0.0000    0.0000    0.0001    0.0000   -0.0000
    0.0000    0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000
    0.0000    0.0000   -0.0000    0.0000    0.0000   -0.0000    0.0000
   -0.0000   -0.0000    0.0000   -0.0000   -0.0000    0.0000   -0.0000

  Column 8

    0.4161
    0.0139
   -0.0043
   -0.0001
    0.0000
   -0.0000
   -0.0000
   -0.0000


B =

   1.0e+06 *

  Columns 1 through 7

    0.0533    0.4297   -0.0000   -0.0000   -0.0000    0.0000    0.0000
    0.4297    5.8394    0.3716    0.0000   -0.0000   -0.0000    0.0000
   -0.0000    0.3716    0.0699    0.0109   -0.0000    0.0000    0.0000
   -0.0000    0.0000    0.0109    0.0102    0.0003    0.0000    0.0000
   -0.0000   -0.0000    0.0000    0.0003    0.0009    0.0000    0.0000
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000   -0.0000   -0.0000    0.0000    0.0000    0.0000
    0.0000   -0.0000    0.0000    0.0000   -0.0000   -0.0000   -0.0000

  Column 8

    0.0000
    0.0000
    0.0000
    0.0000
   -0.0000
   -0.0000
   -0.0000
    0.0000


U =

  Columns 1 through 7

    0.0017   -0.1456    0.5257    0.4215   -0.4417    0.4205   -0.3042
    0.0021   -0.1577    0.4784    0.2322    0.0403   -0.1638    0.3713
    0.0028   -0.1722    0.4297    0.0625    0.3353   -0.4302    0.1991
    0.0015   -0.0755    0.1516   -0.0332    0.1933   -0.1858   -0.0945
    0.0021   -0.0830    0.1306   -0.0803    0.2073   -0.1401   -0.2380
    0.0021   -0.0650    0.0776   -0.0824    0.1340   -0.0471   -0.2082
    0.0042   -0.0994    0.0861   -0.1380    0.1444    0.0101   -0.2502
```

```
  0.0146   -0.2695    0.1579   -0.3702    0.2169    0.1700   -0.3574
  0.0203   -0.2893    0.1003   -0.3657    0.0576    0.2380   -0.0332
  0.0279   -0.3069    0.0438   -0.3338   -0.0972    0.2191    0.2411
  0.0252   -0.2138   -0.0067   -0.1857   -0.1527    0.0827    0.2604
  0.0336   -0.2194   -0.0396   -0.1366   -0.2144   -0.0124    0.2586
  0.0442   -0.2202   -0.0680   -0.0796   -0.2413   -0.1123    0.1704
  0.0572   -0.2148   -0.0905   -0.0197   -0.2283   -0.1871    0.0374
  0.1097   -0.3026   -0.1577    0.0559   -0.2632   -0.3175   -0.1280
  0.1385   -0.2693   -0.1653    0.1271   -0.1348   -0.2576   -0.2200
  0.1729   -0.2198   -0.1554    0.1733    0.0195   -0.1075   -0.1763
  0.4270   -0.3030   -0.2499    0.3647    0.3225    0.1807   -0.0220
  0.2612   -0.0619   -0.0706    0.1416    0.2360    0.2520    0.1887
  0.3166    0.0517    0.0112    0.0370    0.1723    0.2497    0.2565
  0.7614    0.3845    0.2478   -0.2921   -0.2344   -0.1719   -0.0928

Columns 8 through 14

  0.1547    0.0172    0.0449    0.0316    0.0203    0.0058   -0.0014
 -0.3556   -0.0952   -0.1599   -0.1075   -0.0799   -0.0482   -0.0349
  0.0898    0.1812    0.2033    0.1391    0.1360    0.1288    0.1183
  0.2207   -0.3459   -0.3671   -0.2859   -0.3163   -0.2999   -0.2187
  0.2798    0.0721    0.1828    0.1346    0.1040    0.0560    0.0124
  0.1547    0.1859    0.1706    0.0385   -0.0812   -0.2111   -0.3114
  0.0930   -0.0145    0.2248    0.2053    0.1702    0.0899    0.0199
 -0.0534   -0.4287   -0.2721   -0.0324    0.1083    0.1913    0.1887
 -0.1958    0.7177   -0.2806   -0.1639   -0.1209   -0.0651   -0.0076
 -0.1595   -0.2611    0.6691   -0.2272   -0.1938   -0.1314   -0.0587
  0.0066   -0.1262   -0.2092    0.8294   -0.1708   -0.1419   -0.0934
  0.1427   -0.0621   -0.1609   -0.1619    0.8092   -0.1858   -0.1485
  0.2295   -0.0030   -0.0937   -0.1282   -0.1800    0.7988   -0.1840
  0.2179    0.0336   -0.0292   -0.0790   -0.1385   -0.1786    0.8148
  0.1601    0.0595    0.0217   -0.0421   -0.1152   -0.1831   -0.2235
 -0.0710    0.0308    0.0429    0.0149   -0.0219   -0.0750   -0.1302
 -0.2156   -0.0083    0.0263    0.0358    0.0369    0.0132   -0.0332
 -0.2490   -0.0450   -0.0084    0.0334    0.0679    0.0777    0.0490
  0.2240    0.0093   -0.0187   -0.0271   -0.0282   -0.0119    0.0163
  0.5330    0.0581   -0.0040   -0.0546   -0.0886   -0.0829   -0.0331
 -0.1459   -0.0116    0.0039    0.0136    0.0190    0.0151    0.0027

Columns 15 through 21

  0.0061    0.0275    0.0420    0.0476   -0.0388   -0.1101   -0.1190
 -0.0736   -0.1184   -0.1394   -0.1581    0.0870    0.2802    0.4399
  0.1566    0.1305    0.0993    0.1233    0.0106   -0.0739   -0.4711
 -0.1277    0.0826    0.2011    0.2750   -0.1098   -0.3395    0.0124
 -0.0271   -0.0661   -0.1300   -0.4852   -0.3712   -0.3517    0.4270
 -0.5202   -0.4469   -0.2556   -0.0039    0.2093    0.2308   -0.2223
  0.0046    0.0752    0.1976    0.5701    0.2614    0.1589    0.5270
  0.1604   -0.0145   -0.1363   -0.2102    0.0942    0.2760   -0.2052
  0.0577    0.0909    0.0770    0.0381   -0.0561   -0.0881    0.0562
  0.0046    0.0538    0.0465   -0.0178   -0.0795   -0.1082   -0.0383
 -0.0609   -0.0002    0.0220   -0.0011   -0.0520   -0.0859   -0.0345
 -0.1353   -0.0452    0.0144    0.0374   -0.0307   -0.0820   -0.0067
 -0.1984   -0.0946   -0.0037    0.0700    0.0058   -0.0495    0.0287
 -0.2307   -0.1405   -0.0392    0.0648    0.0441    0.0082    0.0450
  0.6690   -0.2567   -0.1311    0.0097    0.0985    0.1059    0.0459
 -0.2533    0.7408   -0.1960   -0.1387    0.0788    0.1549   -0.0057
 -0.1346   -0.2014    0.7874   -0.2865   -0.0076    0.1042   -0.0424
 -0.0281   -0.1648   -0.2901    0.3045   -0.2957   -0.1459   -0.0365
  0.0626    0.0649    0.0028   -0.2590    0.7148   -0.3258    0.0579
  0.0619    0.1486    0.1349   -0.0569   -0.3003    0.5411    0.0344
 -0.0178   -0.0291   -0.0184    0.0213    0.0198   -0.0889   -0.0121
```

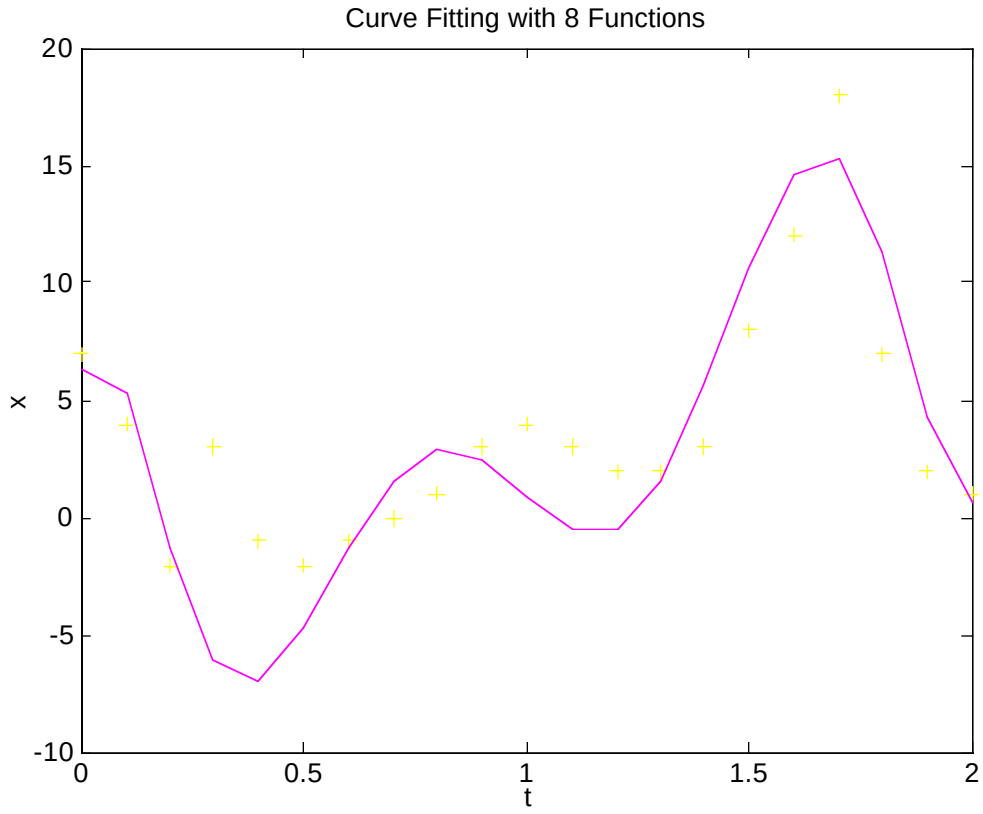
```
S =

   1.0e+03 *

  Columns 1 through 7

    2.4279         0         0         0         0         0         0
         0    0.2559         0         0         0         0         0
         0         0    0.1123         0         0         0         0
         0         0         0    0.0299         0         0         0
         0         0         0         0    0.0031         0         0
         0         0         0         0         0    0.0011         0
         0         0         0         0         0         0    0.0000
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Column 8

         0
         0
         0
         0
         0
         0
         0
    0.0000
         0
         0
         0
         0
         0
         0
         0
         0
         0
         0
         0
         0
         0

V =

  Columns 1 through 7

    0.0732   -0.5238    0.5463   -0.1318    0.2531   -0.1044    0.1281
    0.1315   -0.4206   -0.1841   -0.4021    0.2152    0.0418   -0.7055
    0.2412   -0.3566   -0.3129    0.2621   -0.4610    0.5734   -0.0337
    0.4483   -0.1674   -0.2438    0.5712    0.5678   -0.1328    0.1433
    0.8417    0.3590    0.2246   -0.2781   -0.1803   -0.0389   -0.0218
    0.0127   -0.2478    0.6155    0.3145   -0.2530    0.1699   -0.0393
    0.0685   -0.3025   -0.1781    0.1126   -0.5080   -0.7751   -0.0252
```

```
    0.0705   -0.3344   -0.2222   -0.4869   -0.0061    0.0994    0.6793

  Column 8

   -0.5596
    0.2537
   -0.3250
    0.1572
   -0.0204
    0.6052
    0.0009
    0.3542


ATA =

   1.0e+06 *

  Columns 1 through 7

    0.0533    0.0700    0.1142    0.1975    0.3526    0.0182    0.0387
    0.0700    0.1142    0.1975    0.3526    0.6423    0.0151    0.0618
    0.1142    0.1975    0.3526    0.6423    1.1873    0.0214    0.1051
    0.1975    0.3526    0.6423    1.1873    2.2192    0.0344    0.1848
    0.3526    0.6423    1.1873    2.2192    4.1850    0.0586    0.3321
    0.0182    0.0151    0.0214    0.0344    0.0586    0.0098    0.0087
    0.0387    0.0618    0.1051    0.1848    0.3321    0.0087    0.0340
    0.0404    0.0646    0.1089    0.1905    0.3416    0.0088    0.0355

  Column 8

    0.0404
    0.0646
    0.1089
    0.1905
    0.3416
    0.0088
    0.0355
    0.0375


L =

  Columns 1 through 7

    1.0000         0         0         0         0         0         0
    0.1513    1.0000         0         0         0         0         0
    0.3238    0.3859    1.0000         0         0         0         0
    0.1147    0.3340    0.8259    1.0000         0         0         0
    0.5601    0.2659    0.8049   -0.5572    1.0000         0         0
    0.1097    0.3195    0.6388    0.0965   -0.1485    1.0000         0
    0.1985    0.4906    0.9183    0.8915    0.2282   -0.0389    1.0000
    0.0515    0.6612   -0.5413   -0.8144   -0.3552    0.0149   -0.4178

  Column 8

         0
         0
         0
         0
         0
         0
         0
    1.0000
```

```
U =

   1.0e+06 *

  Columns 1 through 7

    0.3526    0.6423    1.1873    2.2192    4.1850    0.0586    0.3321
         0   -0.0272   -0.0655   -0.1383   -0.2807    0.0093   -0.0116
         0         0   -0.0066   -0.0229   -0.0594   -0.0011    0.0021
         0         0         0    0.0010    0.0043   -0.0001   -0.0004
         0         0         0         0   -0.0001   -0.0000    0.0000
         0         0         0         0         0   -0.0000    0.0000
         0         0         0         0         0         0    0.0000
         0         0         0         0         0         0         0

  Column 8

    0.3416
   -0.0112
    0.0026
   -0.0001
   -0.0000
    0.0000
   -0.0000
    0.0000


P =

     0     0     0     0     1     0     0     0
     1     0     0     0     0     0     0     0
     0     0     1     0     0     0     0     0
     0     0     0     0     0     0     0     1
     0     0     0     1     0     0     0     0
     0     0     0     0     0     0     1     0
     0     1     0     0     0     0     0     0
     0     0     0     0     0     1     0     0


ans =

   1.0e-09 *

  Columns 1 through 7

         0         0         0         0         0         0         0
         0   -0.0146   -0.0146   -0.0291         0         0    0.0073
         0         0         0         0    0.2328         0         0
         0    0.0073   -0.0146         0         0   -0.0018         0
    0.0291         0         0         0         0         0    0.0291
         0    0.0073         0         0         0         0         0
         0   -0.0146         0         0         0    0.0018    0.0073
         0   -0.0018         0         0   -0.0218   -0.0018   -0.0018

  Column 8

         0
         0
         0
    0.0073
         0
         0
```

```
   -0.0073
   -0.0018


ans =

   2.9829e+08


ans =

     8


ans =

  1.0228e-143


ans =

     8


ans =

   5.8947e+06


c =

   1.0e+06 *

    1.0879
   -0.4863
    0.6331
   -0.3075
    0.0399
   -1.1776
   -0.0017
   -0.6967


xc =

    6.2861
    5.3437
   -1.2446
   -6.0343
   -6.9433
   -4.7417
   -1.2840
    1.6410
    2.9509
    2.4951
    0.9583
   -0.4689
   -0.5542
    1.4840
    5.5757
   10.6128
   14.6078
   15.3132
   11.4054
```

```
    4.3050
    0.6731
```




```
cov =

   1.0e+06 *

  Columns 1 through 7

    5.1879   -2.3533    3.0124   -1.4571    0.1893   -5.6103   -0.0079
   -2.3533    1.0721   -1.3657    0.6597   -0.0857    2.5442    0.0038
    3.0124   -1.3657    1.7494   -0.8463    0.1099   -3.2579   -0.0046
   -1.4571    0.6597   -0.8463    0.4096   -0.0532    1.5760    0.0022
    0.1893   -0.0857    0.1099   -0.0532    0.0069   -0.2047   -0.0003
   -5.6103    2.5442   -3.2579    1.5760   -0.2047    6.0673    0.0086
   -0.0079    0.0038   -0.0046    0.0022   -0.0003    0.0086    0.0000
   -3.2822    1.4834   -1.9068    0.9234   -0.1200    3.5503    0.0048

  Column 8

   -3.2822
    1.4834
   -1.9068
    0.9234
   -0.1200
    3.5503
    0.0048
    2.0829
```

```
B =

     1     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0
     0     0     0     0     0     0     0     0
     0     0     0     1     0     0     0     0
     0     0     0     0     1     0     0     0
     0     0     0     0     0     1     0     0
     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     0     1

X =

  Columns 1 through 7

         0    0.0486   -0.5001   -0.1055   -0.1334   -0.0209   -0.5596
         0    0.0326   -0.0802   -0.4362   -0.2067   -0.0217    0.2538
    1.0000    0.9595    0.7372    0.6258    0.7349   -0.5913   -0.3250
         0   -0.0783   -0.0029    0.2395   -0.5898    0.1228    0.1572
         0   -0.2590   -0.1445   -0.1912    0.1429    0.0464   -0.0204
         0    0.0221   -0.4222    0.2943    0.1499   -0.0327    0.6052
         0    0.0310   -0.0241   -0.0376    0.0789    0.7943    0.0009
         0    0.0353   -0.0183   -0.4743   -0.0489   -0.0154    0.3542

  Column 8

    0.1213
   -0.7024
   -0.0376
    0.1452
   -0.0221
   -0.0318
   -0.0252
    0.6836


R =

   1.0e+05 *

  Columns 1 through 4

       ∞ +    NaNi         0                    0                    0
       0              2.1928                    0                    0
       0                   0               0.1797                    0
       0                   0                    0               0.0110
       0                   0                    0                    0
       0                   0                    0                    0
       0                   0                    0                    0
       0                   0                    0                    0

  Columns 5 through 8

       0                   0                    0                    0
       0                   0                    0                    0
       0                   0                    0                    0
       0                   0                    0                    0
  0.0002                   0                    0                    0
       0              0.0000                    0                    0
       0                   0               0.0000                    0
       0                   0                    0               0.0000
```

```
B =

     1     1     0     0     0     0     0     0
     1     2     1     0     0     0     0     0
     0     1     2     1     0     0     0     0
     0     0     1     2     1     0     0     0
     0     0     0     1     2     1     0     0
     0     0     0     0     1     2     1     0
     0     0     0     0     0     1     2     1
     0     0     0     0     0     0     1     1


X =

  Columns 1 through 7

    0.3536    0.6822   -0.5117   -0.1369   -0.2479    0.2615   -0.3239
   -0.3536   -0.4044    0.5421    0.2190    0.3536    0.1380    0.7495
    0.3536    0.3125   -0.3147   -0.1355   -0.5159   -0.3424   -0.0875
   -0.3536   -0.0491    0.4043   -0.2740    0.4889   -0.0010   -0.0759
    0.3536   -0.0722   -0.1841    0.1393   -0.1391    0.0473    0.0129
   -0.3536    0.2389    0.1008    0.0992    0.2833   -0.3683    0.2578
    0.3536   -0.2488   -0.1304   -0.5034   -0.3883    0.6514    0.0237
   -0.3536    0.3831    0.3475    0.7464    0.2408   -0.4829   -0.5028

  Column 8

   -0.5596
    0.2536
   -0.3250
    0.1572
   -0.0204
    0.6052
    0.0008
    0.3543


R =

   1.0e+22 *

  Columns 1 through 7

    1.2524         0         0         0         0         0         0
         0    0.0000         0         0         0         0         0
         0         0    0.0000         0         0         0         0
         0         0         0    0.0000         0         0         0
         0         0         0         0    0.0000         0         0
         0         0         0         0         0    0.0000         0
         0         0         0         0         0         0    0.0000
         0         0         0         0         0         0         0

  Column 8

         0
         0
         0
         0
         0
         0
         0
    0.0000
```