Hint 1 for (2) in Homework 2
February 9, 2000

Using the condition of the solution v set up at an arbitrary time τ

$$v(x,\tau)=0 \ , \ \frac{\partial v}{\partial t}(x,\tau)=f(x,\tau) \ , \ x\in(0,L)$$

differentiation of the function

$$w_f(x,t)=\int_0^t v(x,t;\tau)\,d\tau$$

in time t , yields

$$\frac{\partial}{\partial t}w_f(x,t)=\frac{\partial}{\partial t}\int_0^t v(x,t;\tau)\,d\tau=v(x,t;t)+\int_0^t \frac{\partial}{\partial t}v(x,t;\tau)\,d\tau=\int_0^t \frac{\partial}{\partial t}v(x,t;\tau)\,d\tau$$

and

$$\frac{\partial^2}{\partial t^2}w_f(x,t)=\frac{\partial}{\partial t}\left[\frac{\partial}{\partial t}w_f(x,t)\right]=\frac{\partial}{\partial t}\int_0^t \frac{\partial}{\partial t}v(x,t;\tau)\,d\tau$$
$$=\frac{\partial}{\partial t}v(x,t;t)+\int_0^t \frac{\partial^2}{\partial t^2}v(x,t;\tau)\,d\tau$$
$$=f(x,t)+\int_0^t \frac{\partial^2}{\partial t^2}v(x,t;\tau)\,d\tau$$

Now you can substitute this relation to $\rho\dfrac{\partial^2 w_f}{\partial t^2}$.

Note : I made a mistake in

$$v(x,\tau)=0 \ , \ \frac{\partial v}{\partial t}(x,\tau)=f(x,\tau) \ , \ x\in(0,L)$$

I should have

$$v(x,\tau)=0 \ , \ \rho\frac{\partial v}{\partial t}(x,\tau)=f(x,\tau) \ , \ x\in(0,L)$$