Topology Optimization - 1 - OPTISHAPE

1. Topology Optimization
2. Homogenization Design Method
3. Background of the New Approach
4. Mathematical Formulation
5. Optimality Criteria Method

What is Topology Design ?

- Shape design keeps the initial topology, while the shape of exterior/interior domain is designed.
- If an extra hole is generated, or if two holes are merged to a single one, we say topology has changed.
- Finding the number, location, and shape of the holes is a typical topology problem.

OPTISHAPE

The key idea is to transfer

shape/topology design

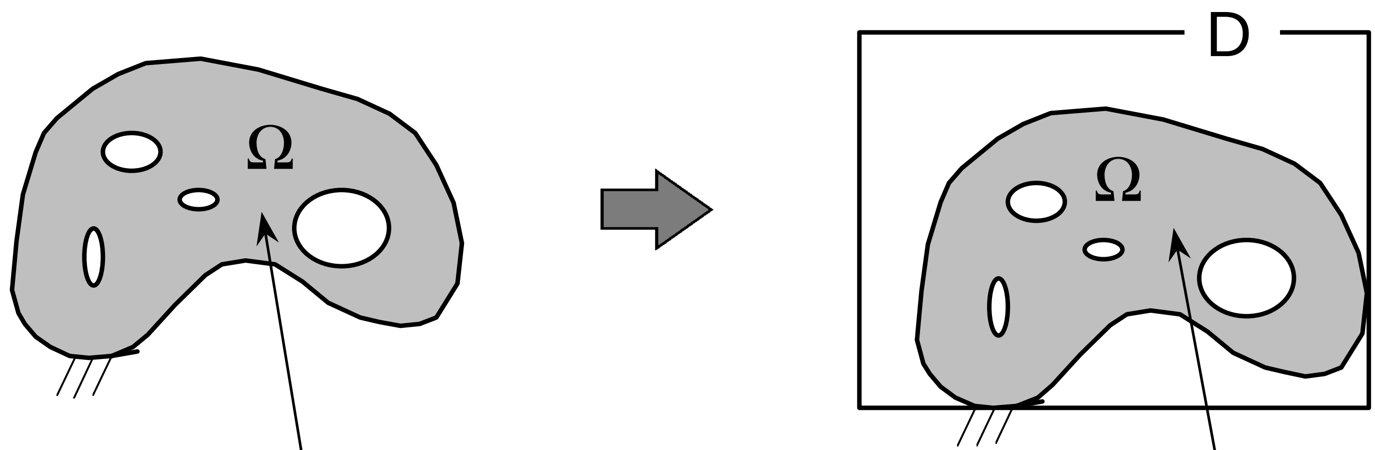
to

Optimum material distribution

with on/off switch condition

Idea in OPTISHAPE

Extension Ω to the fixed domain D



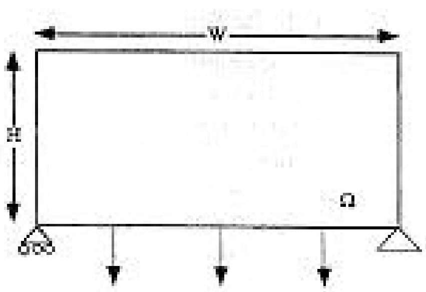
The elasticity tensor $\boldsymbol{E}$

$\mathrm{c}_\Omega \boldsymbol{E}$

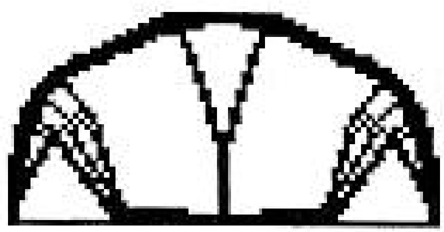
$$\mathrm{c}_\Omega(x) = \begin{cases} 1 & \text{if } \ x \in \Omega \\ 0 & \text{if } \ x \notin \Omega \end{cases}$$

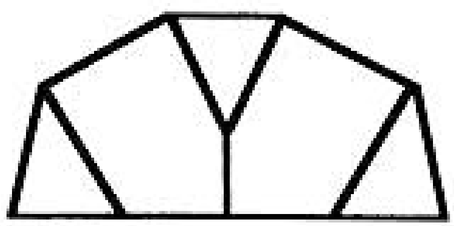
$$\mathrm{c}_\Omega \boldsymbol{E} \in L^\infty(\Omega)$$

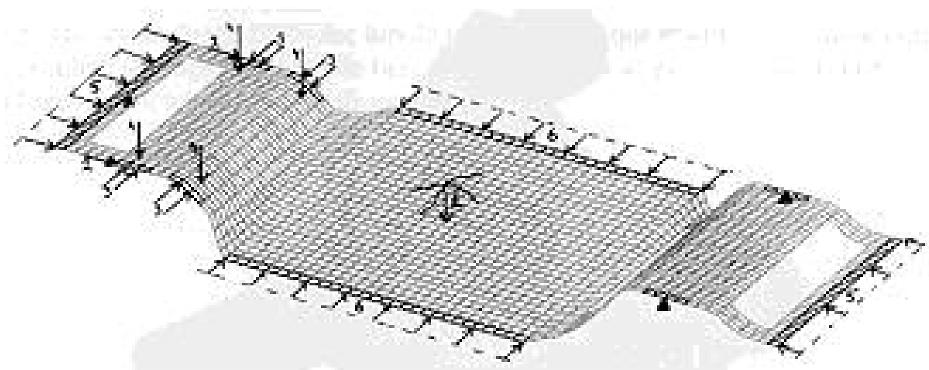
Examples by OPTISHAPE



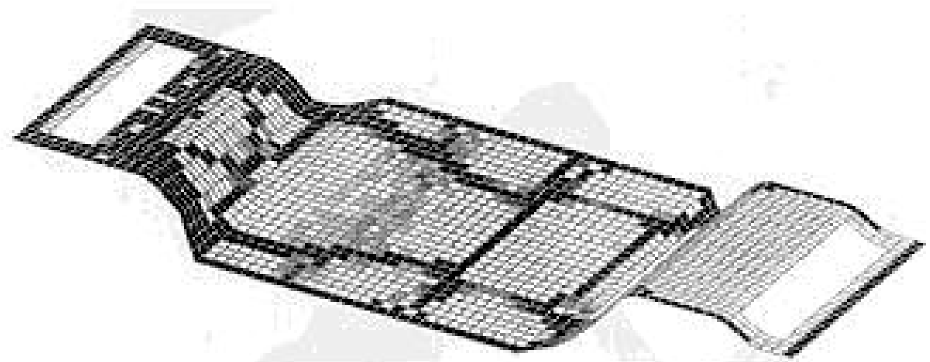
bridge design







½ oor panel r einfor cement



Topology Optimization

- has become an important and well recognized sub-area of structural optimization
  - Design Sensitivity Analysis (1960s & 70s)
    - linear and nonlinear problems
  - Sizing Optimization (1960s)
  - Shape Optimization (1970s & 80s)
  - Topology (Layout) Optimization (90s)
    - Discrete and Continuum Topology Optimization
    - Material Based Optimization
    - Extension to MEMS area

Evidence

- Last Two : 1st and 2nd World Congress on Structural and Multi-disciplinary Optimization (Germany95 & Poland97)
- There are numerous sessions on topology optimization related
- Commercial Codes
  - OPTISHAPE(Japan), OPTISTRUCT(US)
  - MSC-NASTRAN, ANSYS ---- Fall 97

OPTISHAPE : Present

- Maximization of the global stiffness of an elastic structure
- Maximization of the mean eigenvalue problems for free vibration
- Combination of the above two
- Maximization of the dynamic stiffness for frequency response problems
- Heat Conduction/Thermal Loading

OPTISHAPE : Near Future

- include SHAPE OPTIMIZATION capability based on Azekami and Shimoda’s Method (at Mitsubishi Motor) for detailed shape design after the standard topology optimization
- include sensitivity analysis for sizing
- TOPOLOGY + SHAPE + SIZING

OPTISHAPE : Future

- Compliant Mechanism, Mechanism, and Flexible Body Design
  - to control deformation and motion of structures, flexible multi-bodies, compliant mechanisms, and even mechanisms to have integrated synthesis study of mechanical systems
  - toward smart structure design with control
- Material Design
  - Young's and Shear moduli and Poisson's ratios
  - Piezo-electric material design for MEMS

http://www02.so-net.or.jp/~quint

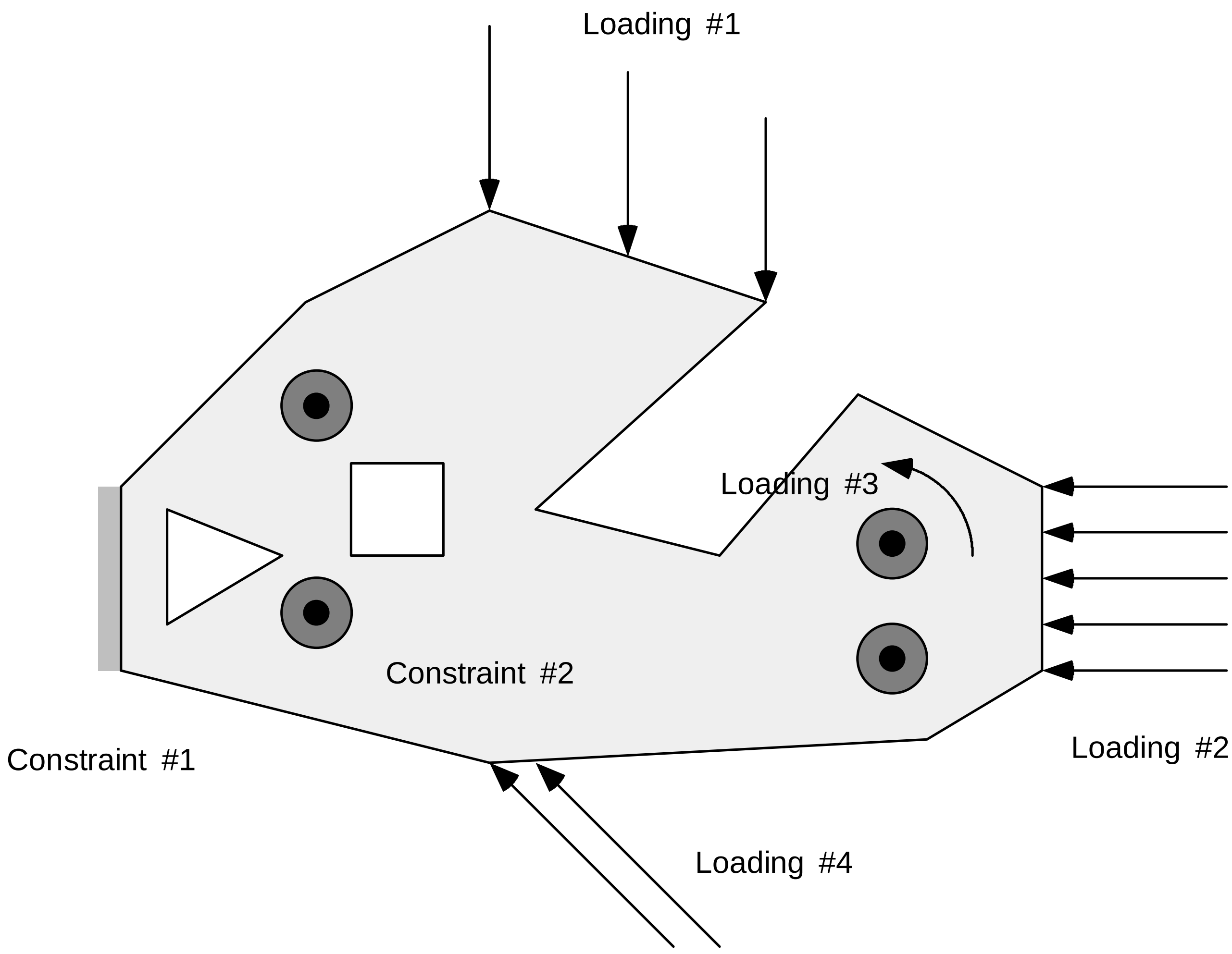
for

more information

Typical Procedure 1

- (1) Define a design domain which contains the final optimum structure
  - geometric restriction for the on/off condition
  - on-flag : solid structure always exists
  - off-flag : void (hole) must be assigned
- (2) Define the loading and displacement constraint
  - multiple loadings and multiple constraints are possible in OPTISHAPE

Multiple Loadings & Multiple Constraints



Typical Procedure 2

- (3) Define the volume (or weight) constraint

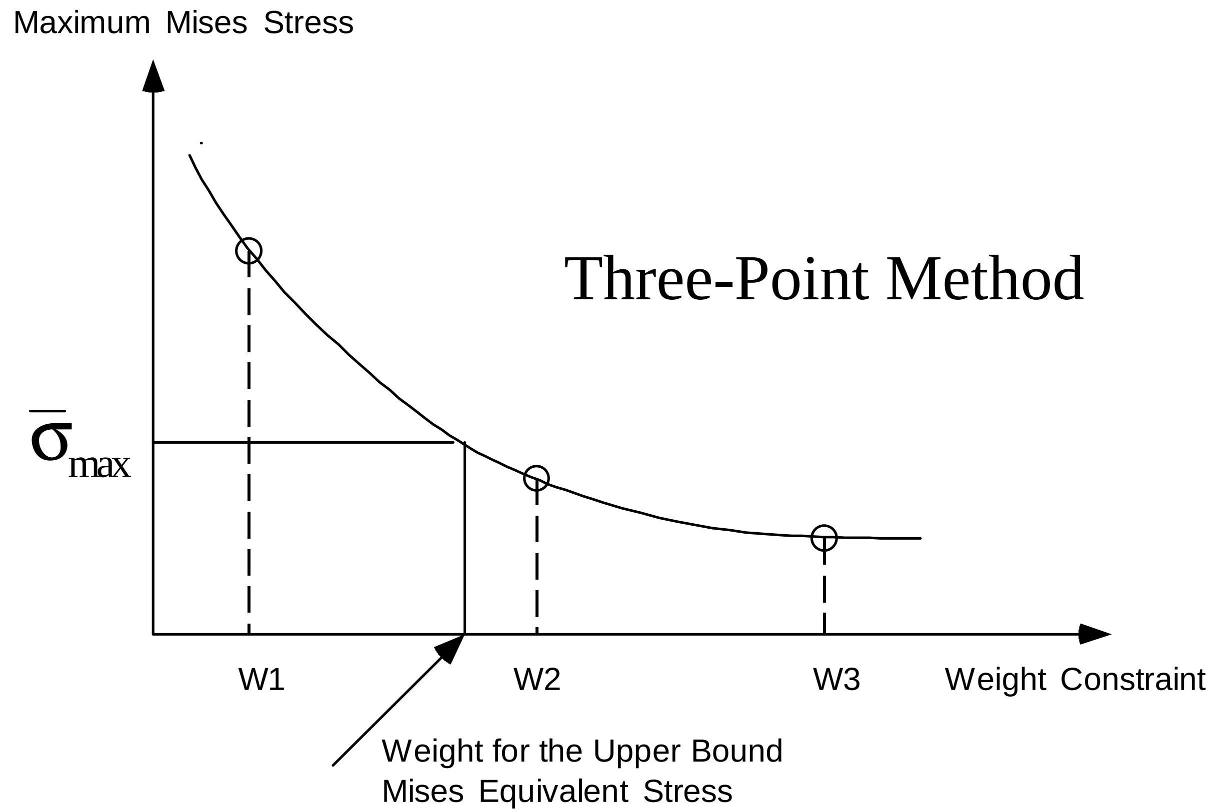
$$\int_{\Omega} \rho \, d\Omega \leq W_1$$

- (4) Applying OPTISHAPE, and obtain
  - the Optimum Layout (Topology & Shape)
  - the Maximum Mises Equivalent Stress
  - the Mean Compliance and Strain Energy Density

Typical Procedure #3

- (5) Repeat the above steps for two more different weight constraints W_2 & W_3
- (6) Obtain the maximum Mises stress and the mean compliance
- (7) Using the quadratic interpolation of
  - Maximum Mises Stresses & Weights
  - compute the weight for the allowable stress constraints

Determination of the Weight Constraint to enforce the stress constraint if exist



Extended Domain by χ_Ω

Internal Virtual Work

$$\int_\Omega \varepsilon(\boldsymbol{v})^T \boldsymbol{E}\,\varepsilon(\boldsymbol{u})\,d\Omega = \int_D \varepsilon(\boldsymbol{v})^T \chi_\Omega \boldsymbol{E}\,\varepsilon(\boldsymbol{u})\,dD$$

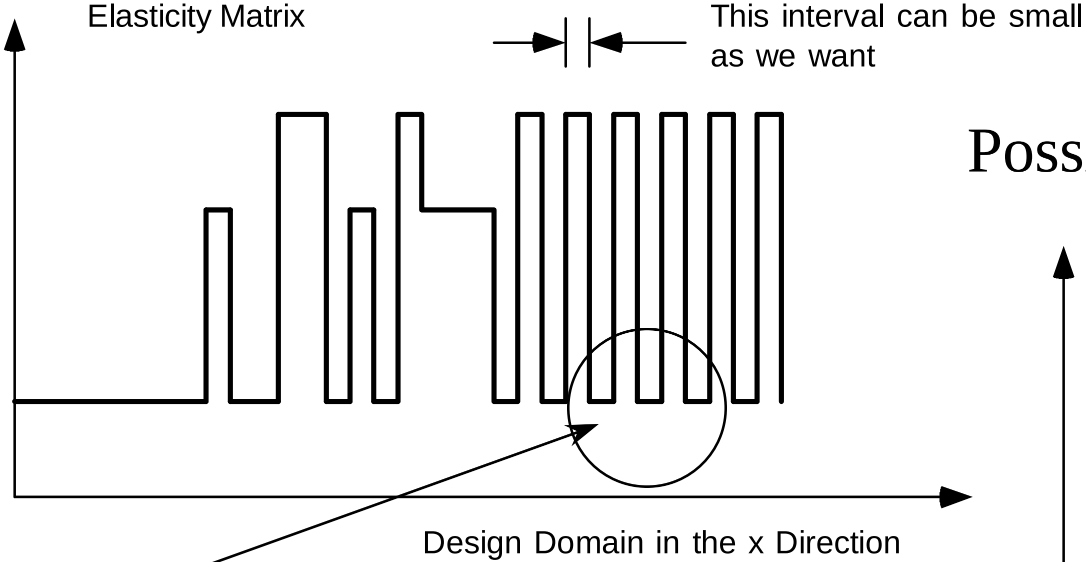
New Material Constants (Extended Elasticity Matrix)

$\chi_\Omega \boldsymbol{E} \in L^\infty(D)$ is very discontinuous

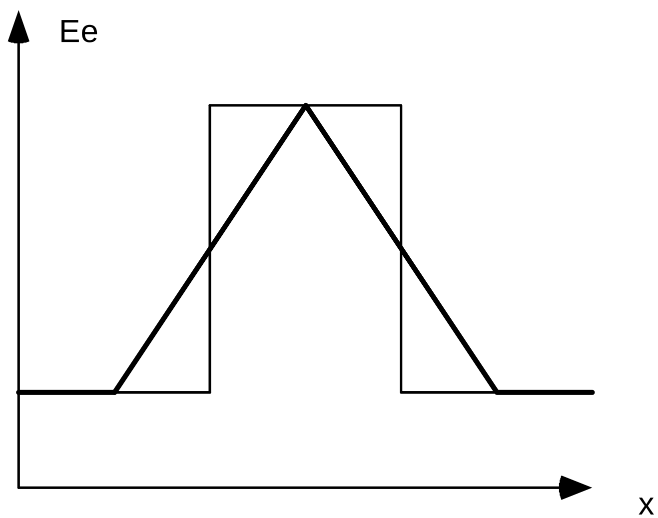
Impossible to take its derivative

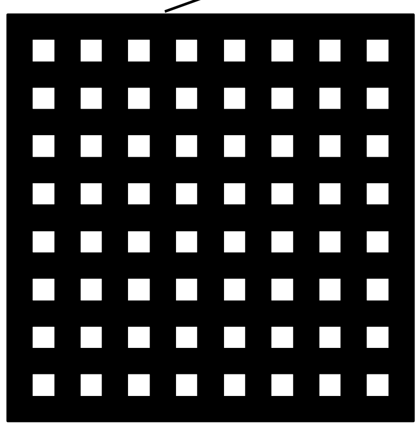
that is, no design sensitivity analysis

Possible Approximation



Possibly Rapidly Varying





Introduce Two Scales
at an arbitrary point

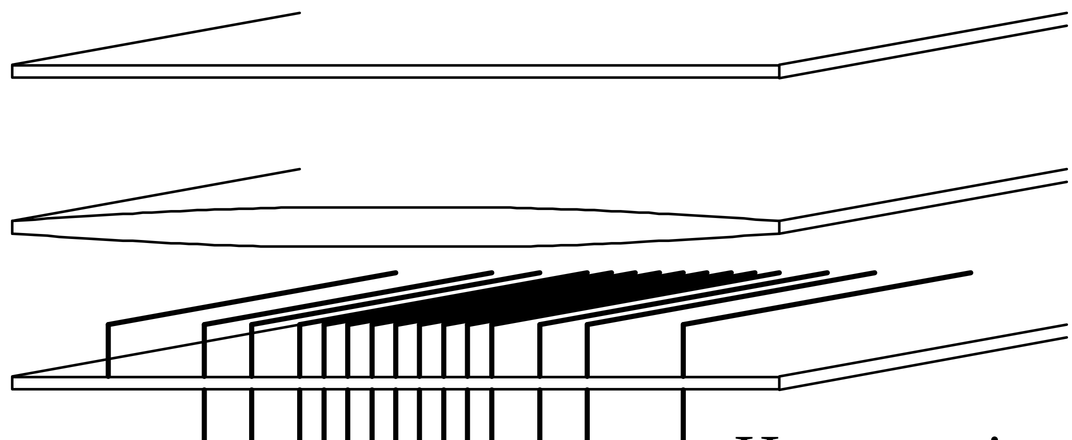
No Way to Approximate
by a Standard Way with
a distribution function

Relaxation

- Very Rapidly Varying Function $\chi_\Omega \boldsymbol{E}$ cannot be approximated by a differentiable function of position $\boldsymbol{x}$ in the standard way
- Introduce the two scales $\left(\boldsymbol{x}, \boldsymbol{y} = \frac{\boldsymbol{x}}{\varepsilon}\right)$ and the micro-scale perforation, and then $\chi_\Omega \boldsymbol{E}$ is approximated by the homogenized average elasticity matrix $\boldsymbol{E}^H$

Origin of the Idea

- G. Cheng and N. Olhoff
  - in plate thickness optimization
  - smoothly varying thickness is not optimum
  - optimum involves rapidly changing ribs

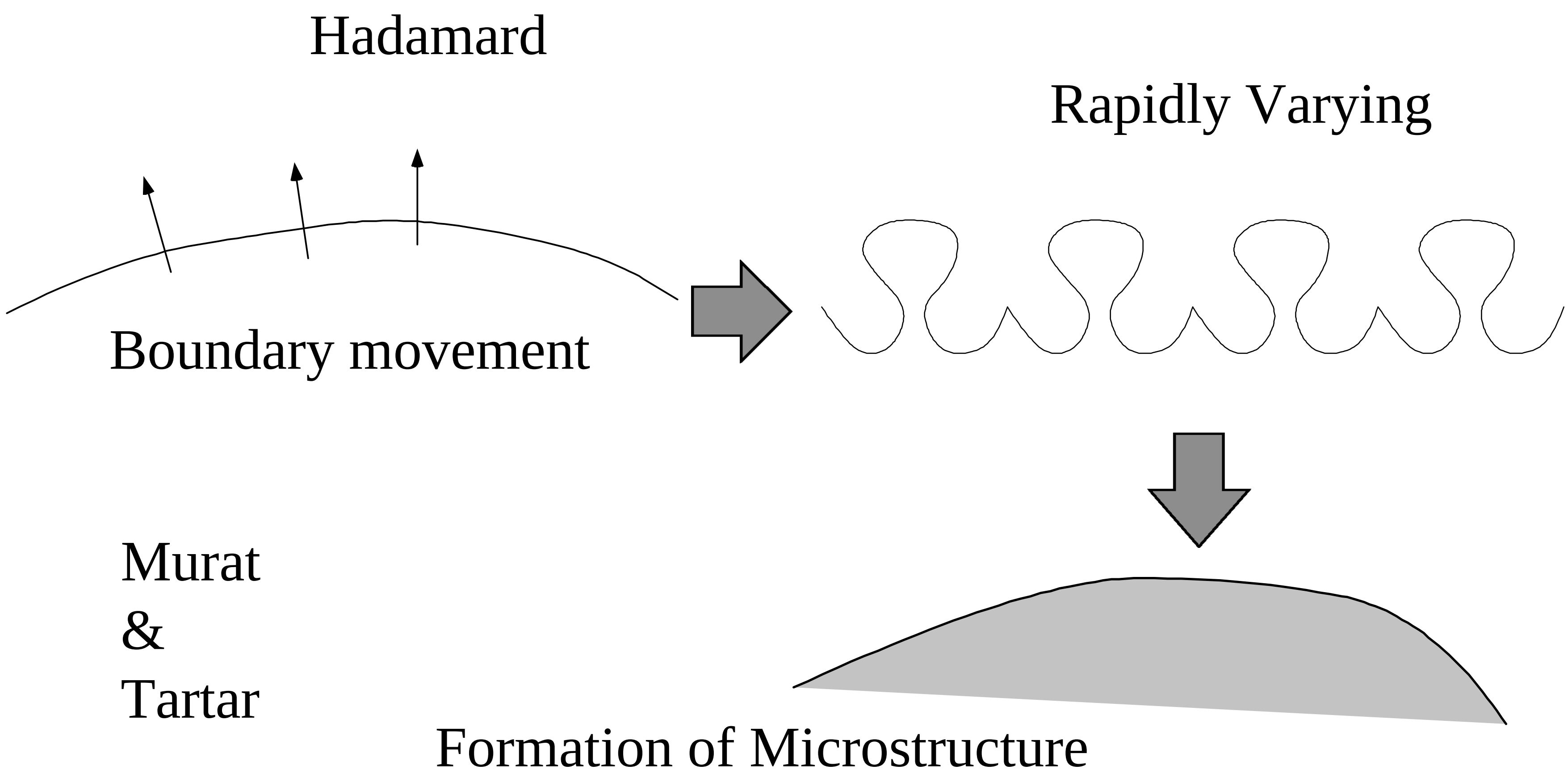


Homogenization is required

Mathematicians

- Lurier, Cherkaev,and Fedrov (1981)
  - Notion of G-convergence that is in the specially designed average sense convergence
- Kohn and Strang (1984)
  - Microscale performation and specialized variational principles
- Murat and Tartar (1983)
  - Homogenization Theory from Hadamard Shape Design Problem

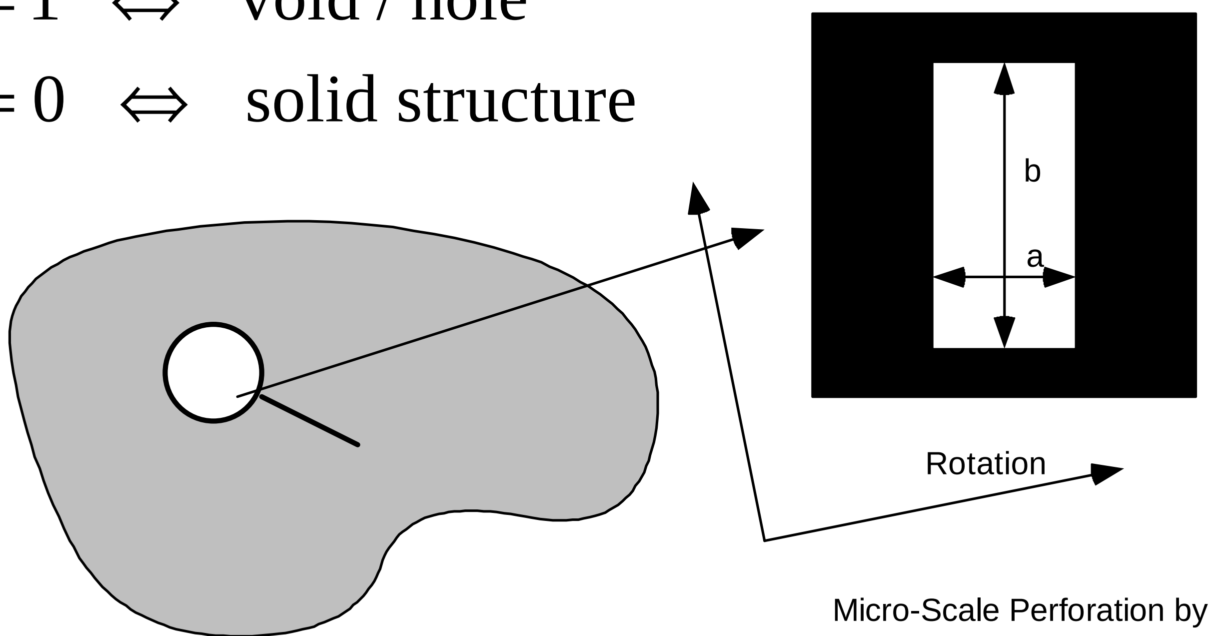
Hadamard v.s. Homogenization



Bendsoe & Kikuchi 1986

$a = b = 1 \quad \Leftrightarrow \quad$ void / hole

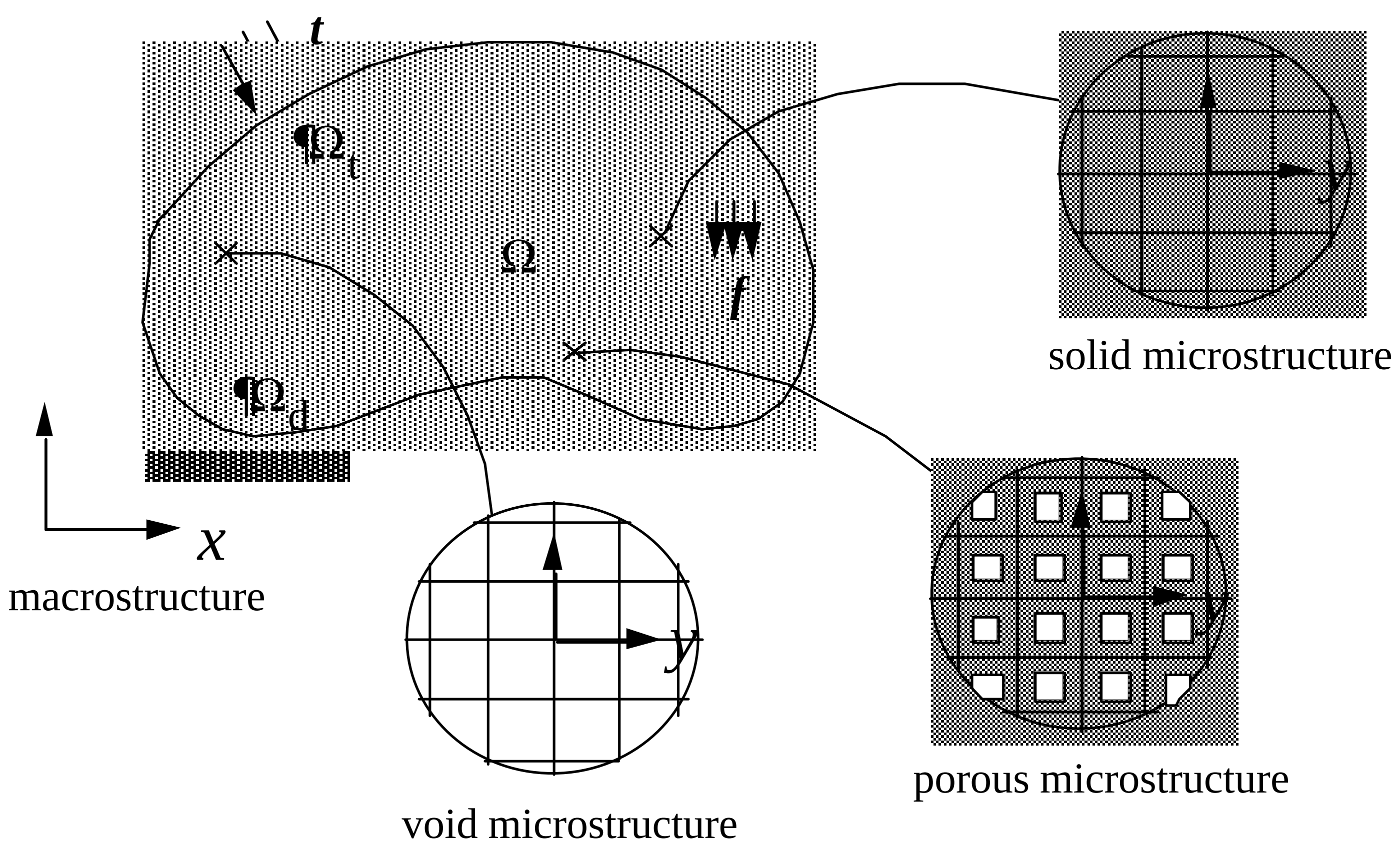
$a = b = 0 \quad \Leftrightarrow \quad$ solid structure



Micro-Scale Perforation by
infinitely many rectangular holes

Design Variables are $\{a, b, \theta\}$ at every where

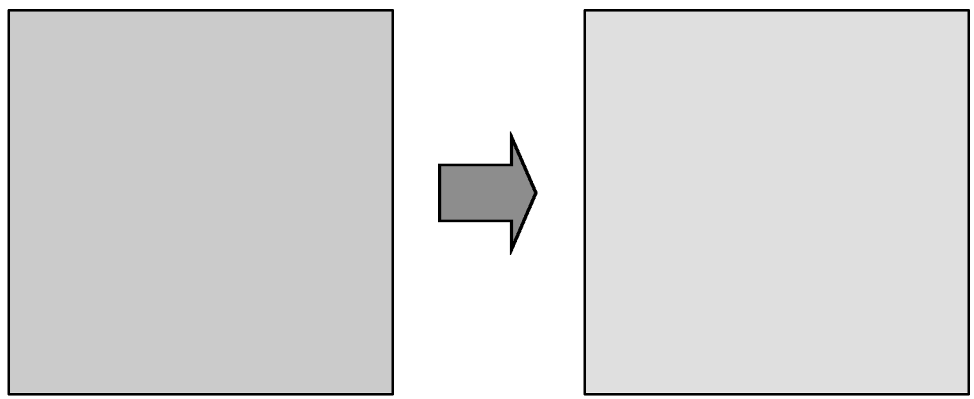
The Homogenization Design



Why Homogenization ?

Small Scale Rapidly
Varying Heterogeneity

This idealization is regarded as the homogenization in theoretical mechanics

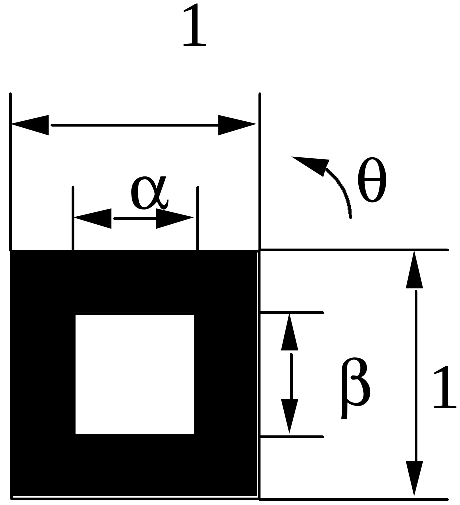


If the exact heterogeneity is used in mechanics, we must introduce so fine finite elements to represent all the detail. This is a difficult task.

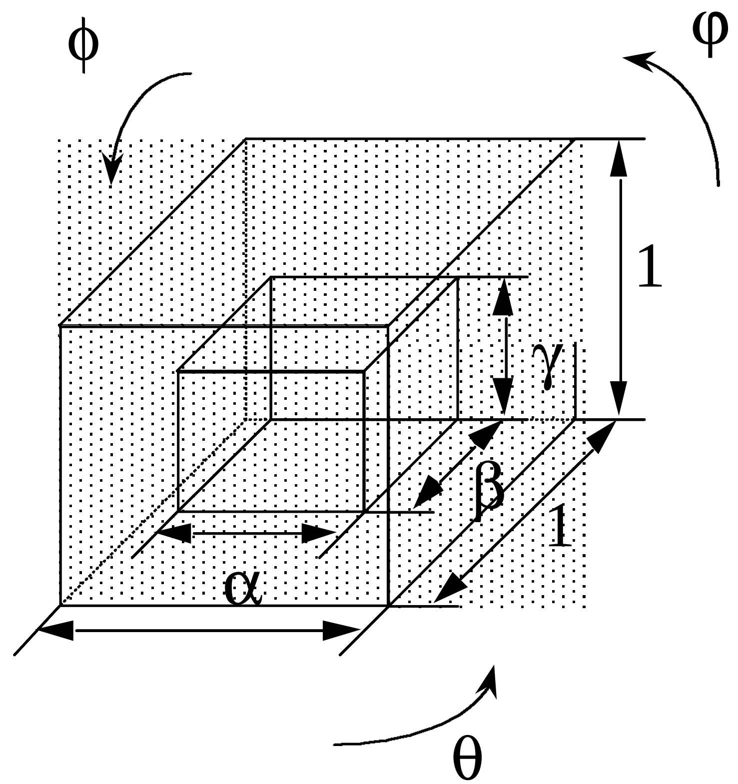
Equivalent Homogeneous
Material

Microstructures

2D



3D



Many Choices

- The key feature is an approximation of the extended elasticity matrix $\chi_{\Omega}\boldsymbol{E} \in L^{\infty}(D)$
- There are infinitely many ways to approximate this by using
  - Generalized Porous Media Constitutive Equations (bio-mechanics, Geo-mechanics)
  - Power Low of Density/Elasticity Constants
  - Rank 1 & Rank 2 Orthotropic Materials
  - Others

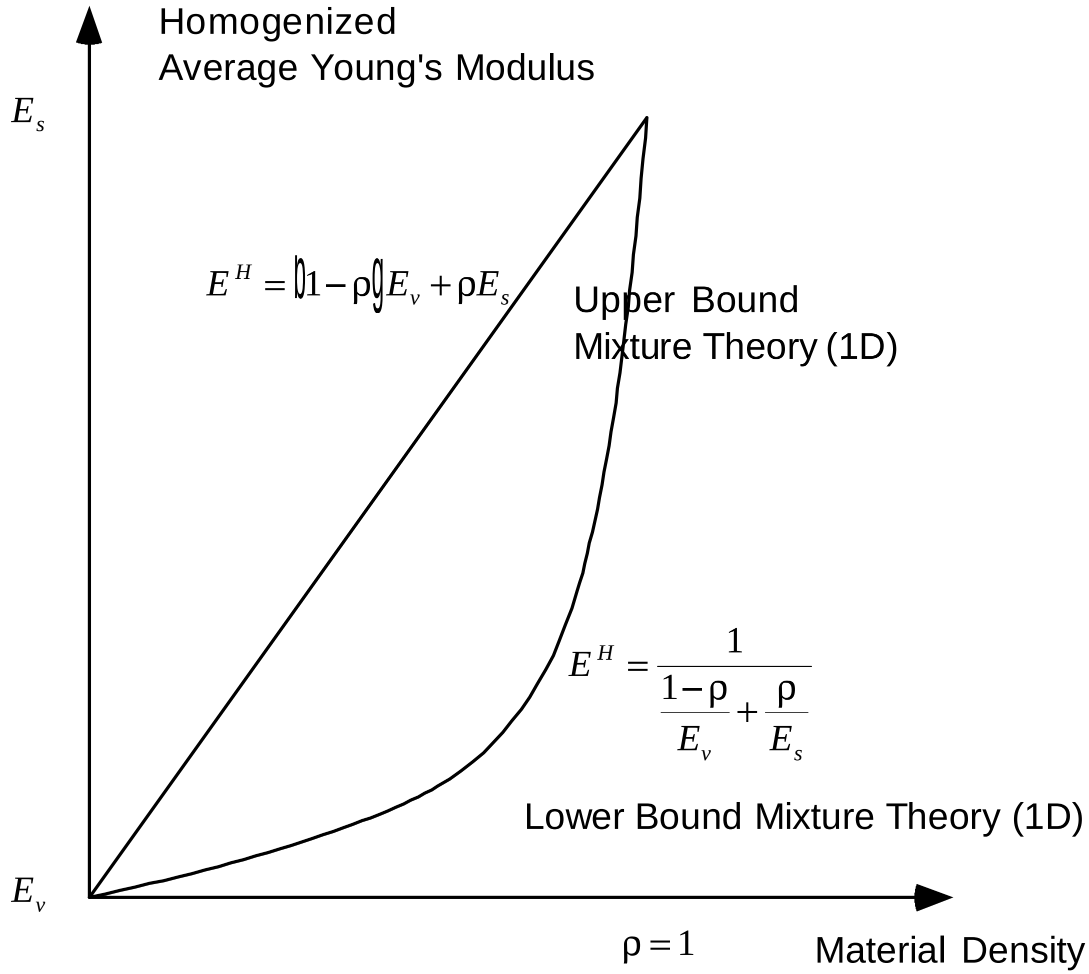
Power Low

Most popular approach at present (Meljek, Yang, ...)
Altair/OPTISTRUCT is now assuming this approach

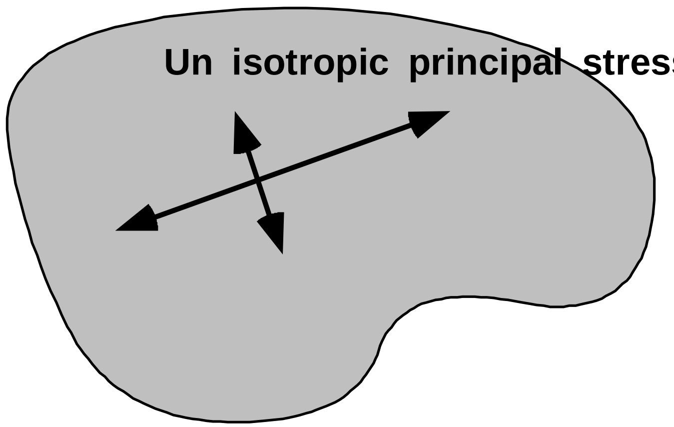
$$\chi_\Omega \boldsymbol{E} \approx \rho^p \boldsymbol{E}$$

For p=2 or 4, the design variable becomes the density ρ such that

$$\rho = \begin{cases} 1 & \text{if} \quad \text{solid structure} \\ 0 & \text{if} \quad \text{void / hole} \end{cases}$$


Homogenized
Average Young's Modulus
E_s
$E^H = (1-\rho)E_v + \rho E_s$
Upper Bound
Mixture Theory (1D)
$E^H = \dfrac{1}{\dfrac{1-\rho}{E_v} + \dfrac{\rho}{E_s}}$
Lower Bound Mixture Theory (1D)
E_v
$\rho = 1$
Material Density

Short Coming of Power Low



Density approch can make only isotropic performation

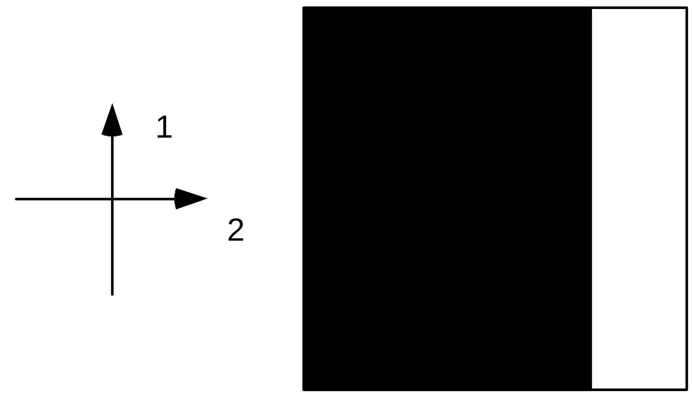
This requires a lot of meshing to have reasonable layout

However, easy programing and handy design variable

Rank 1 & Rank 2 Materials

Rank 1 Lamination

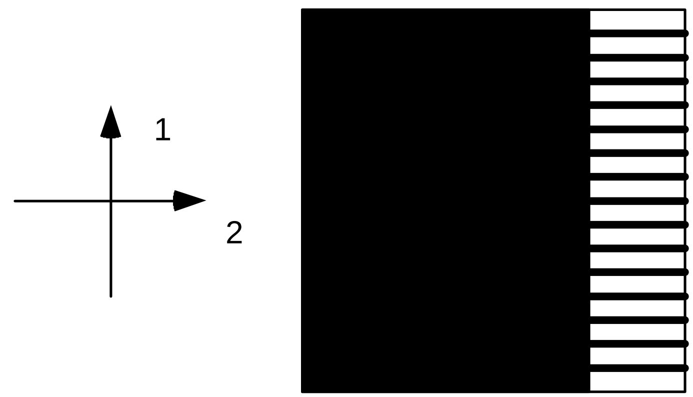
Zero E in 2
Zero Shear



Rank 2 Lamination

Zero Shear

Rectangular hole is very close to Rank 2 Material



Elasticity Matrix

Rank 1 Material

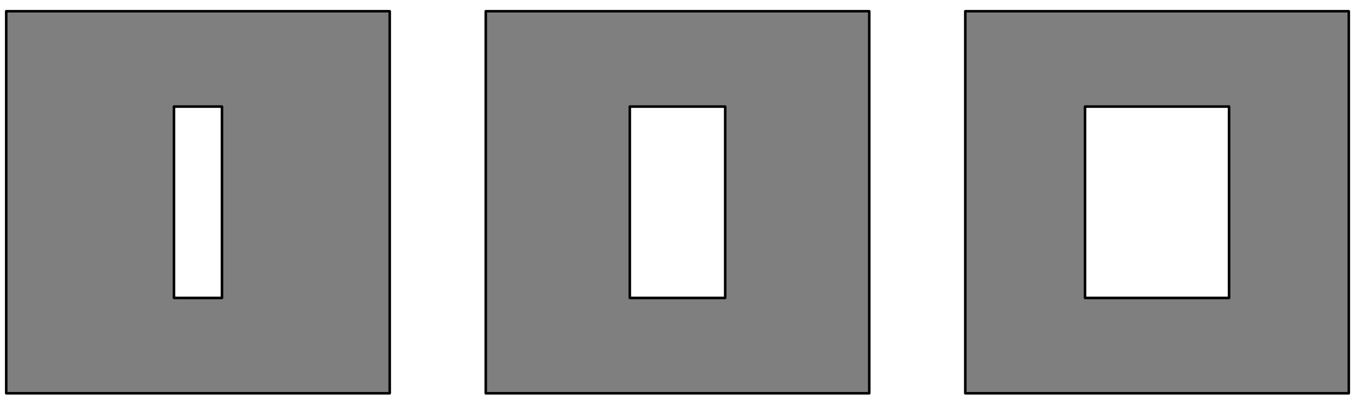
$$\boldsymbol{E}^{H} = \begin{bmatrix} (1-a)E_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank 2 Material

$$\boldsymbol{E}^{H} = \frac{E_s}{(1-\nu^2)[1-ab]} \begin{bmatrix} (1-\nu^2)(1-ab)(1-a)+\nu^2(1-b) & \nu(1-b) & 0 \\ \nu(1-b) & (1-b) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Advantage of Rank 2

Rank 2 elasticity matrix can be computed in the closed form while rectangular hole requires FE calculation



Rank 2 and Rectangular Hole

- Rank 2 material is regarded as the optimum orthotropic material (since its shear modulus becomes zero)
- Then this yield the stable optimum solution mathematically verified in the sense that FE mesh dependency cannot be observed
- However, this results considerably lots of gray perforated medium in the optimum

Penalization

- We would like to have clear segregation of solid and void portions to define precise shape and topology, that is
- No gray scale solution is desirable

$$\chi_{\Omega}\boldsymbol{E} \approx \begin{cases} \rho^{p}\boldsymbol{E} & \text{..... Density Power Low} \\ \boldsymbol{E}^{H} & \text{..... Rank2 \& Rectangular hole} \\ \left(\dfrac{\boldsymbol{E}^{H}}{\boldsymbol{E}}\right)\boldsymbol{E}^{H} & \text{..... Penalization} \end{cases}$$

Multi-Loading 1

Inequality Relation

$$\max_{(a,b,\theta)} \min_{i=1,2,\ldots,i_{\max}} \frac{1}{2} a_i\left(\boldsymbol{u}_i, \boldsymbol{u}_i\right) - f_i\left(\boldsymbol{u}_i\right) = \max_{(a,b,\theta)} \min_{i=1,2,\ldots,i_{\max}} -\frac{1}{2} a_i\left(\boldsymbol{u}_i, \boldsymbol{u}_i\right)$$

$$\leq \max_{(a,b,\theta)} -\frac{1}{2} a_m\left(\boldsymbol{u}_m, \boldsymbol{u}_m\right) = \max_{(a,b,\theta)} \frac{1}{2} a_m\left(\boldsymbol{u}_m, \boldsymbol{u}_m\right) - f_m\left(\boldsymbol{u}_m\right)$$

Minimum Principle to the I-th Load (Equilibrium)

$$\frac{1}{2} a_i\left(\boldsymbol{u}_i, \boldsymbol{u}_i\right) - f_i\left(\boldsymbol{u}\right) = \min_{\boldsymbol{v}_i} \frac{1}{2} a_i\left(\boldsymbol{v}_i, \boldsymbol{v}_i\right) - f_i\left(\boldsymbol{v}\right)$$

Multiple Loading 2

Formtion of a Single Objective Function

$$a_m\left(\boldsymbol{u}_m, \boldsymbol{u}_m\right) = \int_D \max_{i=1,\ldots,i_{\max}} \left(\left(\boldsymbol{u}_i\right)^T \chi_\Omega \boldsymbol{E} \max_{i=1,\ldots,i_{\max}} \left(\boldsymbol{u}_i\right)\right) dD$$

$$f_m\left(\boldsymbol{u}_m\right) = \int_D \max_{i=1,\ldots,i_{\max}} \left(\left(\boldsymbol{u}_i\right)^T \chi_\Omega \boldsymbol{E} \sigma_0\right) dD + \int_D \max_{i=1,\ldots,i_{\max}} \boldsymbol{u}_i^{\,T} \chi_\Omega \rho \boldsymbol{b}\, dD + \ldots..$$

Approximated Design Problem

$$\max_{\left(a,b,\theta\right)} \; \min_{\boldsymbol{v}} \frac{1}{2} a_m\left(\boldsymbol{v}, \boldsymbol{v}\right) - f_m\left(\boldsymbol{v}\right)$$

subject to

$$\int_D \rho\, dD \le W_0$$

Alternate

Weighted Sum Approach

$$\max_{\{a,b,\theta\}} \sum_{i=1}^{i_{\max}} w_i \min_{\boldsymbol{v}} \frac{1}{2} a_i(\boldsymbol{v},\boldsymbol{v}) - f_i(\boldsymbol{v})$$

$$\text{subject to}$$

$$\int_D \rho \, dD \leq W_0$$

Diaz and Bendsoe

OPTISHAPE can do both ways by user's choice

Lagrangian

Optimization Problem

$$\max_{(a,b,\theta)} \; \min_{\boldsymbol{v}} \frac{1}{2}a(\boldsymbol{v},\boldsymbol{v}) - f(\boldsymbol{v})$$

subject to

$$\int_D \rho \, dD \le W_0$$

Taylor & Prager in 1967

Lagrangian

$$L = \frac{1}{2}a(\boldsymbol{v},\boldsymbol{v}) - f(\boldsymbol{v}) - \lambda\left(\int_D \rho \, dD - W_0\right)$$

First Variation

$$\begin{aligned}
\delta L = {} & a(\boldsymbol{v},\delta\boldsymbol{v}) - f(\delta\boldsymbol{v}) - \delta\lambda\left(\int_D \rho \, dD - W_0\right) \\
& + \int_D \left\{ \varepsilon(\boldsymbol{v})^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial a} \varepsilon(\boldsymbol{v}) - \lambda \frac{\partial \rho}{\partial a} \right\} \delta a \, dD \\
& + \int_D \left\{ \varepsilon(\boldsymbol{v})^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial b} \varepsilon(\boldsymbol{v}) - \lambda \frac{\partial \rho}{\partial b} \right\} \delta b \, dD \\
& + \int_D \left\{ \varepsilon(\boldsymbol{v})^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial \theta} \varepsilon(\boldsymbol{v}) \right\} \delta\theta \, dD
\end{aligned}$$

Optimality Condition

Equilibrium

$$a(\boldsymbol{v}, \delta\boldsymbol{v}) = f(\delta\boldsymbol{v}) \qquad \forall \delta\boldsymbol{v}$$

Weight Constraint : Kuhn-Tucker Condition

$$\lambda\left(\int_D \rho\, dD - W_0\right) = 0 \quad , \quad \lambda \le 0 \quad , \quad \int_D \rho\, dD - W_0 \le 0$$

Optimality Condition

$$\int_D (\bar{a} - a)\left\{ \varepsilon(\boldsymbol{v})^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial a} \varepsilon(\boldsymbol{v}) - \lambda \frac{\partial \rho}{\partial a} \right\} dD \ge 0 \quad , \quad 0 \le \forall \bar{a} \le 1$$

$$\int_D (\bar{b} - b)\left\{ \varepsilon(\boldsymbol{v})^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial b} \varepsilon(\boldsymbol{v}) - \lambda \frac{\partial \rho}{\partial b} \right\} dD \ge 0 \quad , \quad 0 \le \forall \bar{b} \le 1$$

$$\varepsilon(\boldsymbol{v})^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial \theta} \varepsilon(\boldsymbol{v}) = 0$$

Optimality Criteria Method 1

$$\int_D \left(\bar{a} - a\right)\left[\{\boldsymbol{v}\}^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial a}\{\boldsymbol{v}\} - \lambda \frac{\partial \rho}{\partial a}\right] dD \geq 0 \quad , \quad 0 \leq \forall \bar{a} \leq 1$$

$$\Leftrightarrow \{\boldsymbol{v}\}^T \frac{\partial \chi_\Omega \boldsymbol{E}}{\partial a}\{\boldsymbol{v}\} - \lambda \frac{\partial \rho}{\partial a} = 0 \quad \text{if} \quad a \neq 0 \quad \text{and} \quad a \neq 1$$

$$\Leftrightarrow \frac{\{\boldsymbol{v}\}^T \dfrac{\partial \chi_\Omega \boldsymbol{E}}{\partial a}\{\boldsymbol{v}\}}{\lambda \dfrac{\partial \rho}{\partial a}} = 1 \quad \text{if} \quad a \neq 0 \quad \text{and} \quad a \neq 1$$

$$\Rightarrow a^{(k+1)} = a^{(k)} \left(\frac{\left\{\boldsymbol{v}^{(k)}\right\}^T \dfrac{\partial \chi_\Omega \boldsymbol{E}}{\partial a}\left(a^{(k)}, b^{(k)}, \theta^{(k)}\right)\left\{\boldsymbol{v}^{(k)}\right\}}{\lambda^{(k)} \dfrac{\partial \rho}{\partial a}\left(a^{(k)}, b^{(k)}, \theta^{(k)}\right)} \right)^{\alpha} \quad , \quad 0 < \alpha \leq 1$$

Optimality Criteria Method 2

With design constraint $0 \le a \le 1$

$$a^{(k+1)} = \max\left\{0, \min\left\{1, a^{(k)}\left[\frac{\left(\boldsymbol{v}^{(k)}\right)^T \dfrac{\partial \chi_\Omega \boldsymbol{E}}{\partial a}\left(a^{(k)}, b^{(k)}, \theta^{(k)}\right)\left(\boldsymbol{v}^{(k)}\right)}{\lambda^{(k)} \dfrac{\partial \rho}{\partial a}\left(a^{(k)}, b^{(k)}, \theta^{(k)}\right)}\right]^{\alpha}\right\}\right\}$$

Algorithm of the optimality criteria method is very similar with the fully stressed design

Choice of Parameter

$$\alpha \approx 0.75$$

Lagrange Multiplier

$\lambda^{(k)}$ is computed by the bisection method

$$\int_D \rho\left(a^{(k)}, b^{(k)}\right) dD = W_0$$

based the implicit function theorem

Volume constraint is always saturated.
This is correct, but for eigenvalue related problems. this is not true.

Optimum Angle : Pedersen

Noting that

$$\varepsilon(\boldsymbol{v})^T(\chi_\Omega \boldsymbol{E})\varepsilon(\boldsymbol{v}) = \sigma(\boldsymbol{v})^T\left[\chi_\Omega \boldsymbol{E}^{-1}\right]\sigma(\boldsymbol{v})$$

$$= \{\sigma_1 \quad \sigma_2 \quad 0\}\,\boldsymbol{T}(\phi)^T\left[\chi_\Omega \boldsymbol{E}^{-1}\right]\boldsymbol{T}(\phi)\begin{Bmatrix}\sigma_1\\ \sigma_2\\ 0\end{Bmatrix}$$

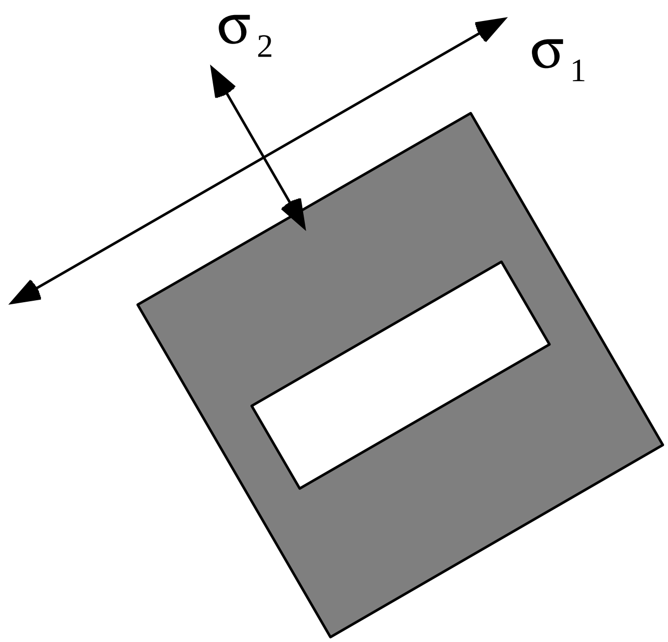
we have

$$\varepsilon(\boldsymbol{v})^T\frac{\partial \chi_\Omega \boldsymbol{E}}{\partial\theta}\varepsilon(\boldsymbol{v}) = 0 \quad\Rightarrow\quad \frac{\partial}{\partial\theta}\boldsymbol{T}(\phi)^T\left[\chi_\Omega \boldsymbol{E}^{-1}\right]\boldsymbol{T}(\phi) = 0 \quad\Leftrightarrow\quad \phi = 0$$

Optimum angle is the one for the principal stress !

Engineering Idea

Optimality Condition can be explained as



Rectangular hole should be aligned in the principal stress direction

Large hole can be assumed in the small principal stress direction

Small hole must be placed in the large principal stress direction

Elaboration

Using the relation

$$\rho = 1 - ab \quad \Rightarrow \quad a = \frac{1-\rho}{b}$$

Habor
Jog
Bendsoe

we can change the design variable

$$\left\{a, b, \theta\right\} \quad \Rightarrow \quad \left\{\rho, b, \theta\right\}$$

and then we can show

$$\max_{\{\rho, b, \theta\}} \min_{\boldsymbol{v}} \frac{1}{2} a\left(\boldsymbol{v}, \boldsymbol{v}\right) - f\left(\boldsymbol{v}\right)$$

$$\Leftrightarrow \quad \max_{\rho} \min_{\boldsymbol{v}} \max_{\{b, \theta\}} \frac{1}{2} a\left(\boldsymbol{v}, \boldsymbol{v}\right) - f\left(\boldsymbol{v}\right)$$

Solve b and angle analytically, then apply optimality criteria method only to the density

Monotonic Convergence

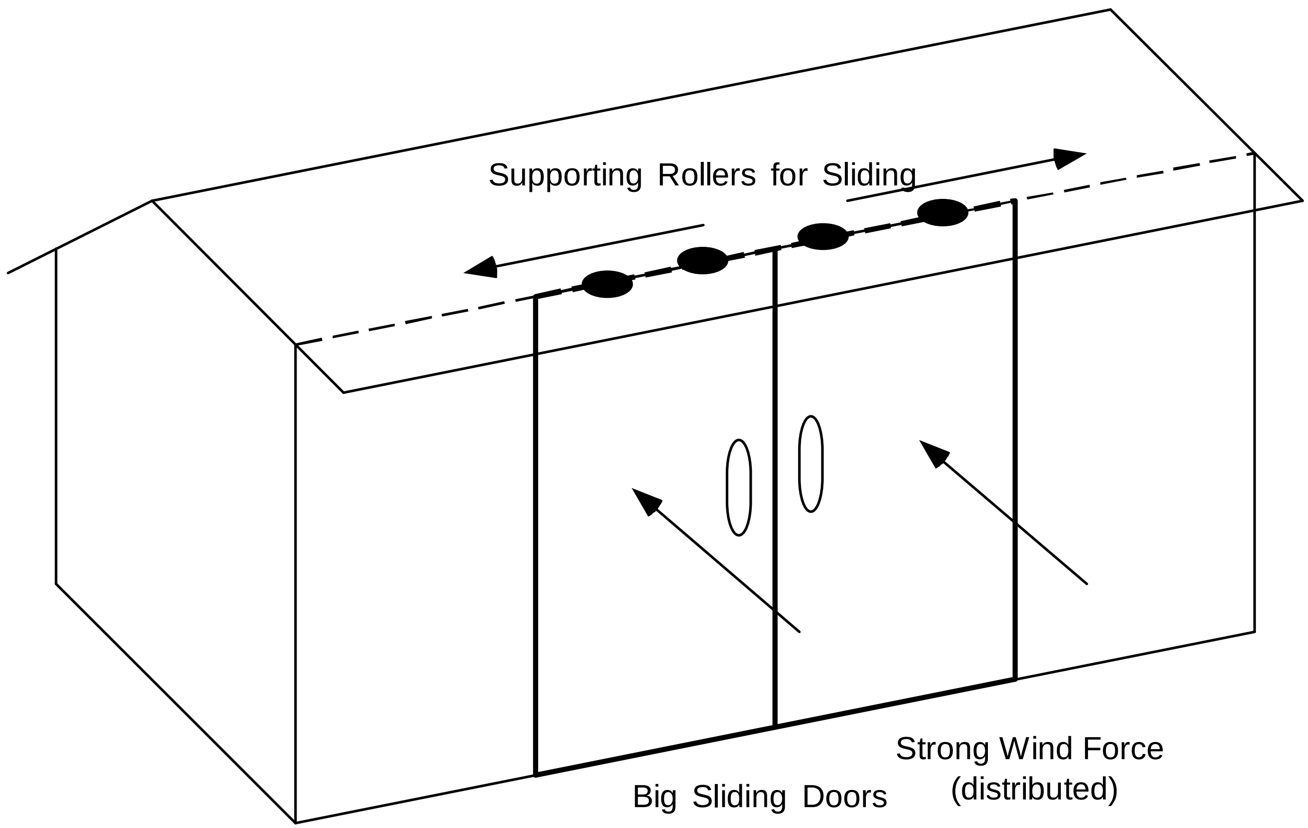
- Optimality criteria method is monotonically converging to a local optima
- The local optima obtained may be strongly dependent of the initial condition
  - Uniformly Biased Initial Condition in OPTISHAPE
- The local optima may depend on the FE mesh

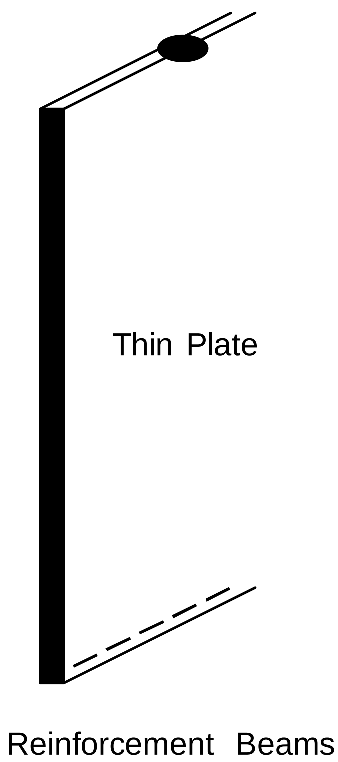
Many Problems

but

It is so powerful !

Excersise #11





Find a pattern of reinforcement of a thin plate subjected to a strong wind force by using OPTISHAPE

Topology Optimization -2- OPTISHAPE

1. Extension of HMD
2. Free Vibration Problem
3. Frequency Response Problem
4. Buckling Problem
5. Flexible Bodies

Extension of OPTISHAPE

- Free Vibration Problem
  - Maximization of lowest frequency
  - Maximization of the distance of two frequencies
  - Inverse frequency identification problem
- Frequency Response Problem
- Buckling Problem for Stability
- Flexible Multi-Body Design
- Material Microstructure Design

Eigenvalue Problem

- Maximizing the lowest eigenvalue

$$\max_{\{a,b,\theta\}} \lambda_{\min} \qquad \boldsymbol{Ku} = \lambda \boldsymbol{Mu}$$

- Several eigenvalues are crashing while optimization is performed

One eigenvalue with n number of eigenvectors
----- Ultimate Optima

Free Vibration

Discrete Free Vibration Problem

$$\boldsymbol{M}\frac{d^2\boldsymbol{x}}{dt^2}+\boldsymbol{K}\boldsymbol{x}=\boldsymbol{0}$$

Separation of variable

$$\boldsymbol{x}(t)=\exp(i\omega t)\boldsymbol{u}$$

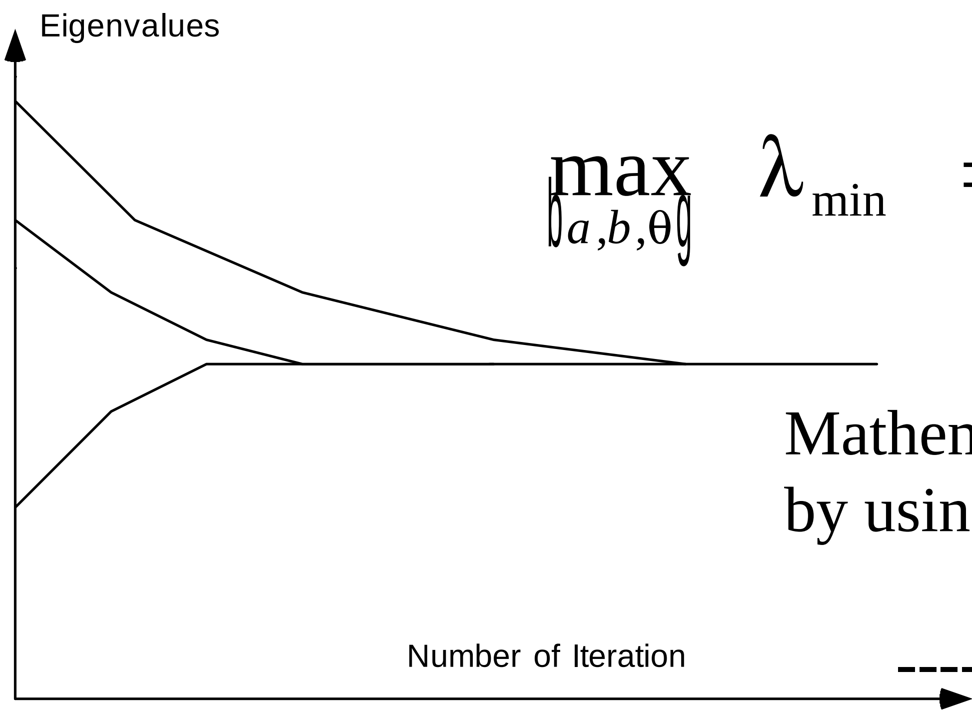
$$-\omega^2\boldsymbol{M}\exp(i\omega t)\boldsymbol{u}+\boldsymbol{K}\exp(i\omega t)\boldsymbol{u}=\boldsymbol{0}$$

$$\left(-\omega^2\boldsymbol{M}\boldsymbol{u}+\boldsymbol{K}\boldsymbol{u}\right)\exp(i\omega t)=0$$

$$-\omega^2\boldsymbol{M}\boldsymbol{u}+\boldsymbol{K}\boldsymbol{u}=\boldsymbol{0}$$

$$\boldsymbol{K}\boldsymbol{u}=\omega^2\boldsymbol{M}\boldsymbol{u}=\lambda\boldsymbol{M}\boldsymbol{u}$$

Modification by Ma, Cheng, Kikuchi



$$\max_{\{a,b,\theta\}} \lambda_{\min} \Rightarrow \max_{\{a,b,\theta\}} \sum_{i=1}^{i_{\max}} w_i \lambda_{\min i}$$

Mathematics can be constructed by using sub-differentiability

---- Bendsoe and Rodriges

$i_{\max}$ = number of crashing eigenvalues

w_i = weights , $i = 1, \ldots, i_{\max}$

Further Modification

$$\max_{\{a,b,\theta\}} \lambda_{\min} \Rightarrow \max_{\{a,b,\theta\}} \sum_{i=1}^{i_{\max}} w_i \lambda_{\min i} + \sum_{j=1}^{j_{\max}} z_j \lambda_l$$

$\lambda_{\min i}$ = minimum crashing eigenvalue

$\lambda_l = 2nd, 3rd\ldots\ldots$, higher eigenvalues

We shall maximize not only the lowest frequency
but also several eigenvalues at once

.... yields sub-optima, but make sense in engineering

Maximization of Distance

$$\max_{\{a,b,\theta\}} \sum_{j=1}^{j_{\max}} w_j \lambda_j - \sum_{i=1}^{i_{\max}} w_i \lambda_i$$

$$\sum_{j=1}^{j_{\max}} w_j \lambda_j = \text{higher eigenvalue}$$

$$\sum_{i=1}^{i_{\max}} w_i \lambda_i = \text{lower eigenvalue}$$

Inverse Frequency Problem

$$\max_{\{a,b,\theta\}} \sum_{i=1}^{i_{\max}} w_i \frac{1}{2}\left(\lambda_i - \lambda^{\text{target}}{}_i\right)^p$$

Idendify the structural configuration so that it has specified eigenvalues for the first small set of eigenvalues

Frequency Response Problem

$$\max_{\{a,b,\theta\}} \min_{\boldsymbol{v}} \frac{1}{2}\boldsymbol{v}^T \boldsymbol{K}\boldsymbol{v} - \frac{1}{2}\omega^2 \boldsymbol{v}^T \boldsymbol{M}\boldsymbol{v} - \boldsymbol{v}^T \boldsymbol{b}$$

$\boldsymbol{K}$ = stiffness matrix

$\boldsymbol{M}$ = mass matrix

ω = specified frequency

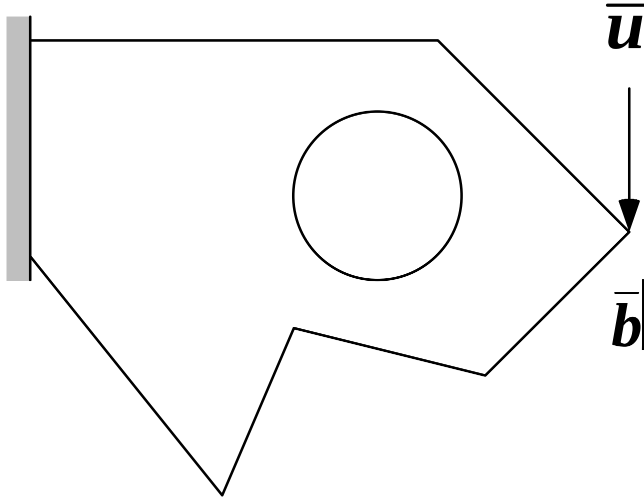
$\boldsymbol{b}$ = excited force with ω frequency

Discrete Equilibrium

$$\left(\boldsymbol{K}-\omega^2\boldsymbol{M}\right)\boldsymbol{u}=\boldsymbol{b}$$

$$\bar{\boldsymbol{b}}(t)=\exp(i\omega t)\boldsymbol{b}=\text{exciting force}$$

$$\bar{\boldsymbol{u}}(t)=\exp(i\omega t)\boldsymbol{u}=\text{dynamical response}$$



$$\bar{\boldsymbol{b}}(t)=\exp(i\omega t)\boldsymbol{b}$$

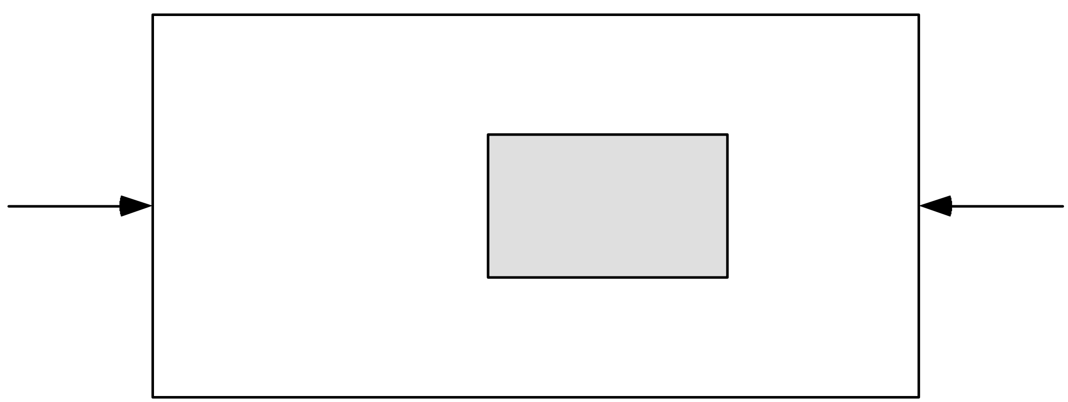
Buckling Problem

$$\max_{\{a,b,\theta\}} \lambda_{\min} \qquad \boldsymbol{K}\boldsymbol{u} = \lambda \boldsymbol{K}_G \boldsymbol{u}$$

$\boldsymbol{K}$ = stiffness matrix

$\boldsymbol{K}_G$ = geometric stiffness matrix

Rodrigues and Gedes



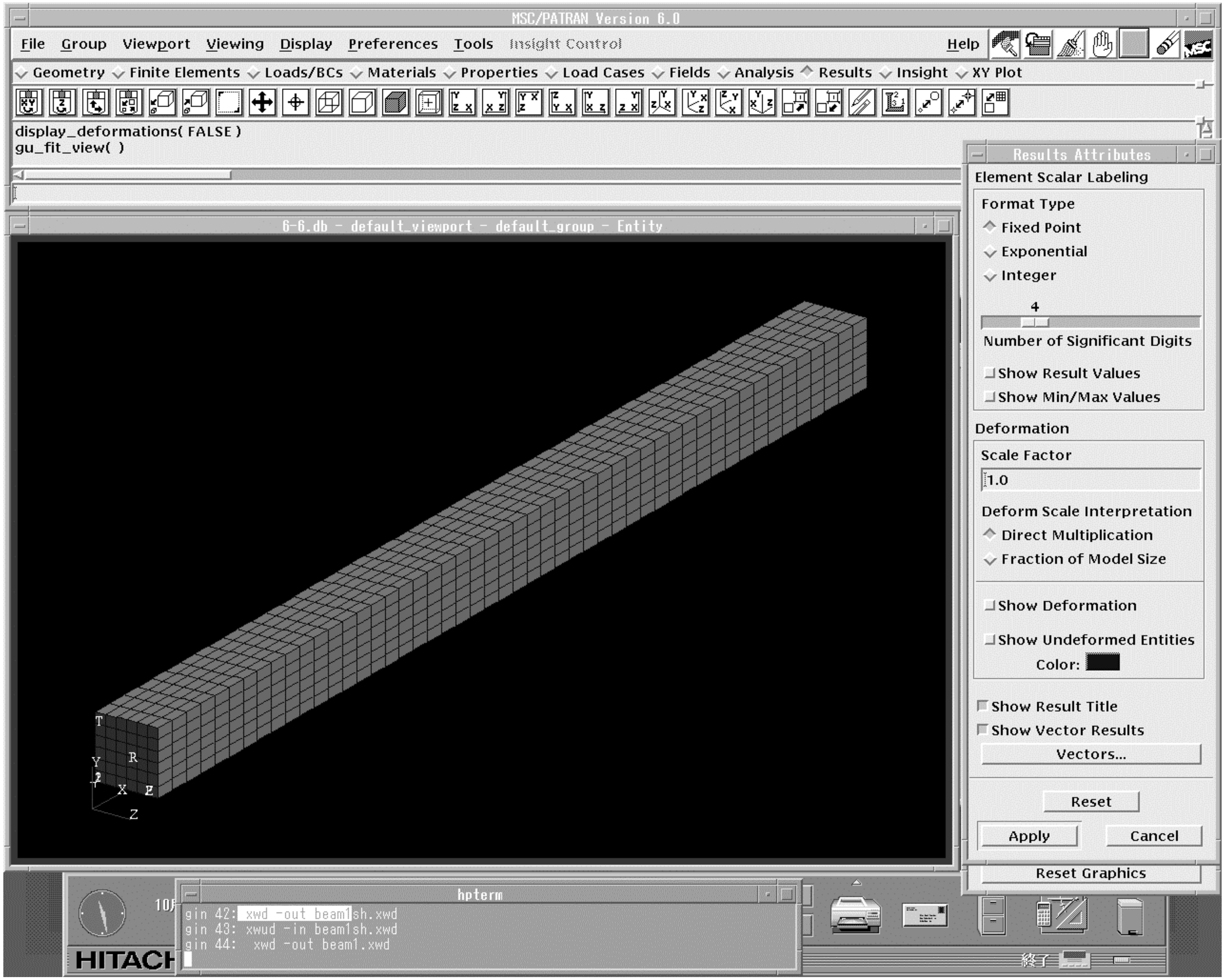
possibility of local buckling modes

Linear Combinations of the above cases

OPTISHAPE

OPTISHAPE in Practice

- OPTISHAPE is currently integrated into SDRC/I-DEAS. In future an icon of OPTISHAPE will be added into I-DEAS option menu
- OPTISHAPE design models can be developed by MSC/PATRAN and the input and output data are fully compatible with MSC/NASTRAN


MSC/PATRAN Version 6.0
File Group Viewport Viewing Display Preferences Tools Insight Control Help
Geometry Finite Elements Loads/BCs Materials Properties Load Cases Fields Analysis Results Insight XY Plot
display_deformations(FALSE)
gu_fit_view( )
6-6.db - default_viewport - default_group - Entity
Results Attributes
Element Scalar Labeling
Format Type
Fixed Point
Exponential
Integer
4
Number of Significant Digits
Show Result Values
Show Min/Max Values
Deformation
Scale Factor
1.0
Deform Scale Interpretation
Direct Multiplication
Fraction of Model Size
Show Deformation
Show Undeformed Entities
Color:
Show Result Title
Show Vector Results
Vectors...
Reset
Apply
Cancel
Reset Graphics
hpterm
gin 42: xwd -out beam1sh.xwd
gin 43: xwud -in beam1sh.xwd
gin 44: xwd -out beam1.xwd
HITACHI

OPTISHAPE to CAD/CAM

- OPTISHAPE can translate the image data of the optimum topology/configuration into a STL file (as well as SLC file in near future) so that smoothed 3D surfaces can be plotted to show the concept of the design (NC Link)
- OPTISHAPE can produce a set of wire frame models of sliced models perpendicular to a specified direction (CAD Link)

OPTISHAPE

More in Practice

Current Development Research

OPTISHAPE

is

constantly enhanced

Flexible Body Design

compliant mechanism design

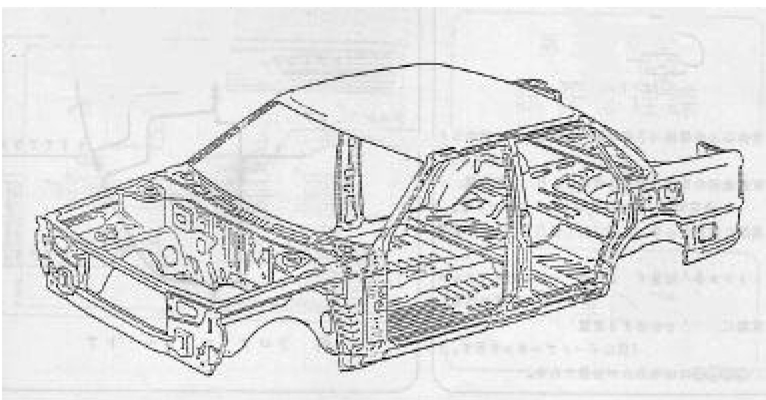
by

Mary Frecker & Shinji Nishiwaki

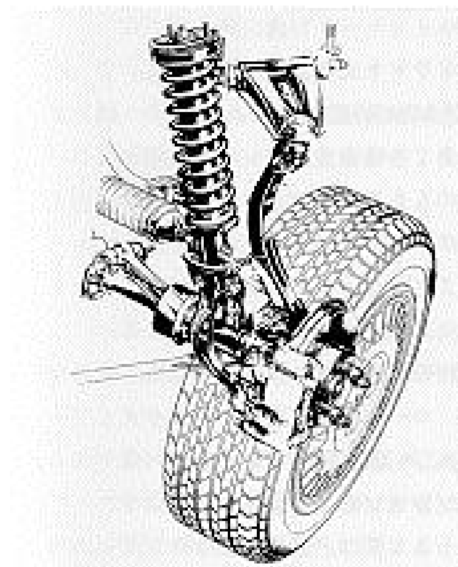
Shinji Ejima

Mechanical Design

Structure Design

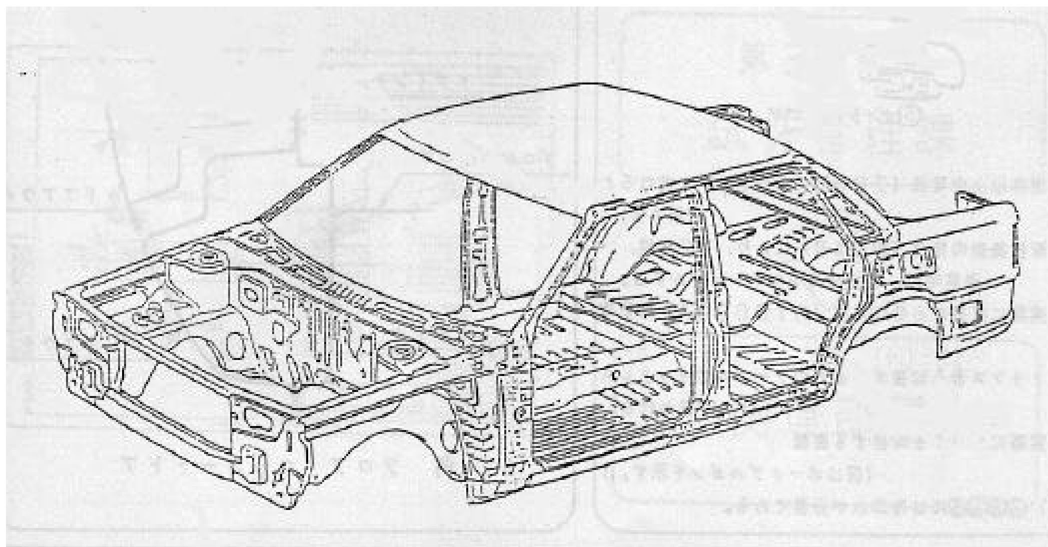


Mechanism Design



New Design Based on Flexibility

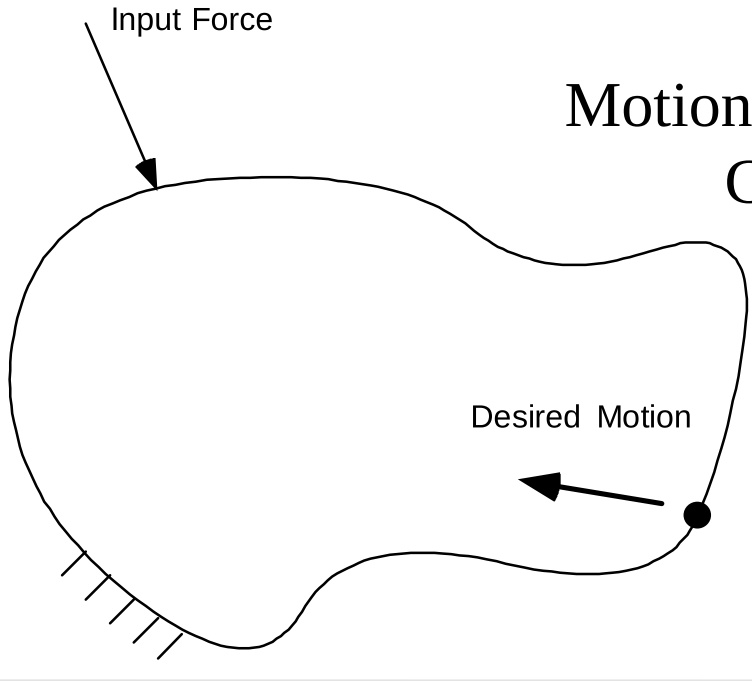
Structure Design



Flexible Structure ⟺ Stiff Structure

Flexible Body Design

- Design a structure that moves to the specified direction as much as possible when input forces are given



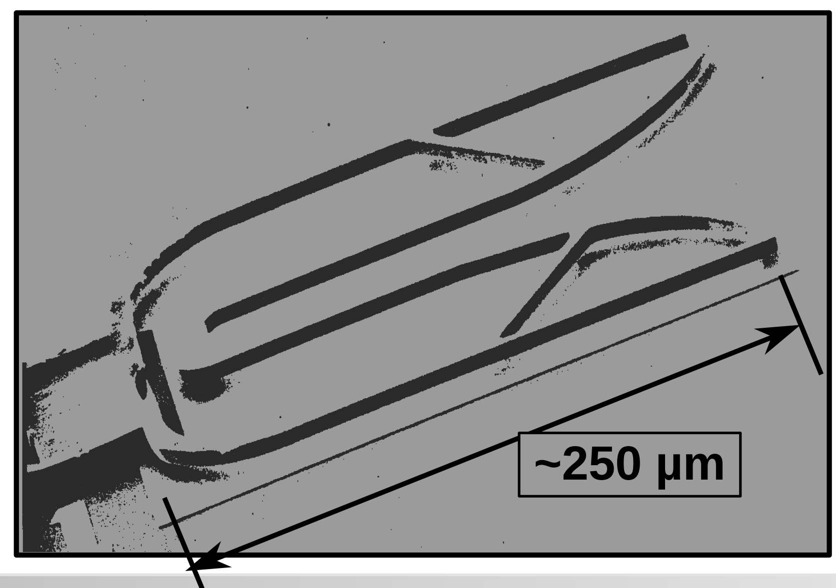
Motion/Kinematics Control

Compliant Mechanism

- MEMS(Micro Electro Mechanical System)
- Basic Ideas & Clues for

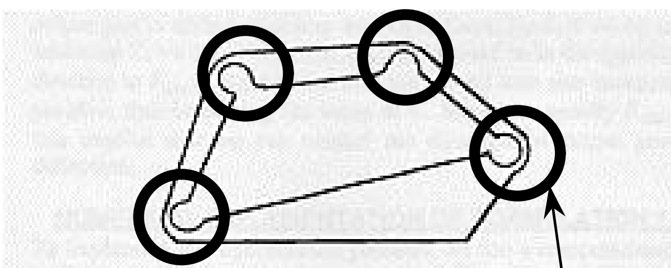
  Rigid Link Mechanism Design

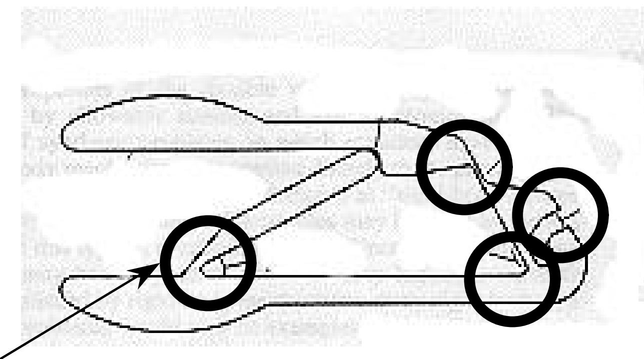
Microcompliant crimping mechanism



Kinematic Synthesis

Based on traditional rigid body kinematics

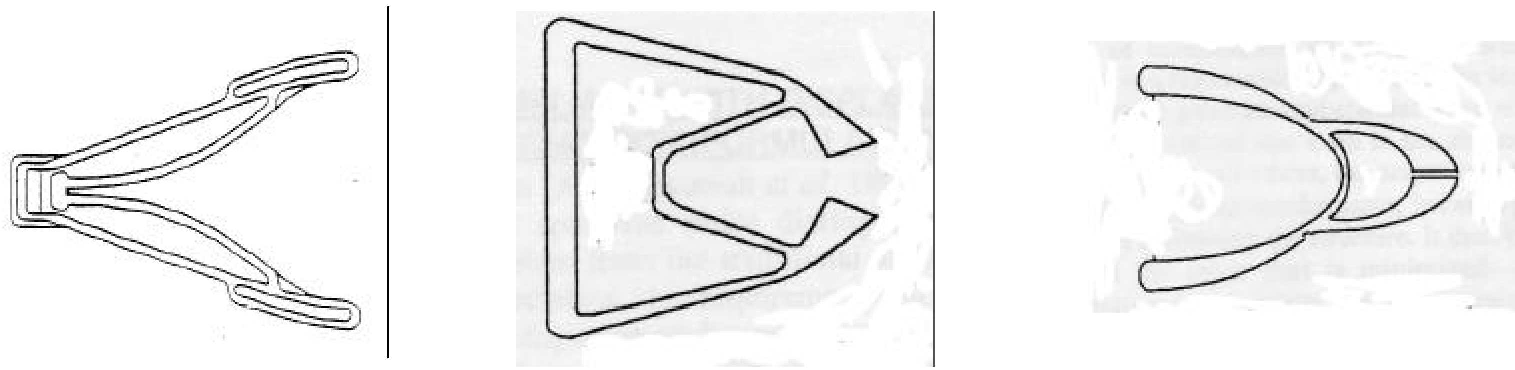




Lumped compliance (Pivot)

I. Her and A. Midha (1986),
L. L.Howell and A. Midha (1994), (1996)

Continuum Synthesis



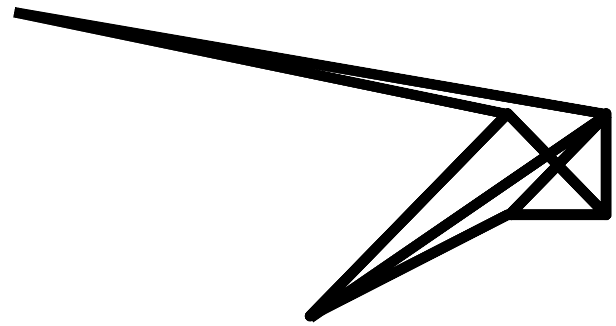
Distributed compliance

O. Sigmund (1995), (1996)
U. D. Larsen, O. Sigmund and S. Bouswstra (1996)

Design of Flexible Structures

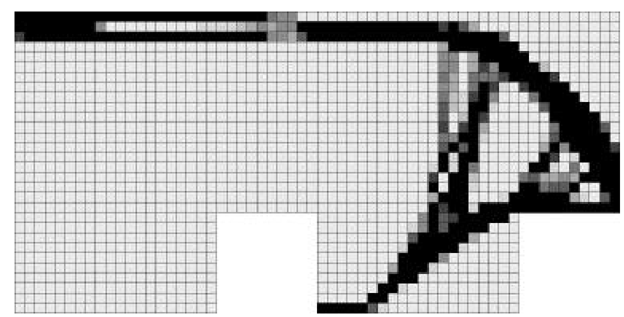
- Truss Approach

  Configuration by Trusses



- Continuous Approach

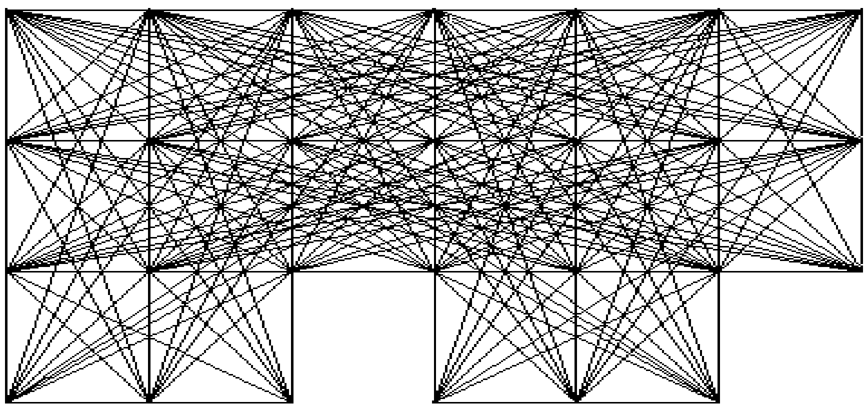
  Distribution of Materials



Truss Approach

Based on Ground Structure Design

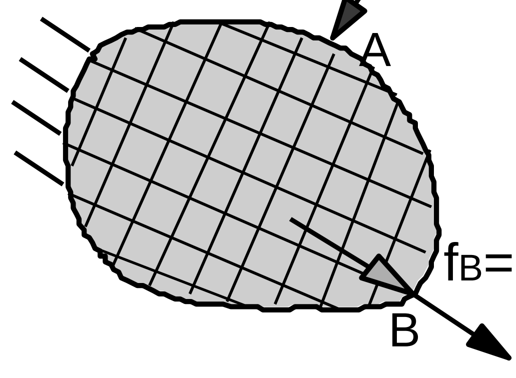
Compliant Mechanism Design



By Mary I. Frecker

Problem Statement 1

Kinematic Function



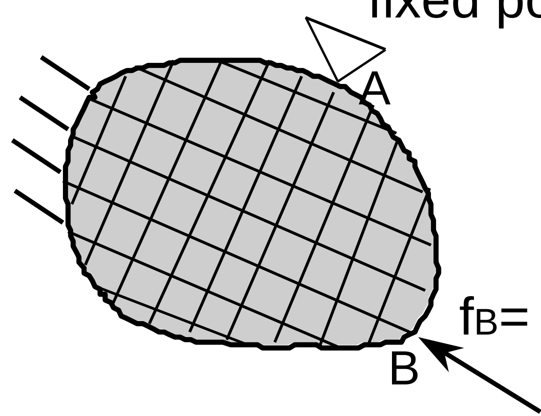
$$\max\,(\textit{mutual energy}) \Leftrightarrow \max\,(\mathbf{v}_B^{T}\,\mathbf{K}\,\mathbf{u}_A)$$

$$\text{subject to: } \mathbf{K}\,\mathbf{u}_A = \mathbf{f}_A$$

$$\mathbf{K}\,\mathbf{v}_B = \mathbf{f}_B$$

Problem Statement 2

Structural Function



$$\min(\textit{strain energy}) \Leftrightarrow \min(\mathbf{u}_B^T \mathbf{K}\mathbf{u}_B)$$

$$\text{subject to}: \mathbf{K}\mathbf{u}_B = -\mathbf{f}_B$$

Multicriteria Optimization

$$\max \left[\frac{\textit{mutual energy}}{\textit{strain energy}} \right] \Leftrightarrow \max \left[\frac{v_B^T K u_A}{u_B^T K u_B} \right]$$

subject to : $K u_A = f_A$

$K v_B = f_B$

$K u_B = -f_B$

total resource constraint

lower and upper bounds

A Simple Design Problem

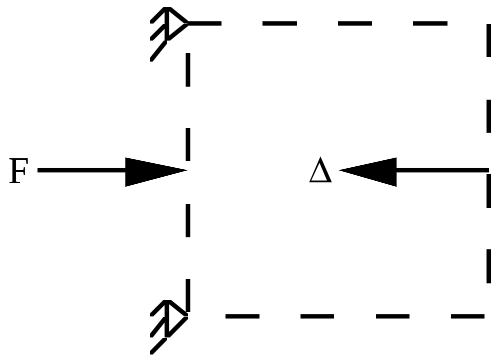


Figure 5a. Design Problem.

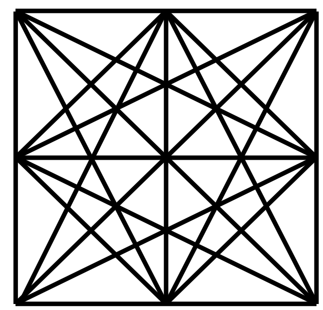


Figure 5b. Initial Guess.

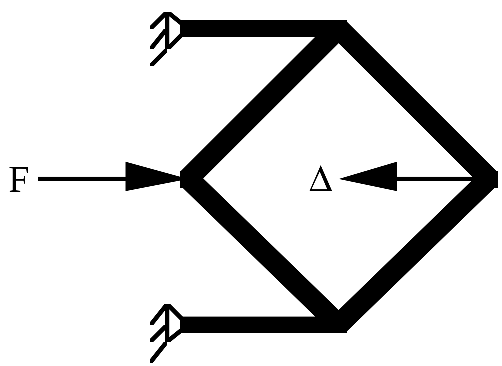


Figure 5c. Compliant Mechanism Solution.

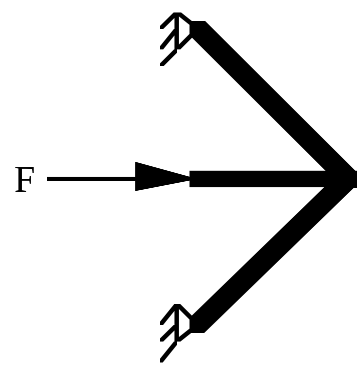
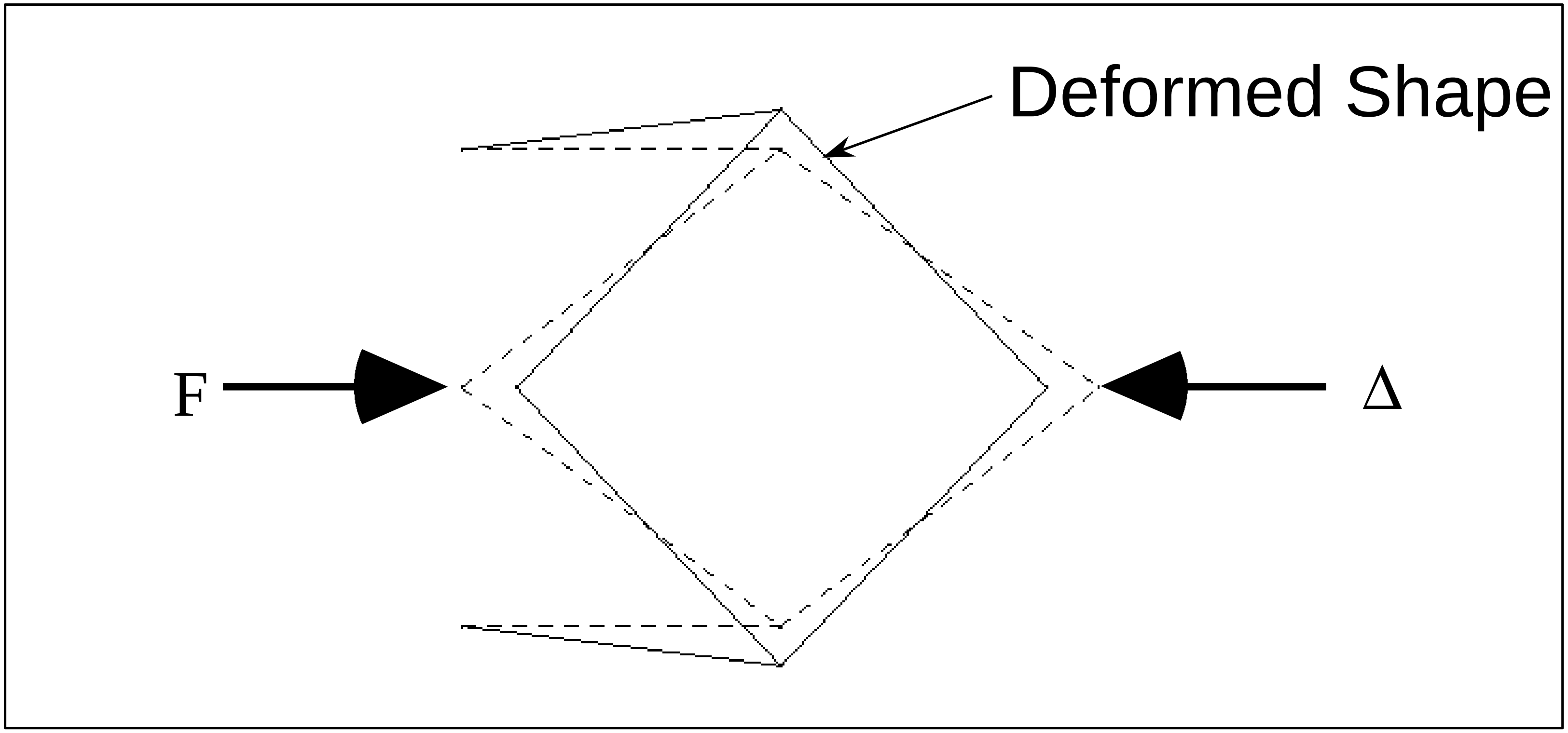


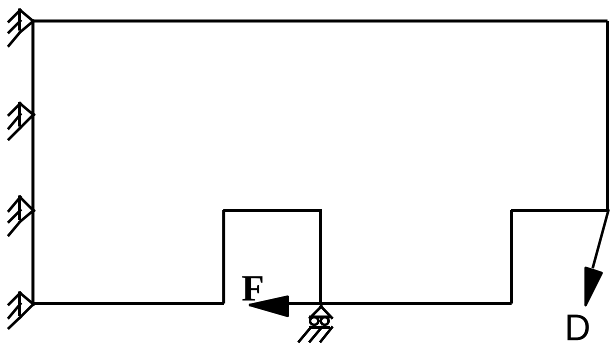
Figure 5d. Stiffest Structure Solution.

Verification of Function

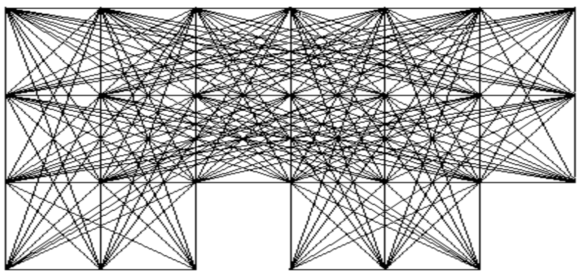


This can satisfy the original objective

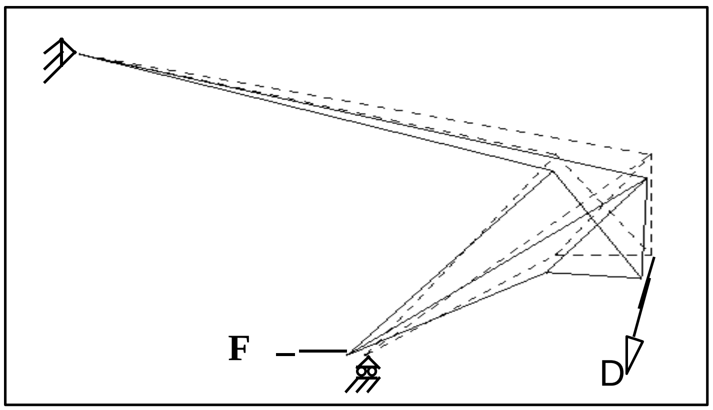
Compliant Gripper



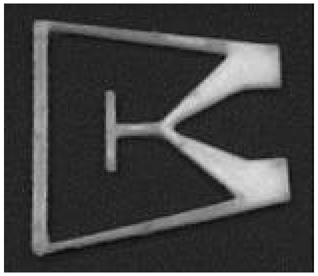
Design Problem

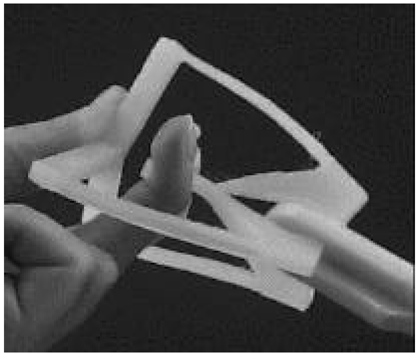


Initial Guess



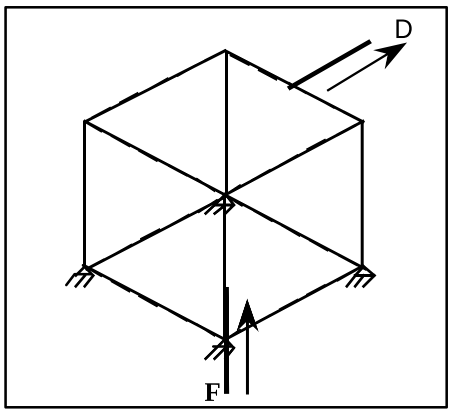
Solution and Finite Element Model



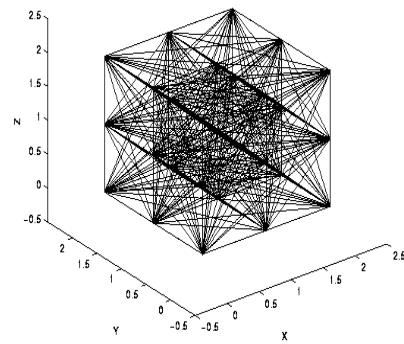


Compliant Grippers

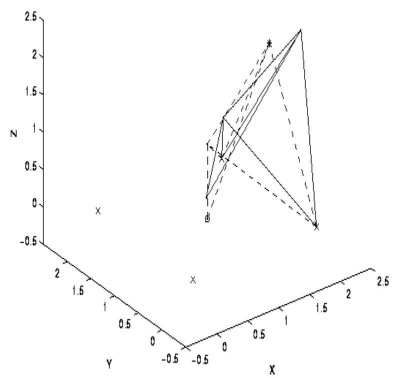
3D Compliant Gripper



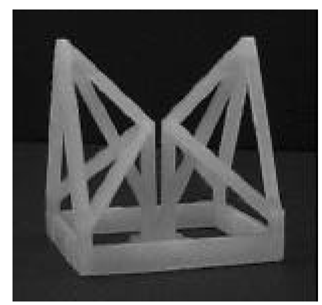
Design Problem



Initial Guess



Solution and Finite Element Model

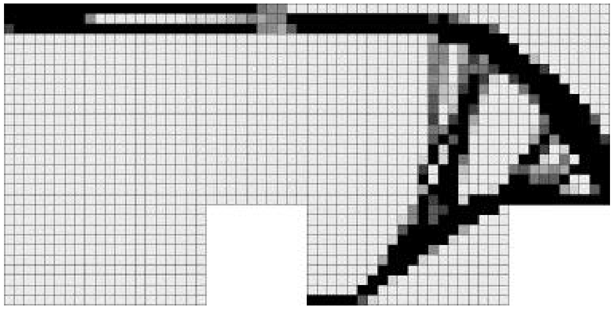


Three Dimensional
Compliant Mechanism

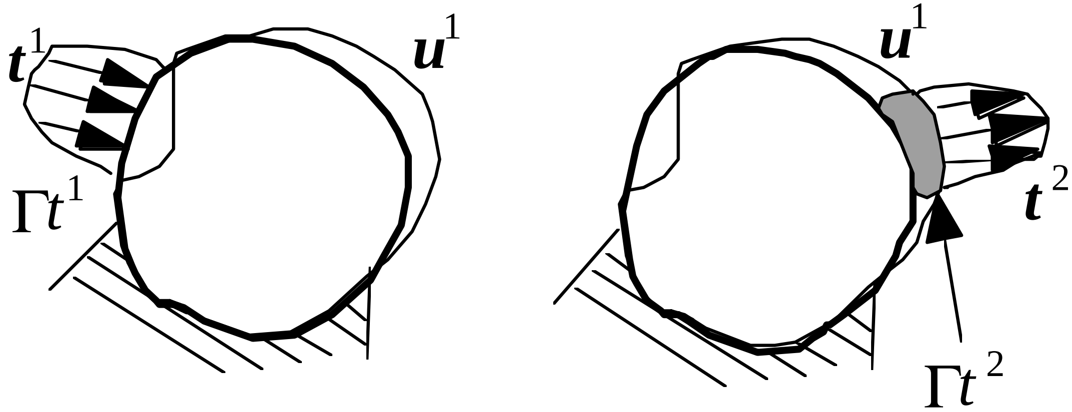
Continuous Approach

Based on
Homogenized Design Method

- Fixed Grid / Voxel Mesh Method
- Homogenization Method



Formulation of Flexibility



Maximize $L^2(\boldsymbol{u}^1) = \int_{\Gamma t^2} \boldsymbol{t}^2 \bullet \boldsymbol{u}^1 d\Gamma$ (Mutual Mean Compliance)

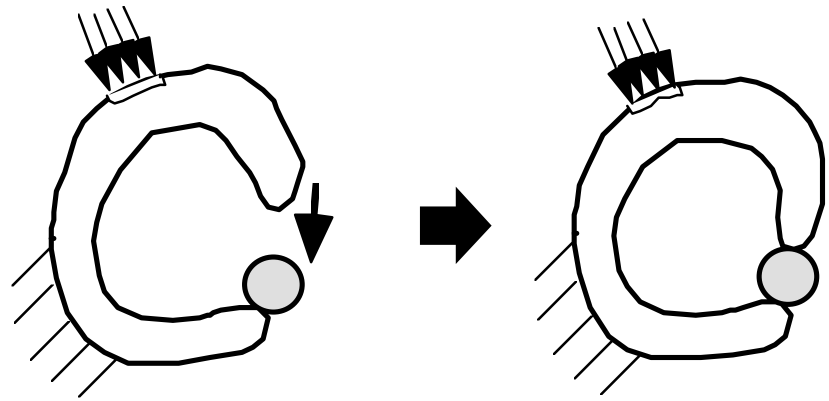
⇨ Flexibility at Γt^2

Minimize $L^1(\boldsymbol{u}^1) = \int_{\Gamma t^1} \boldsymbol{t}^1 \bullet \boldsymbol{u}^1 d\Gamma$ (Mean Compliance)

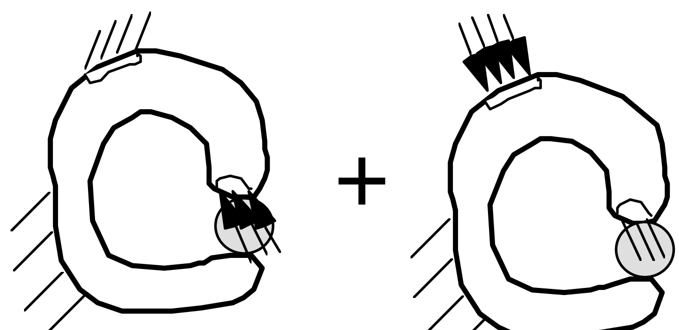
⇨ Stiffness at Γt^1

Design of Flexible Structures

- Kinematic function
  - ➡ Flexibility



- Structural function
  - ➡ Stiffness



Reaction force + Applied force + Defamation Constraint ⇩ Constraine
d
Motion

Multicriteria Optimization

Flexibility ⇨ Max *Mutual Mean Compliance*

⇕ Trade Off

Stiffness ⇨ Min $\sum$ *Mean Compliance*

➡ Compromise Solutions

Multi-Objective Functions

(1)

$$\text{Max } \frac{\textit{Mutual Mean Compliance}}{\sum \textit{Mean Compliance}}$$

(2)

$$\text{Max } w \,\text{Log}(\textit{Mutual Mean Compliance}) - (1-w)\text{Log}(\sum \textit{Mean Compliance})$$

where *w* is a weighting Coefficient

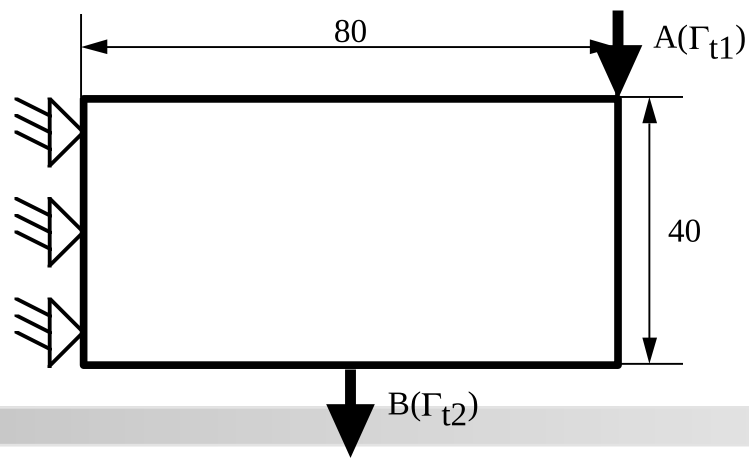
Weighting Method vs. Ratio Formulation

- Max w_1 *Mutual Mean Compliance*
  $-(1-w_1)\sum$ *Mean Compliance*

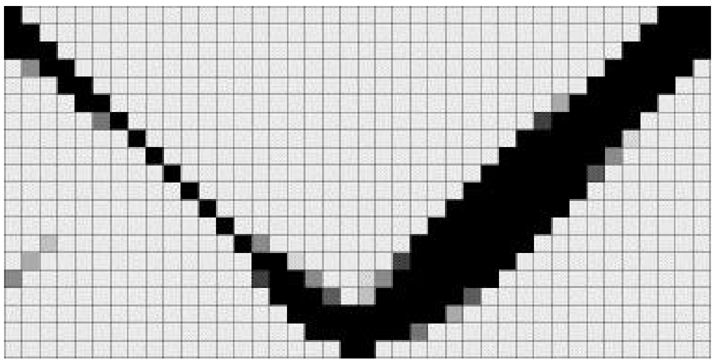
  where w_1 is a weighting Coefficient

- Max $$\frac{\textit{Mutual Mean Compliance}}{\sum \textit{Mean Compliance}}$$

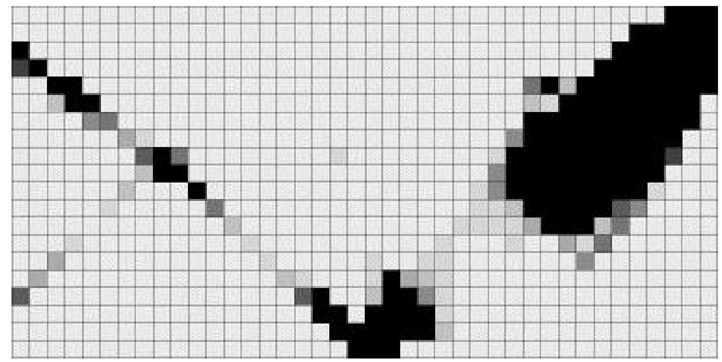
Example



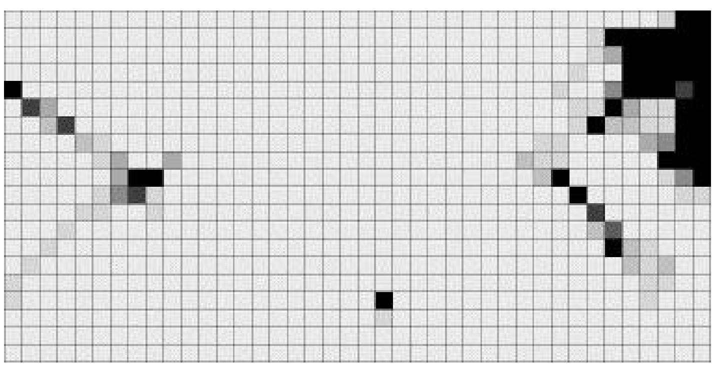
Weighting Method vs. Ratio Formulation



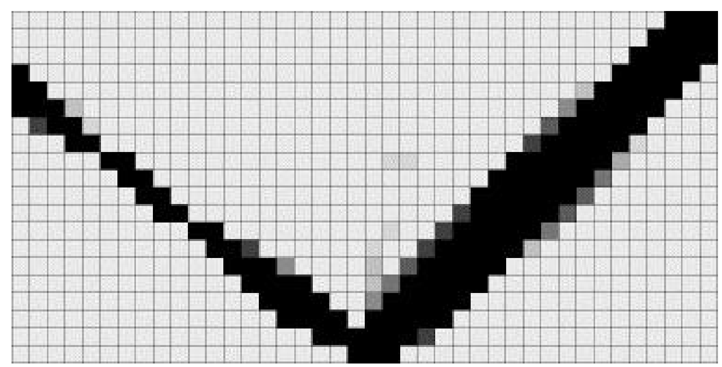
w_1 =0.001



w_1 =0.01

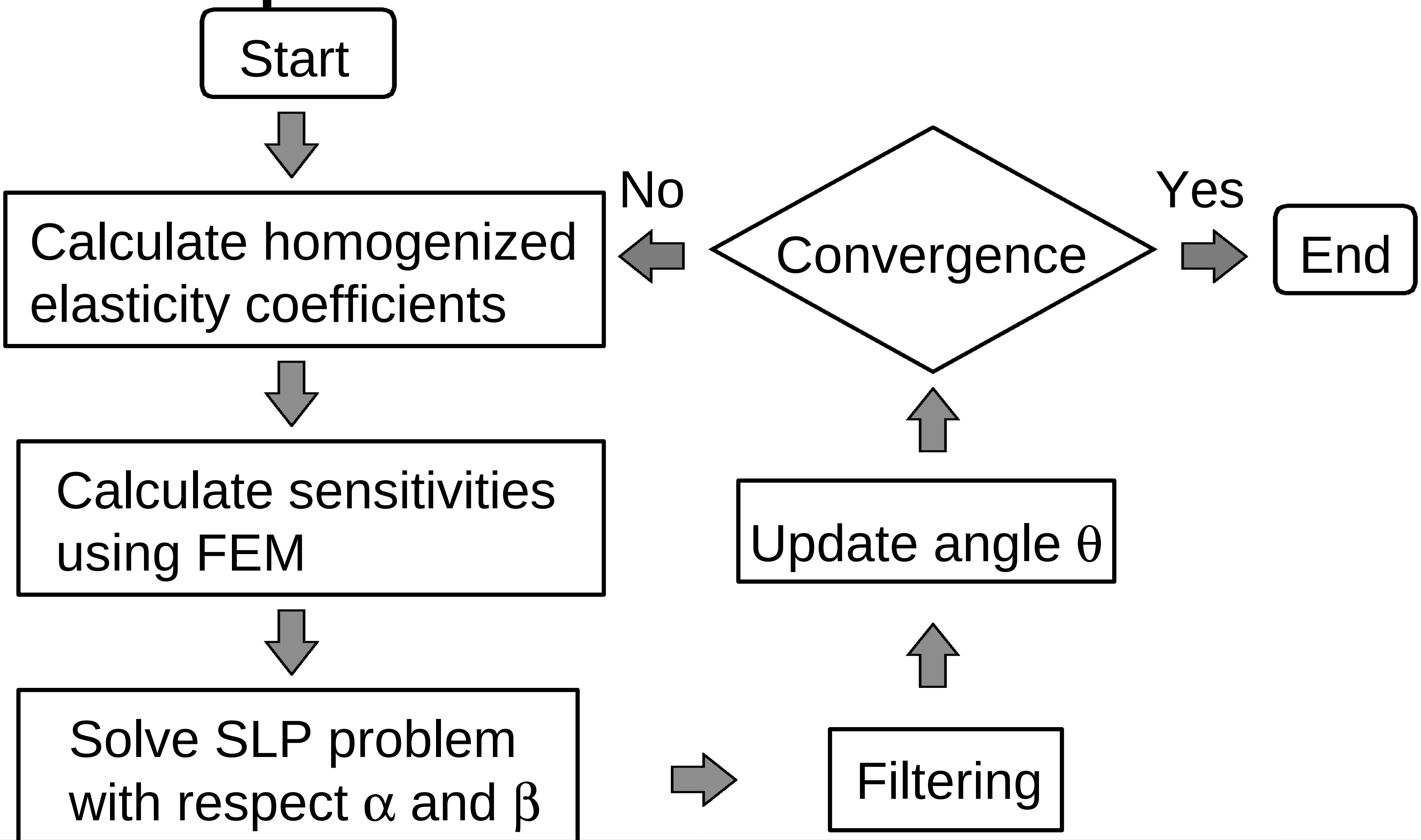


w_1 =0.1



Ratio Formulation

Optimization Procedure



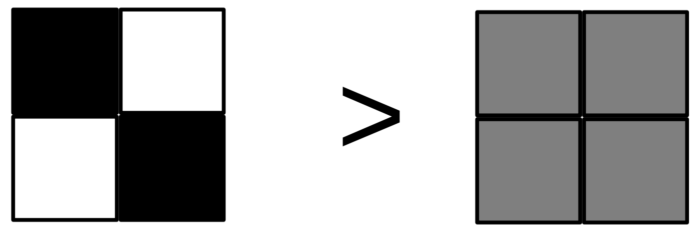
SLP vs. OC

- Sequential Linear Programming (SLP)
  - Linear Approximation + Simplex Method
  - Slow Convergence
  - Easy Implementation for Any Objective Functions
- Optimality Criteria Method (OC)
  - KKT-Conditions + Heuristics
  - Quick Convergence
  - Difficult to Construct Heuristics for General Objective Functions

Checkerboard Pattern

$\rho_1 = \rho_4 = 1$

$\rho_2 = \rho_3 = 0$



$\rho_1 = \rho_2 = 1/2$

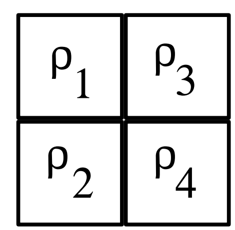
$\rho_3 = \rho_4 = 1/2$

Artificially stiffer

Filtering scheme

$$\bar{\rho}_1 = \frac{1}{4}\left(3\rho_1 + \rho_2 + \rho_3 - \rho_4\right) \qquad \bar{\rho}_2 = \frac{1}{4}\left(\rho_1 + 3\rho_2 - \rho_3 + \rho_4\right)$$

$$\bar{\rho}_3 = \frac{1}{4}\left(\rho_1 - \rho_2 + 3\rho_3 + \rho_4\right) \qquad \bar{\rho}_4 = \frac{1}{4}\left(-\rho_1 + \rho_2 + \rho_3 + 3\rho_4\right)$$

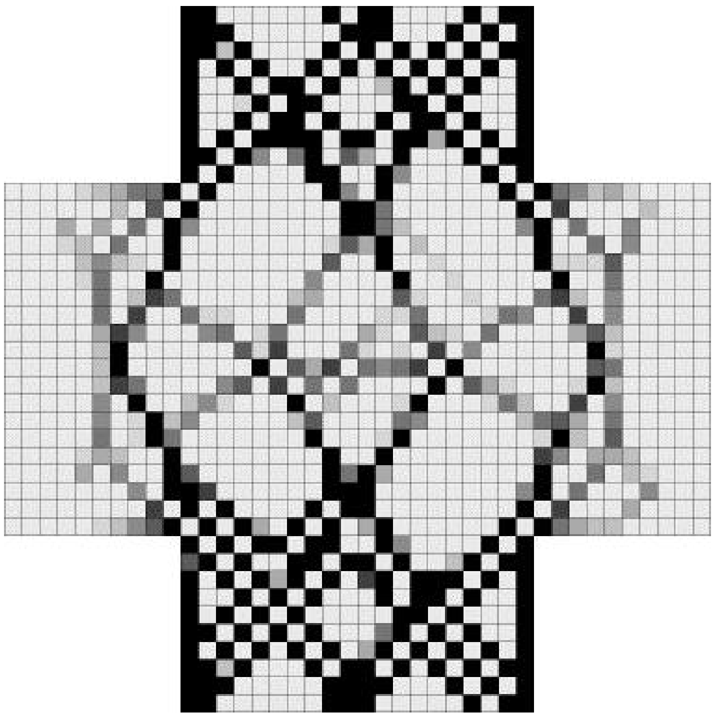


One group

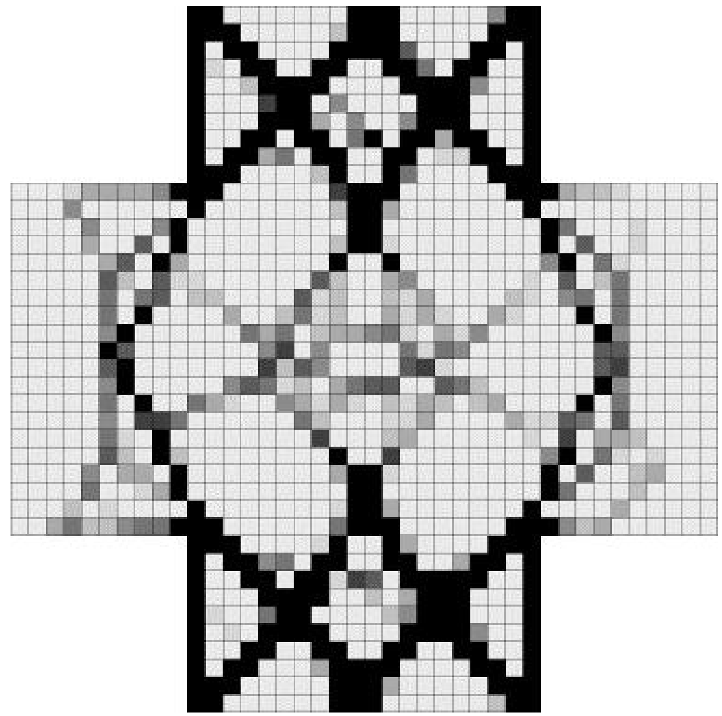
where $\rho_i = 1 - \alpha_i \beta_i \quad (i = 1, \ldots, 4)$

Elimination of Checkerboard

Eigen-frequency problem

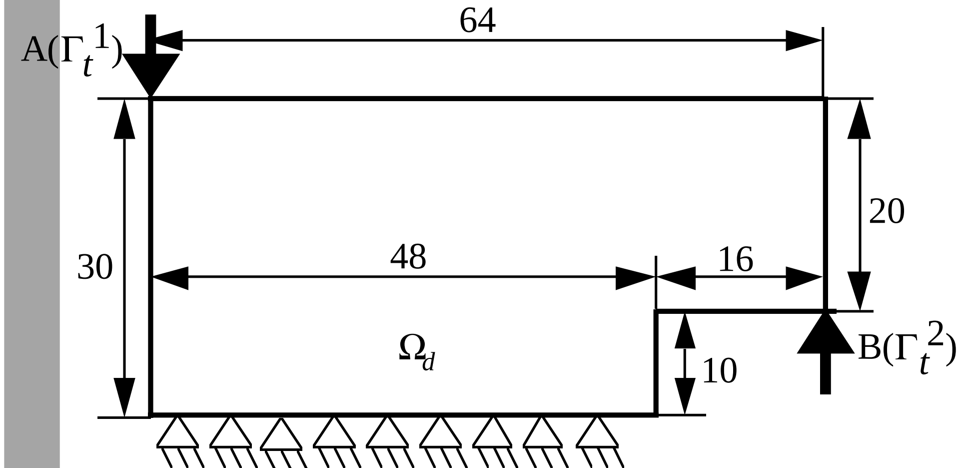


No Filtering

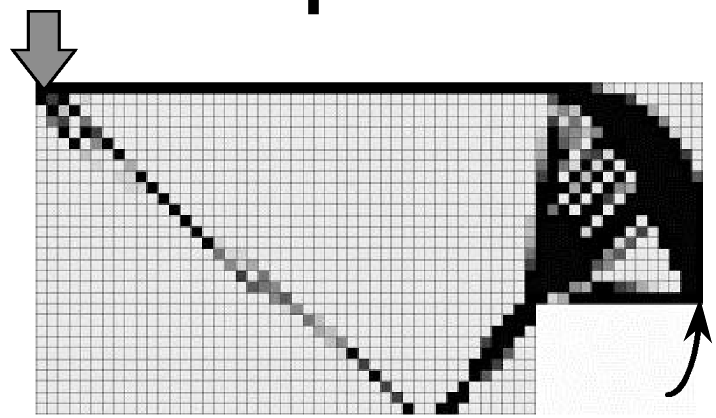


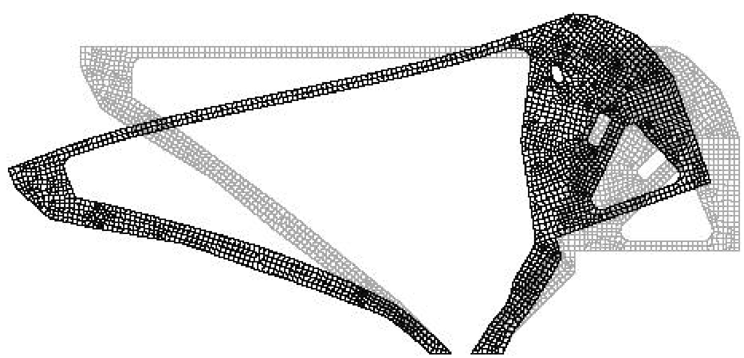
Filtering

Compliant Clamp



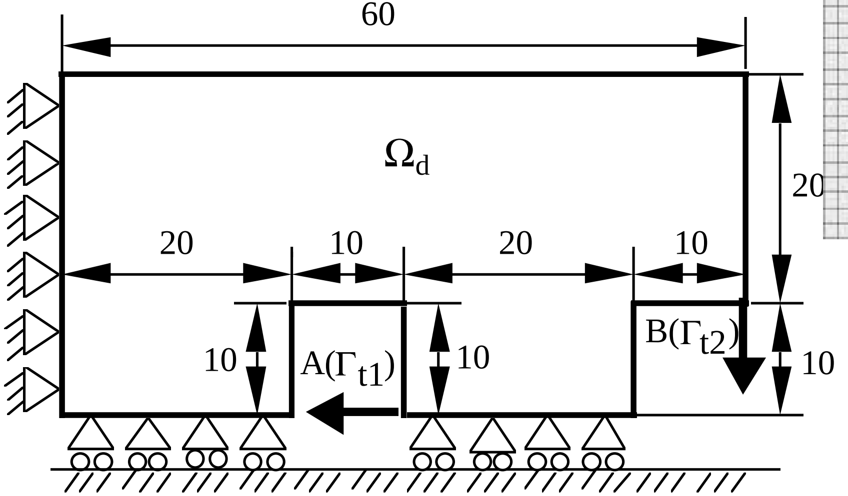
Design domain



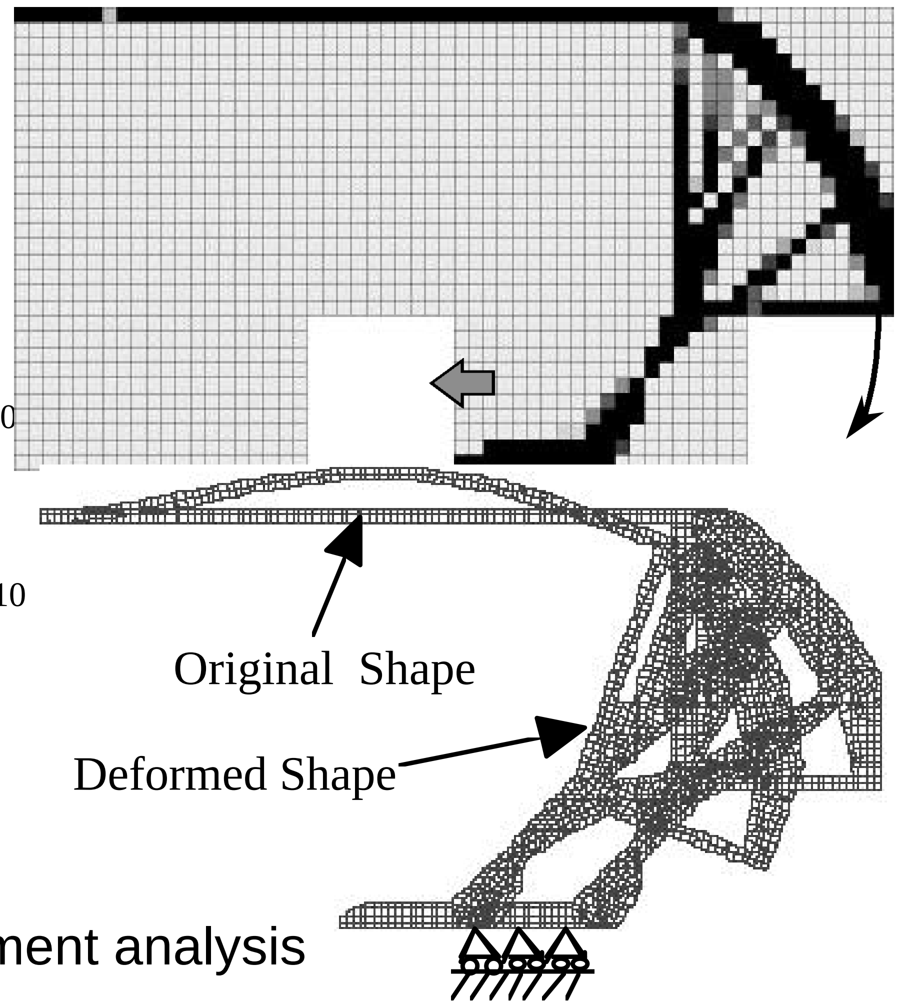


Large displacement analysis

Compliant Gripper

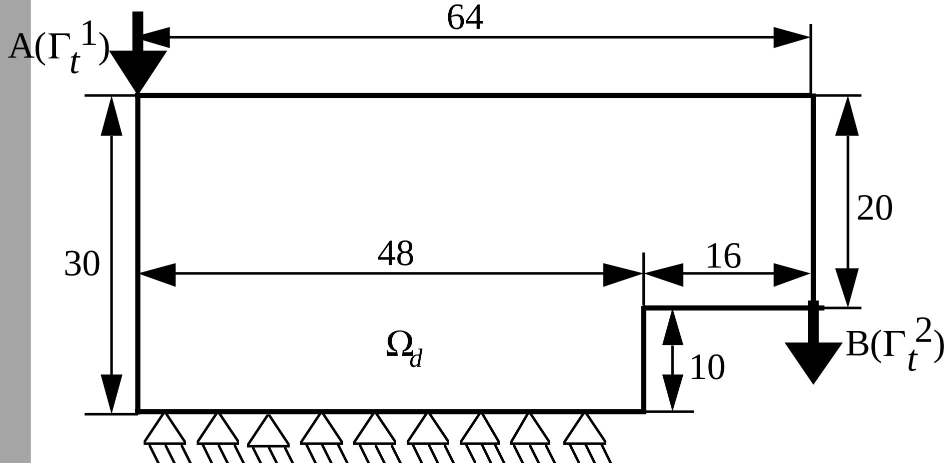


Design domain

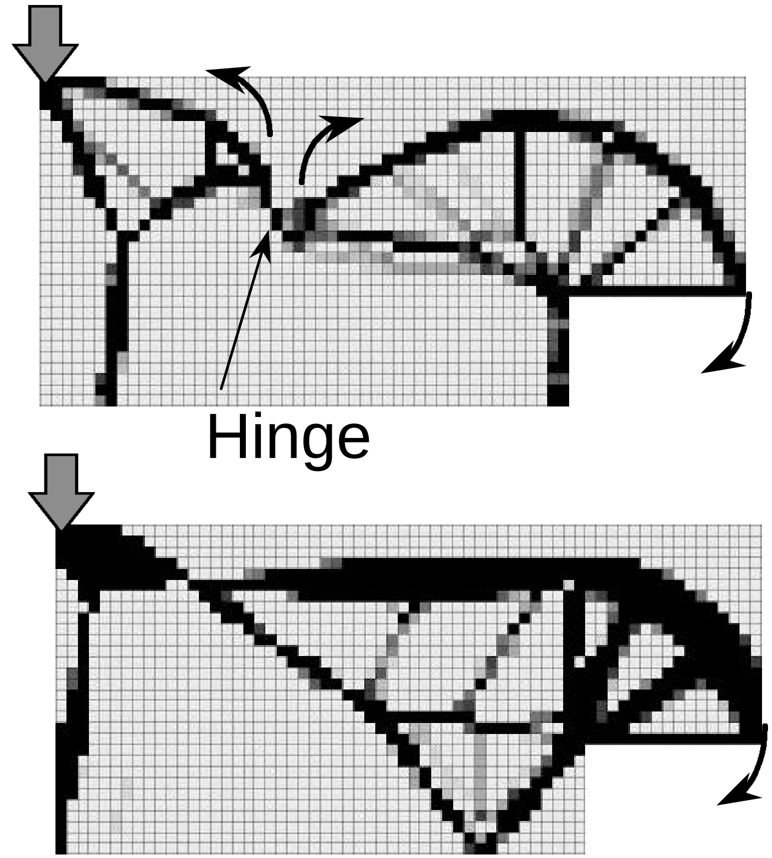


Large displacement analysis

Compliant Pliers

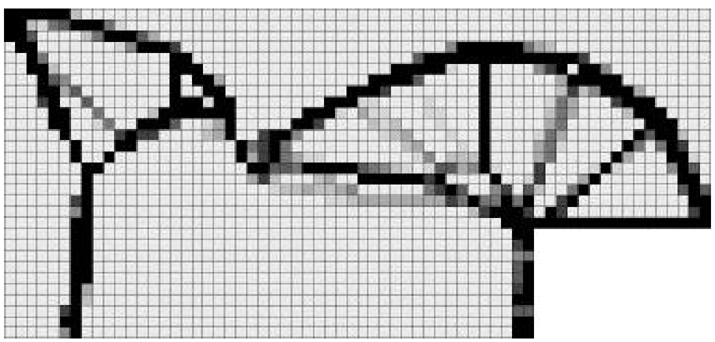


Design domain

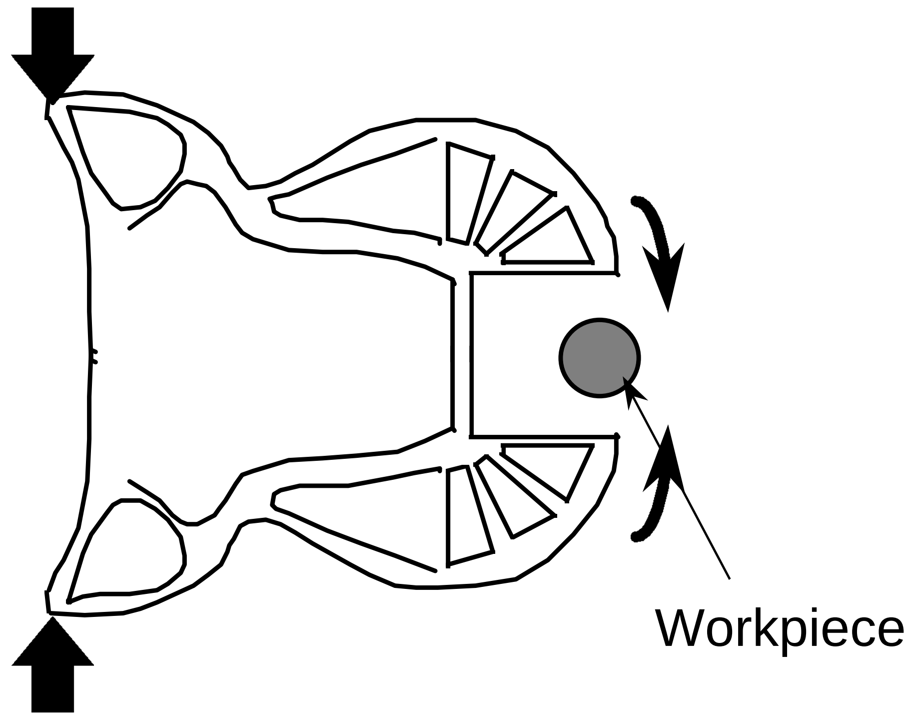


Optimal configurations

Final Configuration

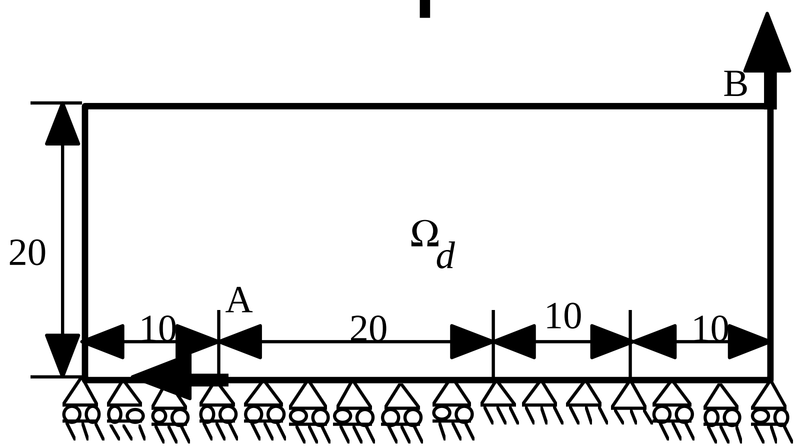


Optimal configuration

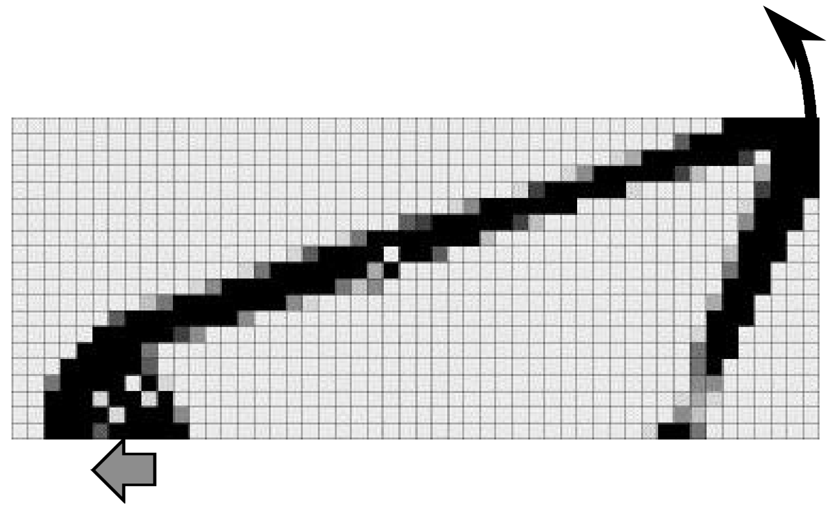


Final configuration

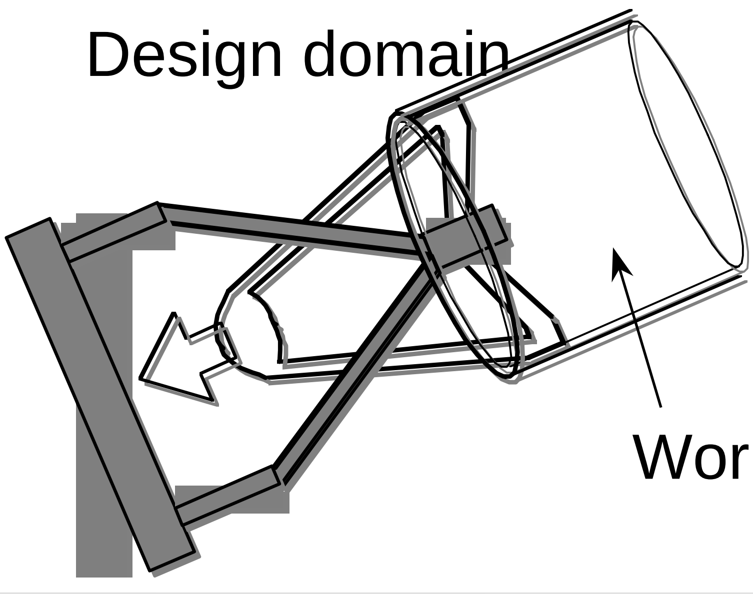
Compliant Internal Gripper



Design domain



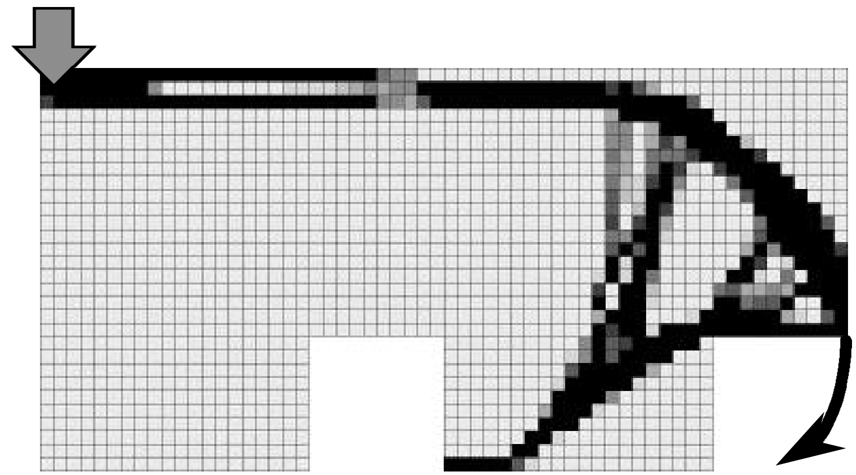
Optimal configuration

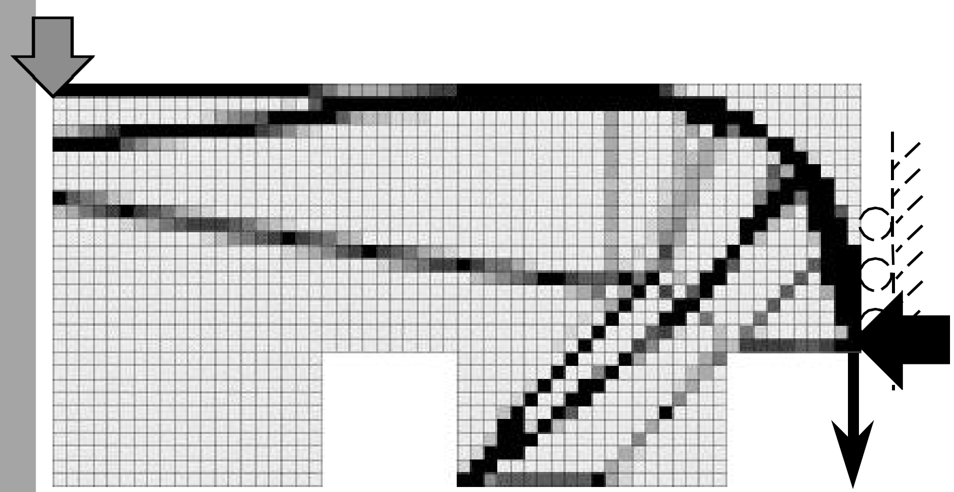


Workpiece

Constrained Motion

Compliant Gripper

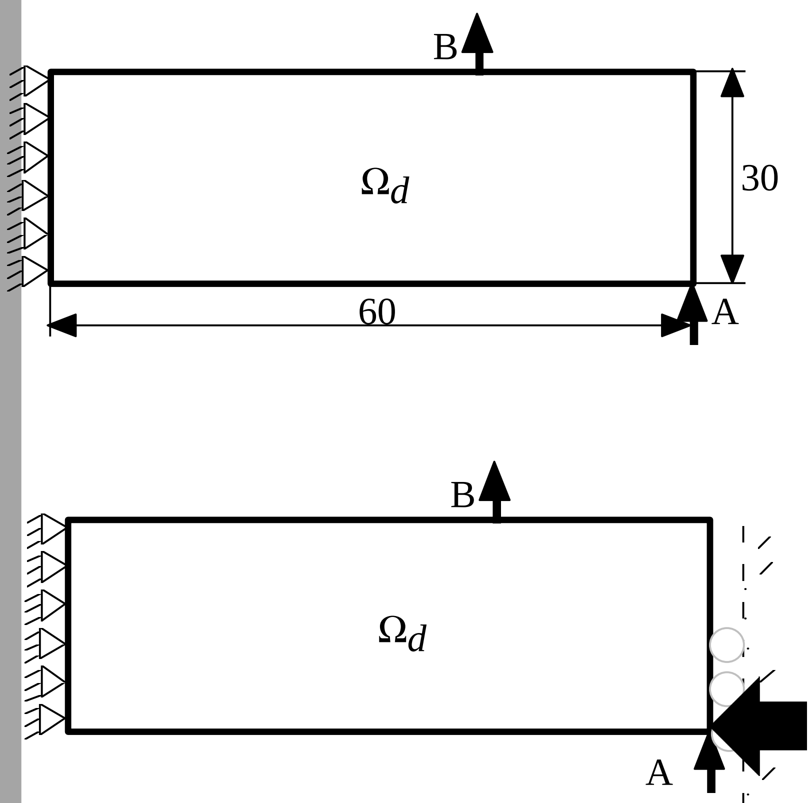




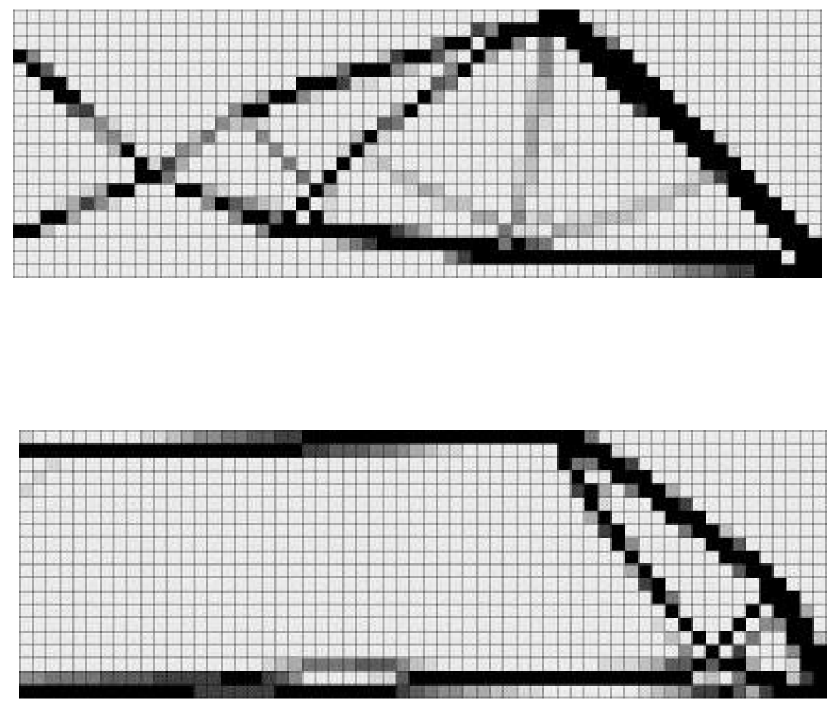
Adding Mutual Stiffness

Transitional Motion

Suspension Design



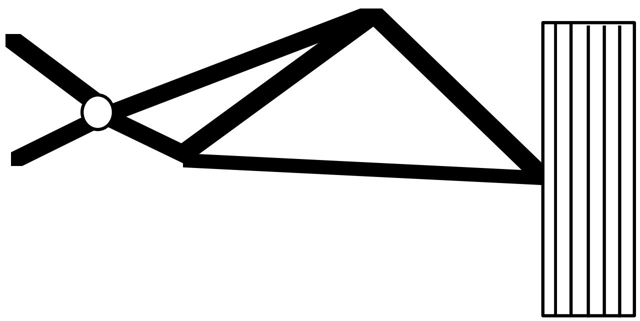
Design domains



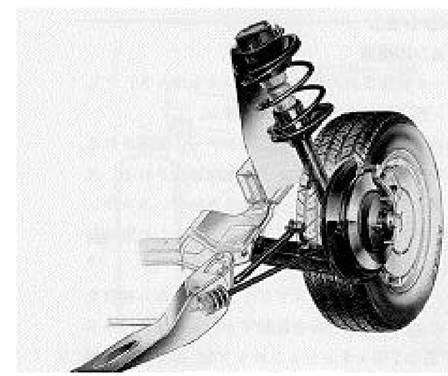
Optimal configurations

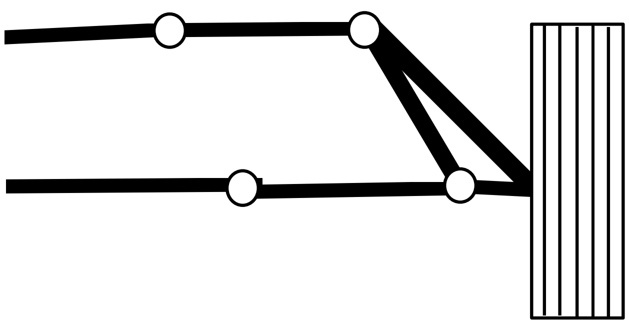
Interpretation

Optimal Configuration + Deformed Shape

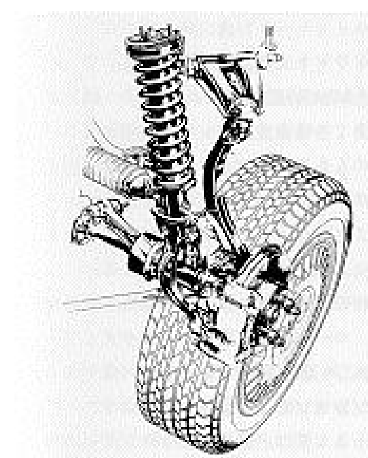


Strut Type



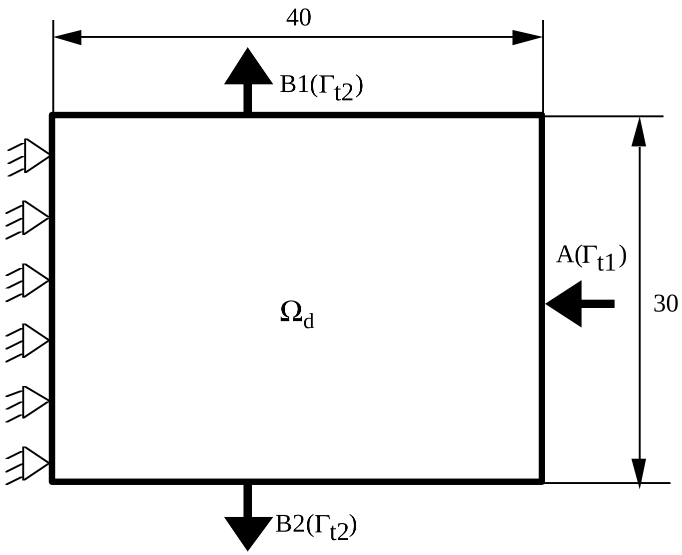


Double wish-bone Type

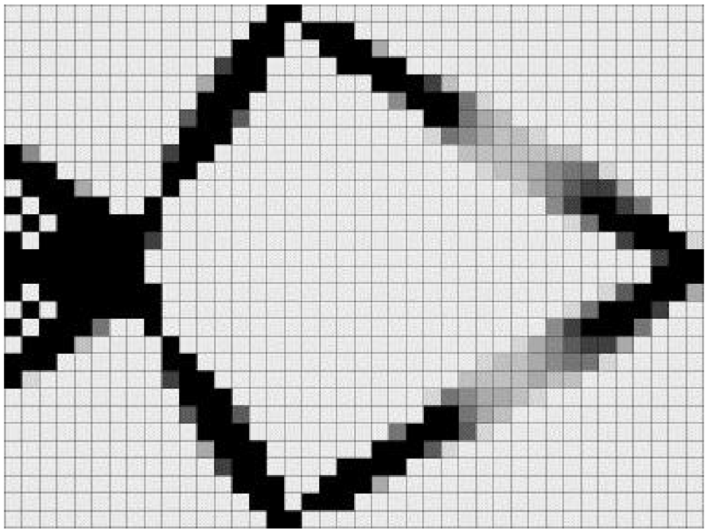


Rigid Link Mechanisms

Two Displacement Outputs



Design domain



Optimal configuration

Extension to Multi-Flexibility

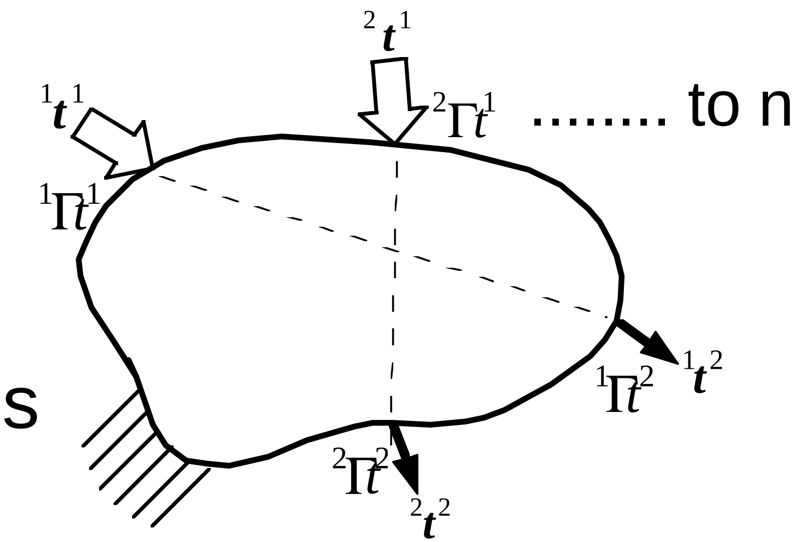
Necessary for Many Performance Criteria

⇨ Automotive Body Design

n Flexibilities Required

⇨

n Mutual Mean Compliances Should be **Positive**



Multi-criteria Optimization

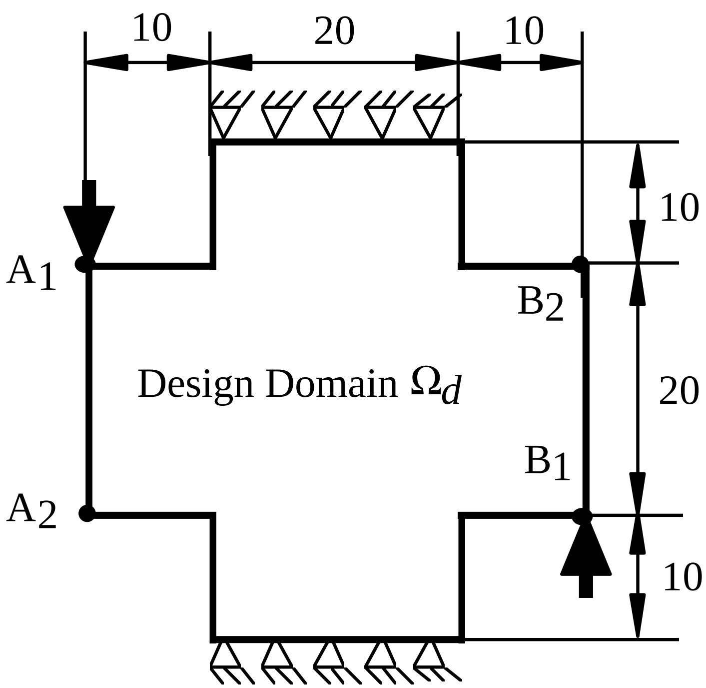
Max i-th Mutual Mean Compliance ($_i MMC$)
(for i=1,...,n)

Min i-th $\sum$ Mean Compliance ($_i MC$)
(for i=1,...,n)

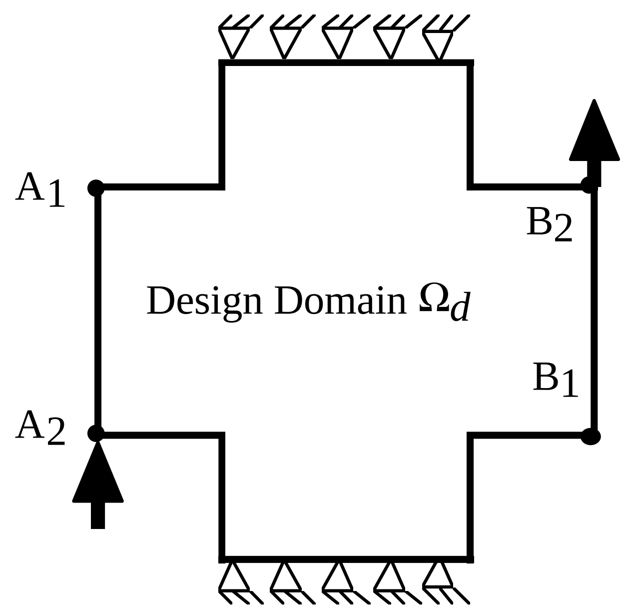
- Multi-Objective Function

$$\text{Max} \quad \frac{-1/C_f \, \text{Log}\left(\sum \text{Exp}(-C_f \, {}_i MMC)\right)}{1/C_s \, \text{Log}\left(\sum \text{Exp}(C_s \, {}^i MC)\right)}$$

Multi-flexiblity Design

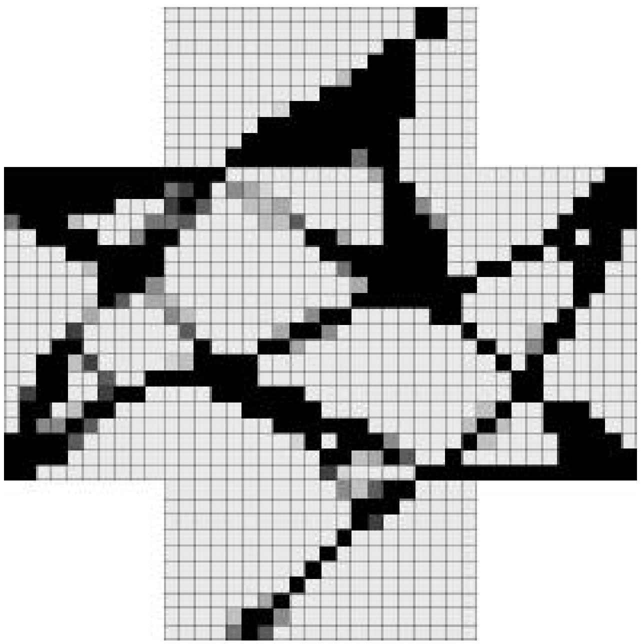


First flexible mode

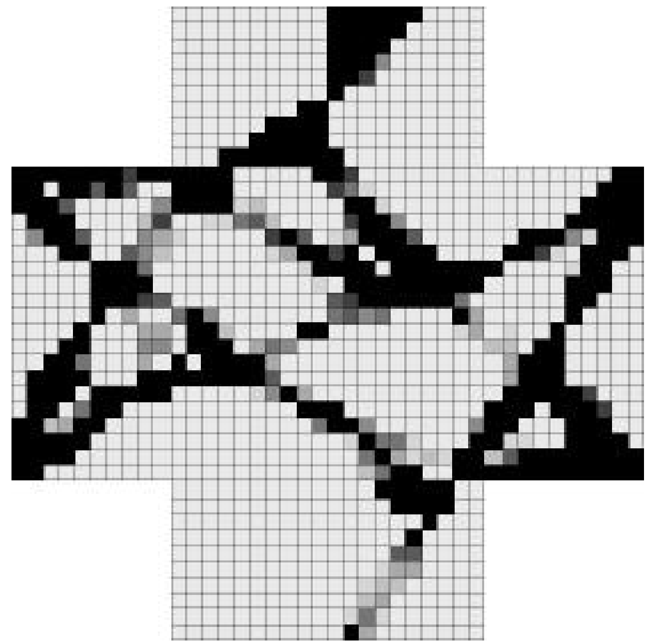


Second flexible mode

Optimal Configurations

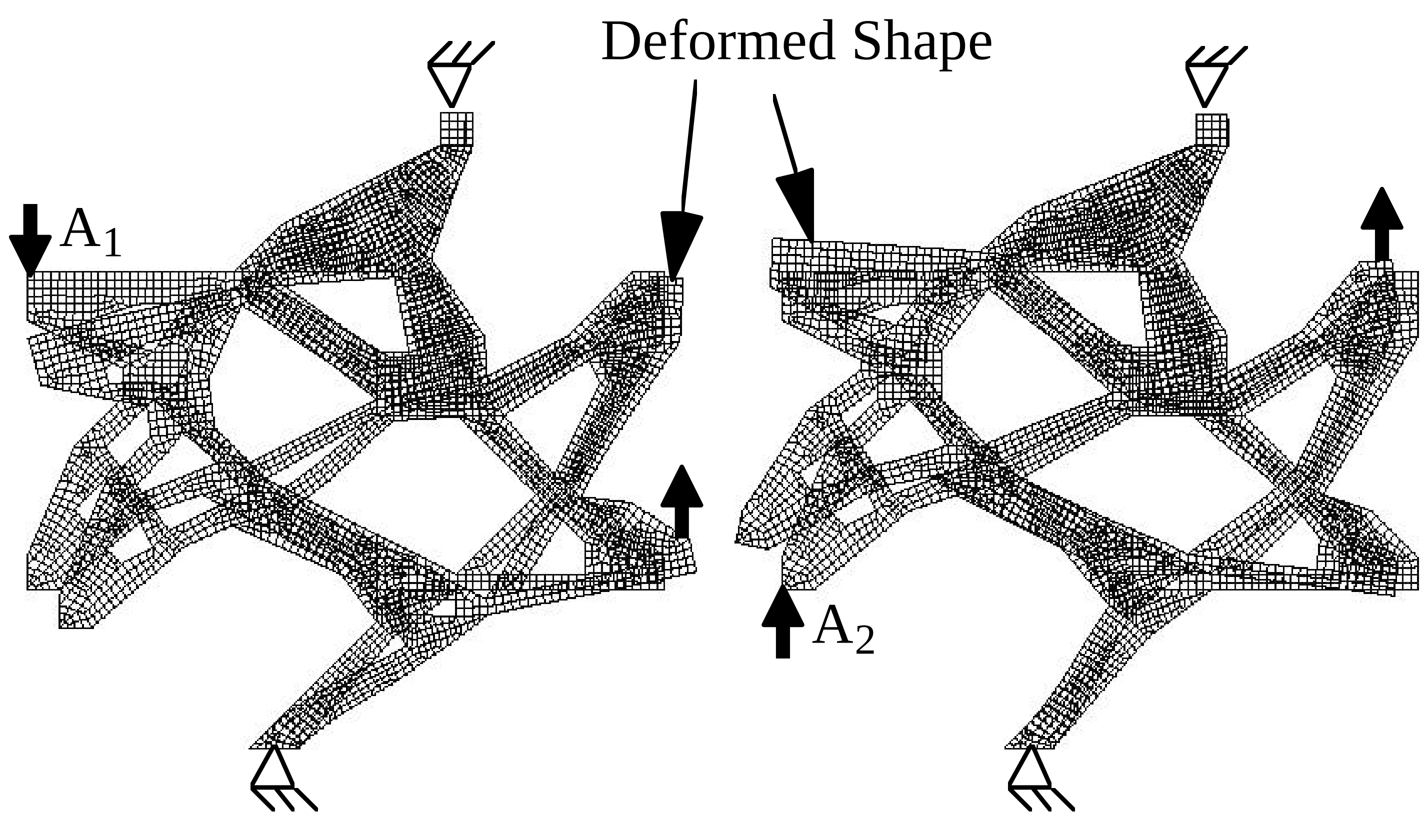


$C_f = C_s = 3$



$C_f = C_s = 10$

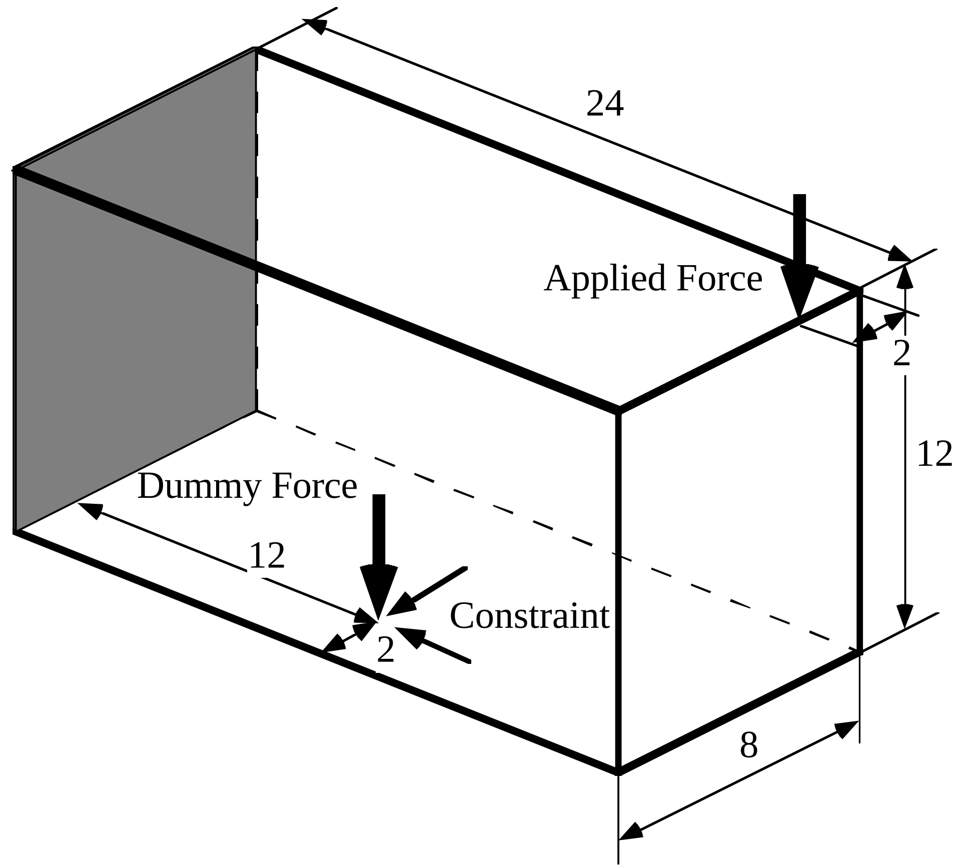
Verification of Performance



Extension

- 3D Compliant Mechanism Design
  - ➡ New 3D element is used
- Complaint Mechanism Design with a Displacement Constraint
  - ➡ New formulation is introduced
    - \+ Image based design

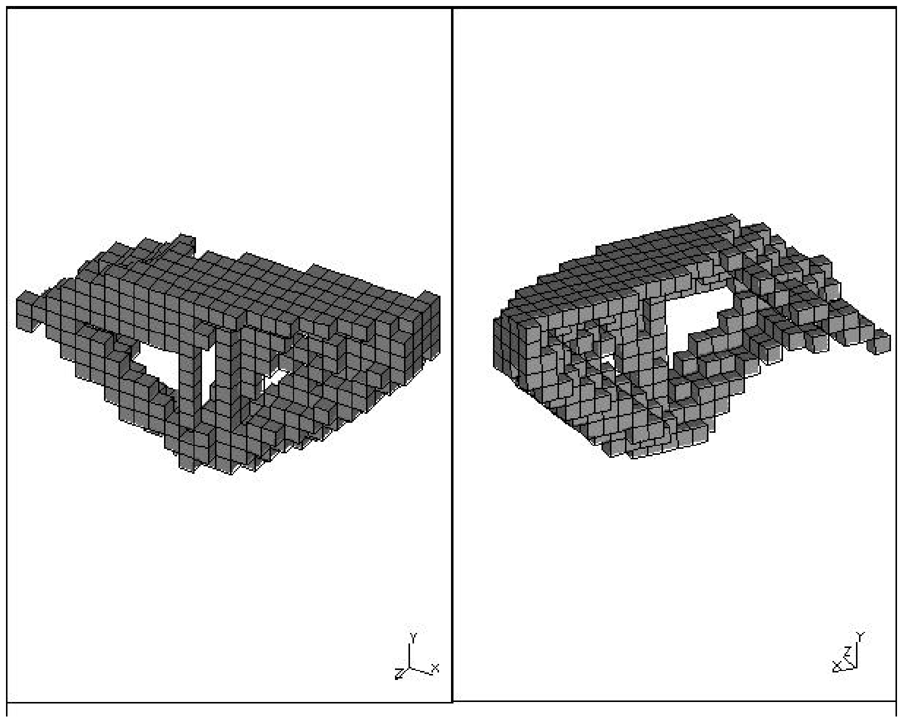
Design of Simple Model



Design Domain

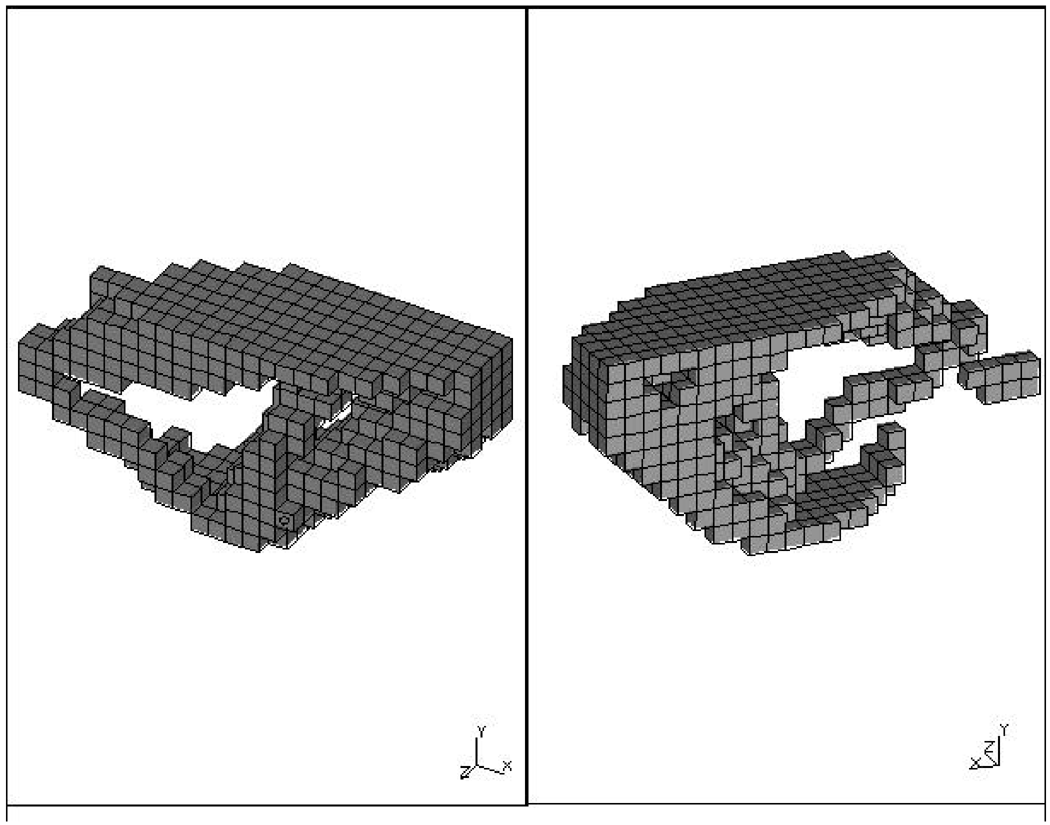
Optimal Configurations (1)

Unconstrained Case



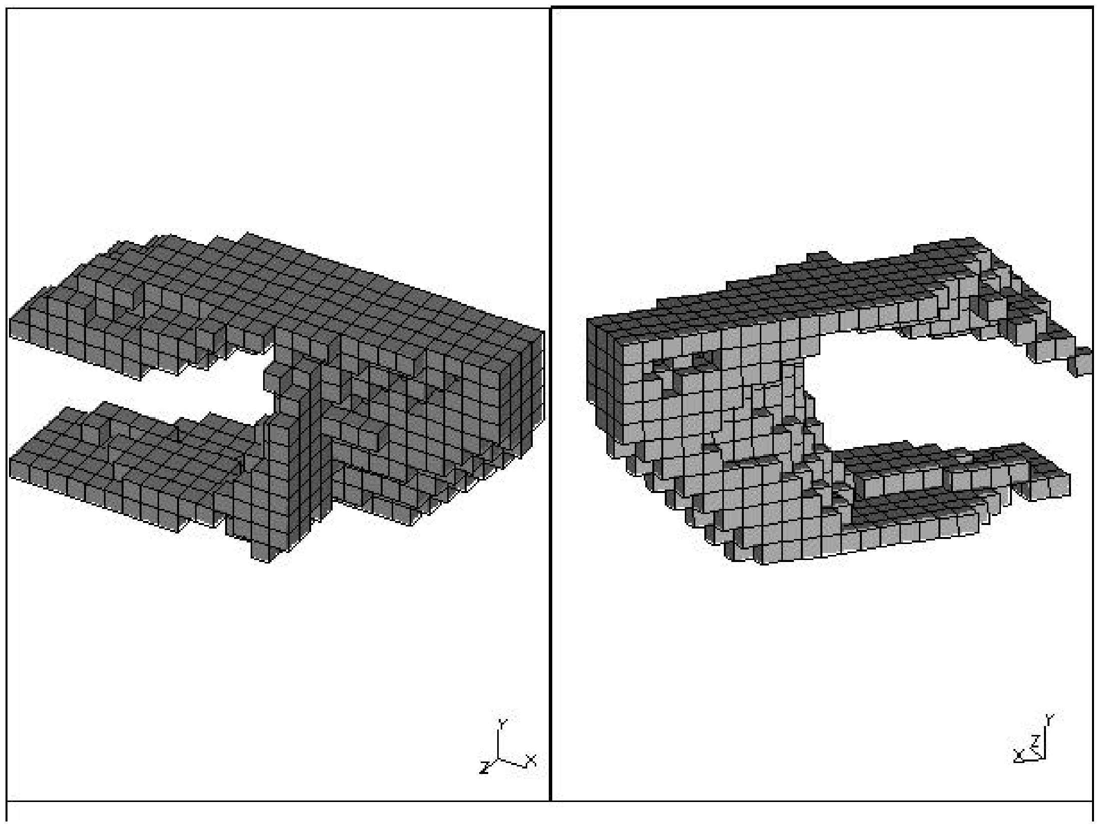
Total Volume =20%

One Constraint Case



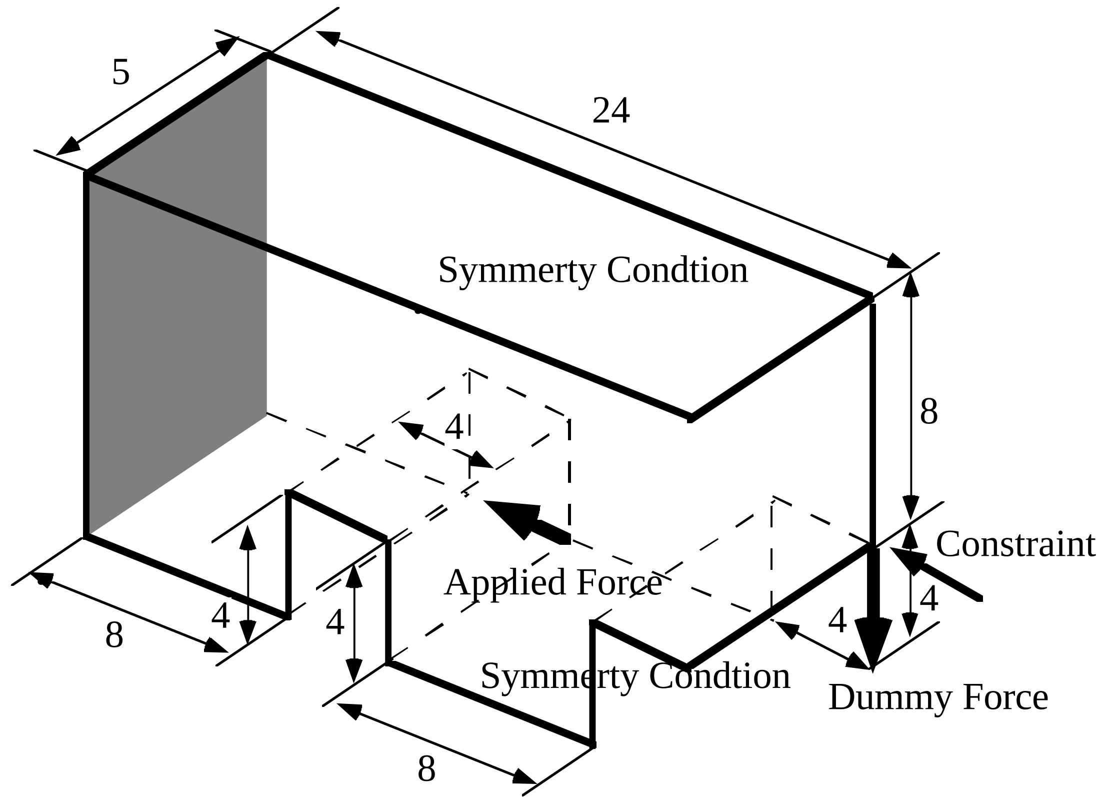
(Total Volume =30%)

Two Direction Constraint



(Total Volume =30%)

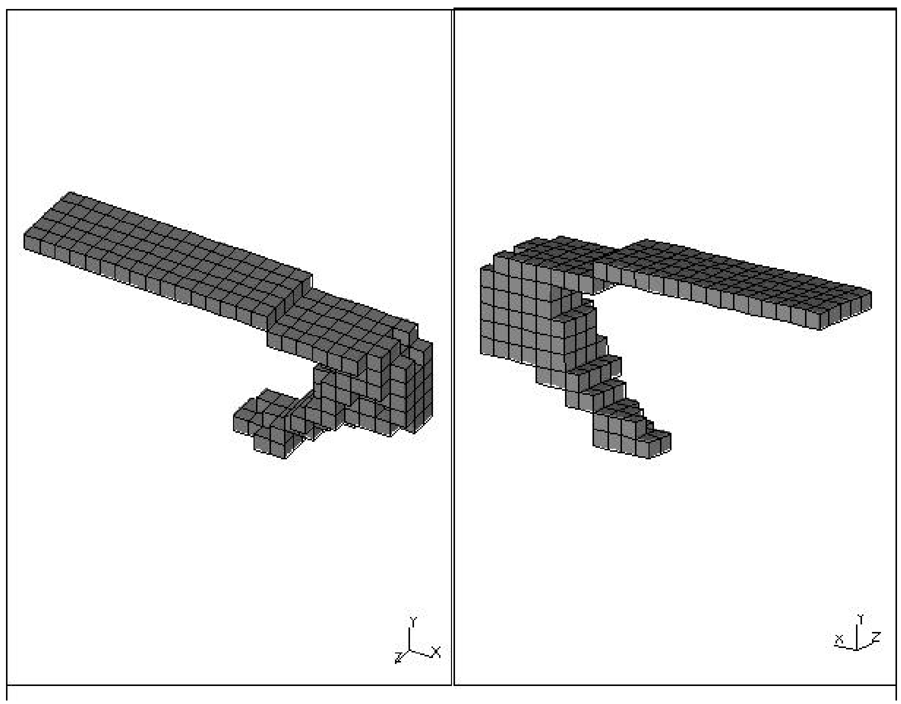
Design of Gripper



Design Domain

Optimal Configurations (1)

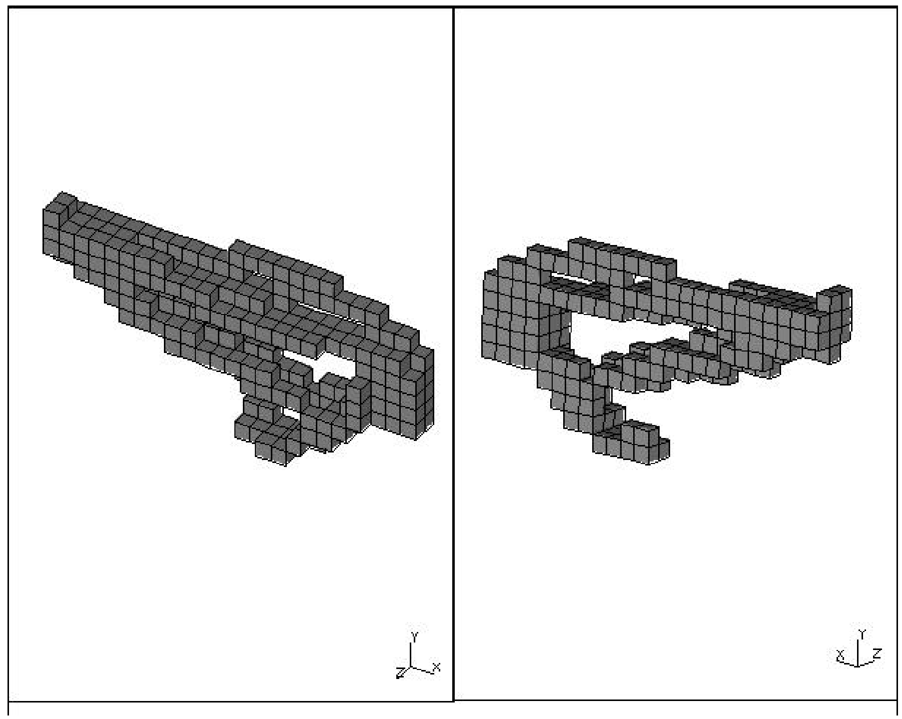
Unconstrained Case



Total Volume =20%

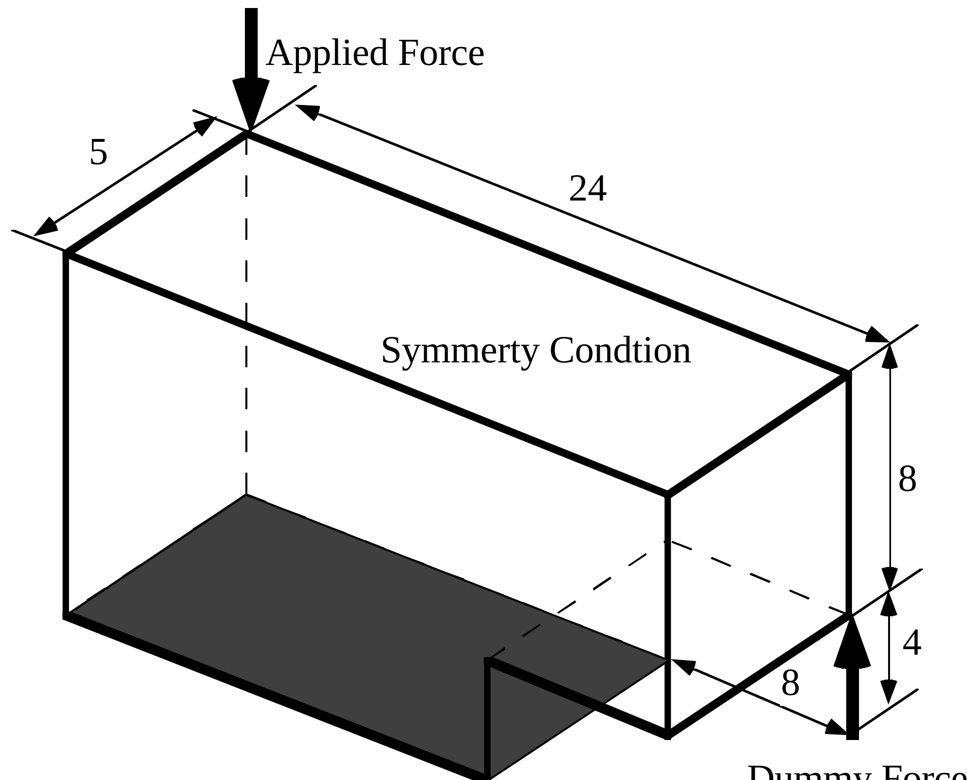
Optimal Configurations (2)

Constrained Case



Total Volume =20%

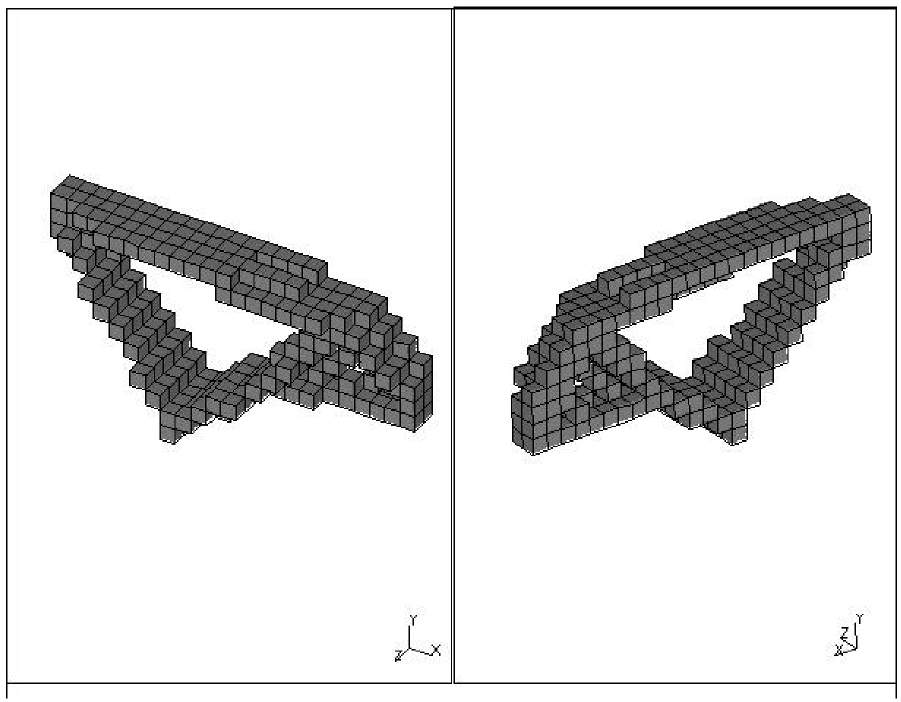
Design of Clamp



Design Domain

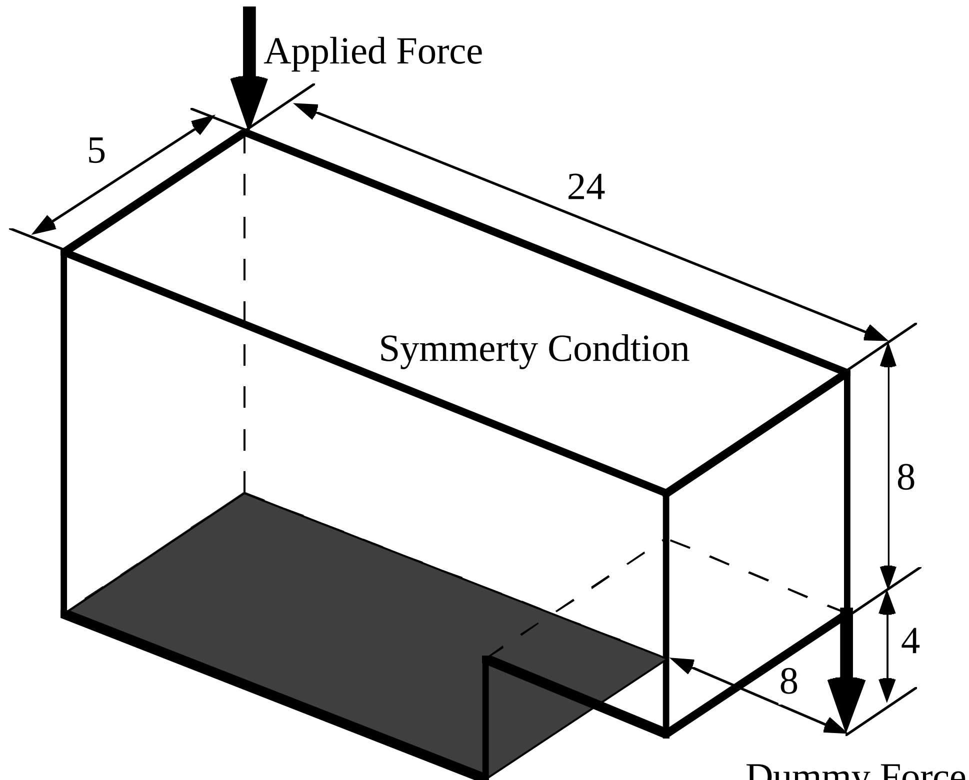
Optimal Configurations

Unconstrained Case



Total Volume =20%

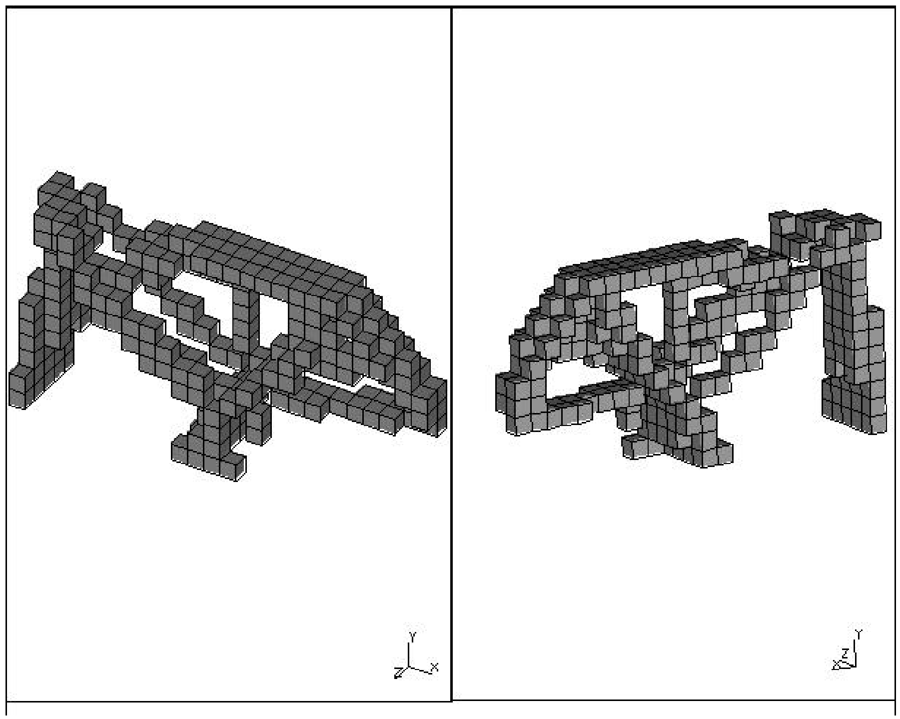
Design of Pliers



Design Domain

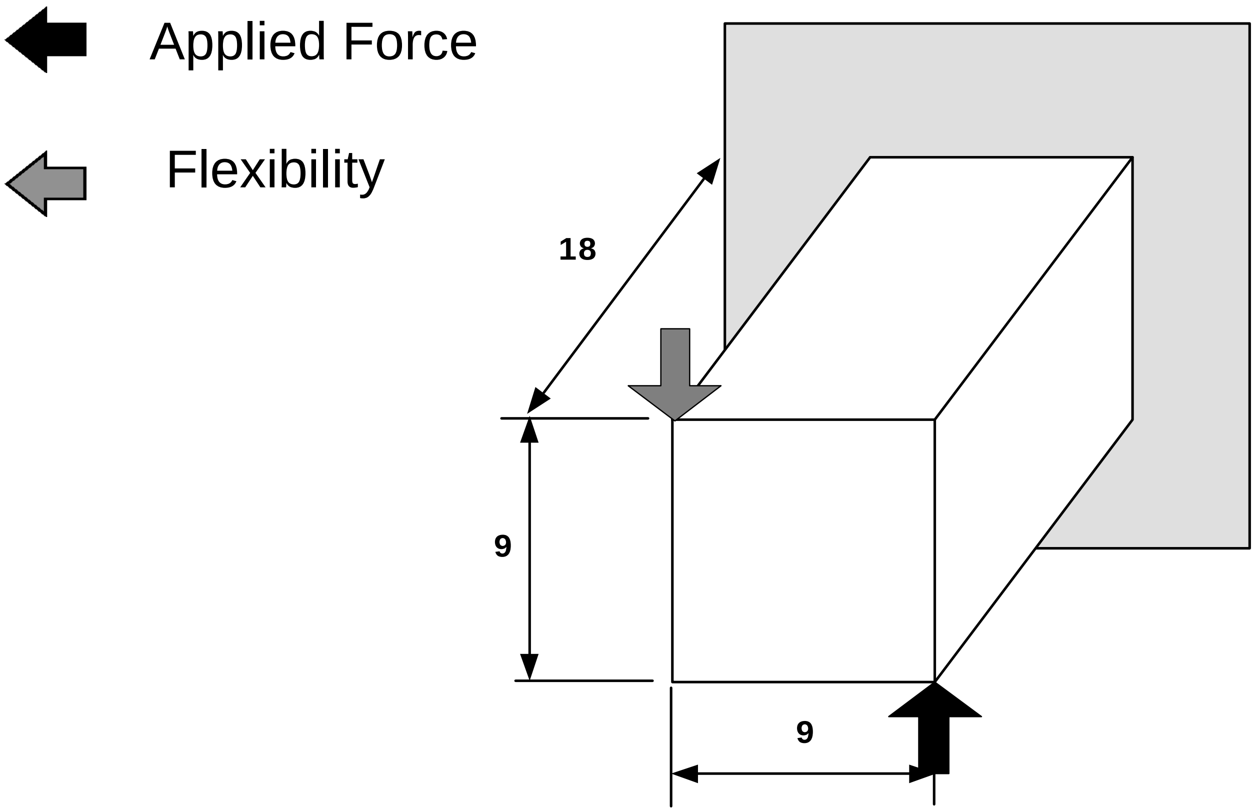
Optimal Configurations

Unconstrained Case



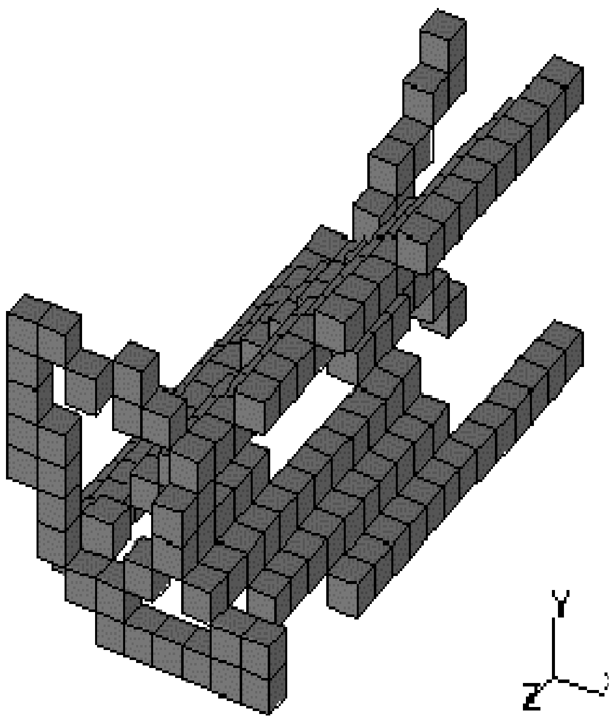
Total Volume =20%

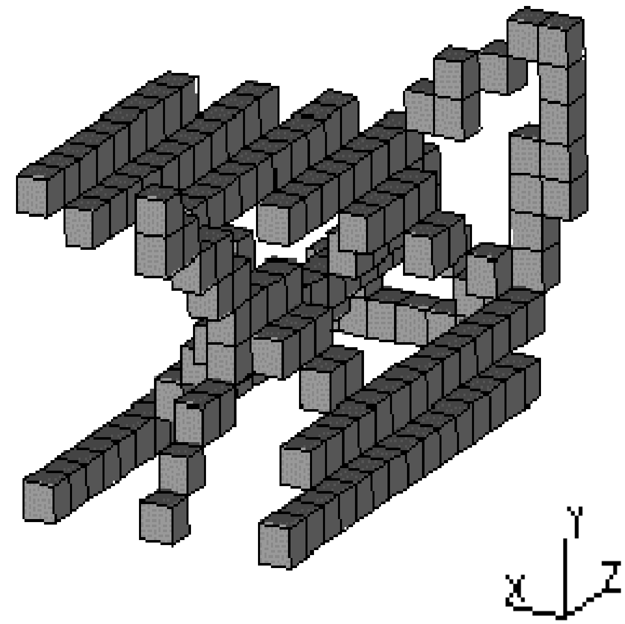
Design of Torsion Bar



Design Domain

Optimal Configuration



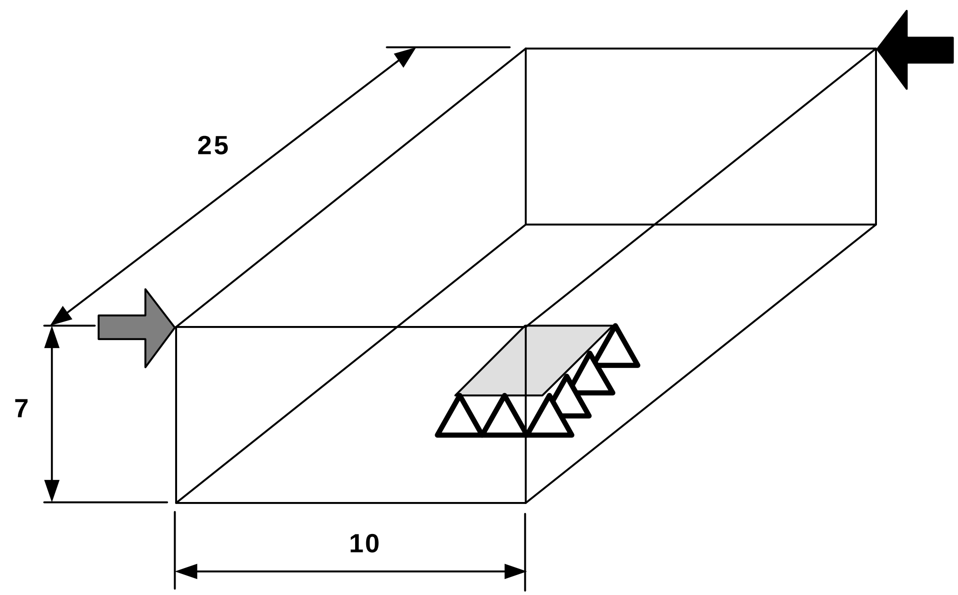


Total Volume =10%

Design of Torsion Plate (1)

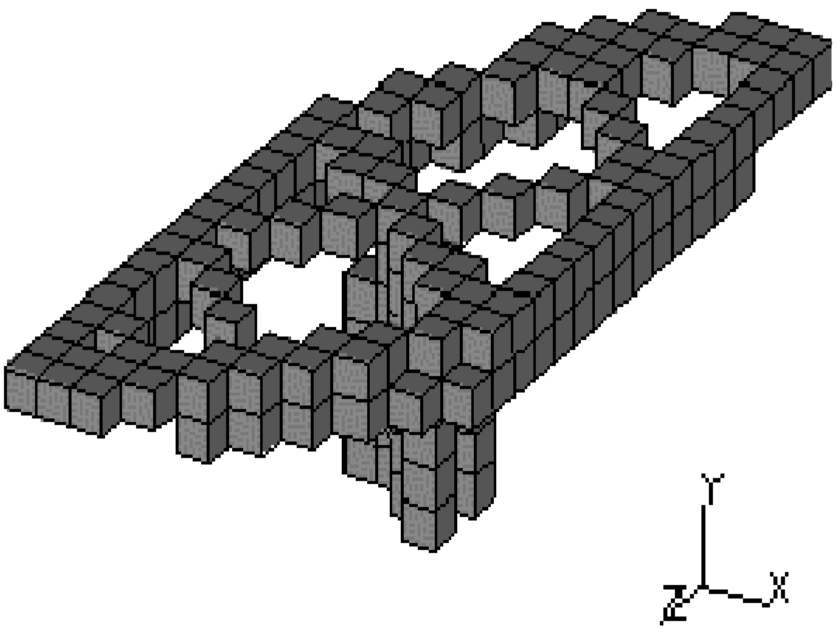
Applied Force

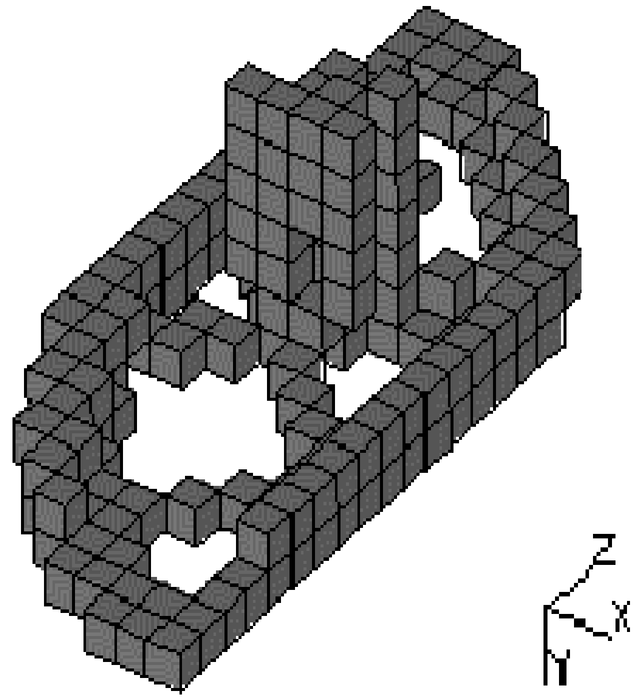
Flexibility



Design Domain

Optimal Configuration (1)



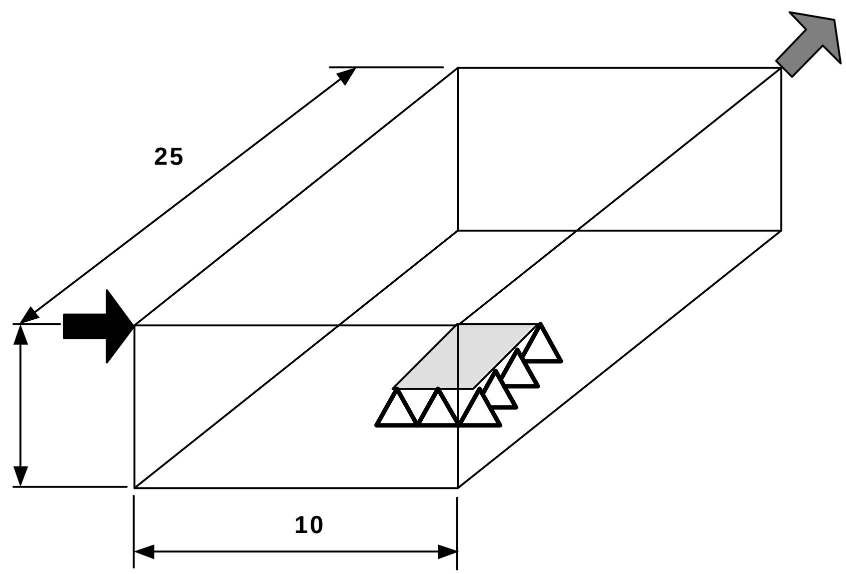


Total Volume =10%

Design of Torsion Plate (2)

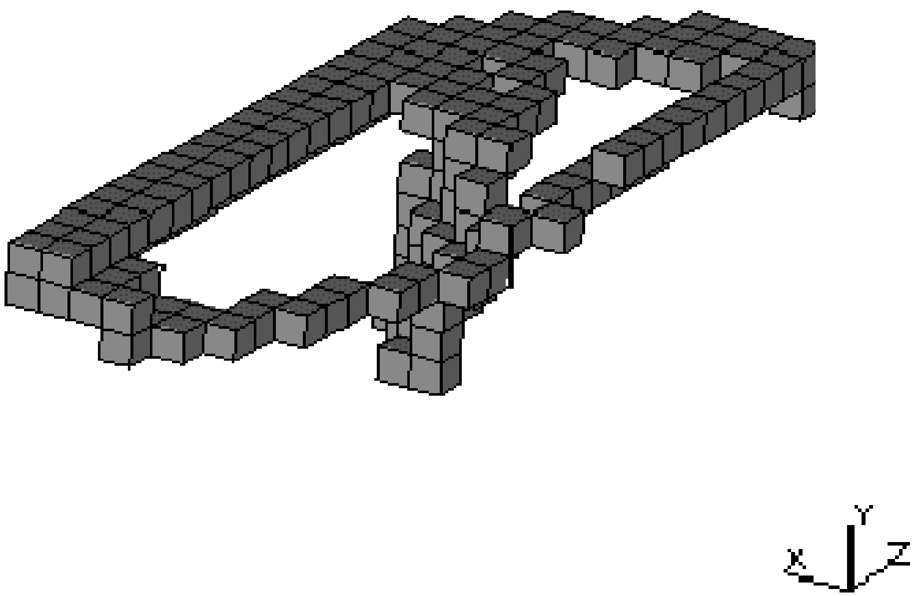
Applied Force

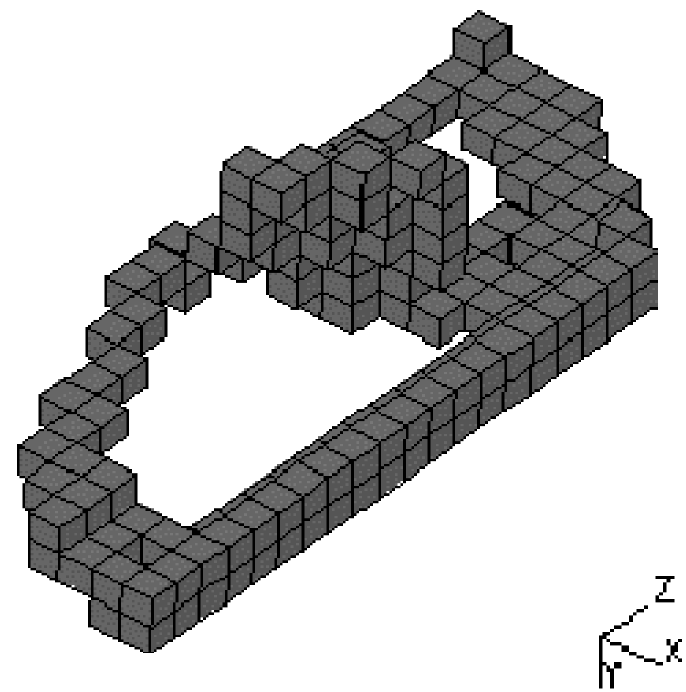
Flexibility



Design Domain

Optimal Configurations (2)



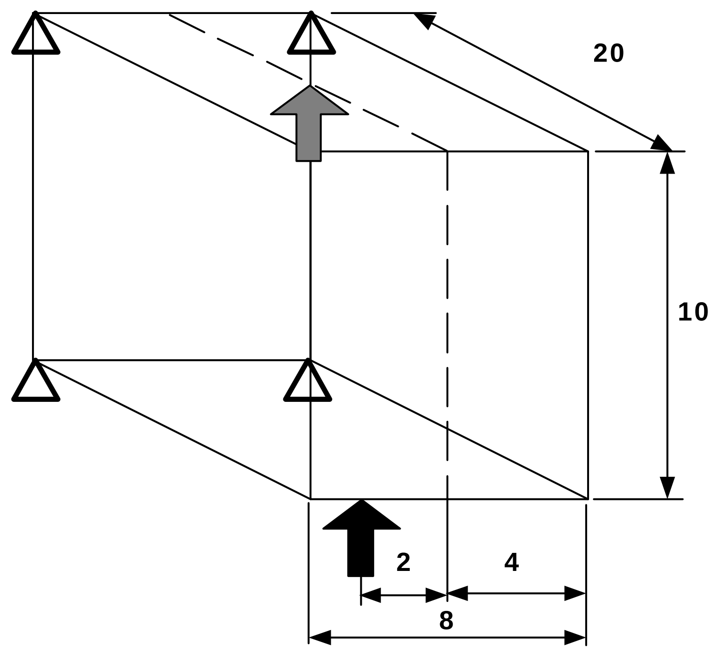


Total Volume =10%

Design of Sus-Like Structure

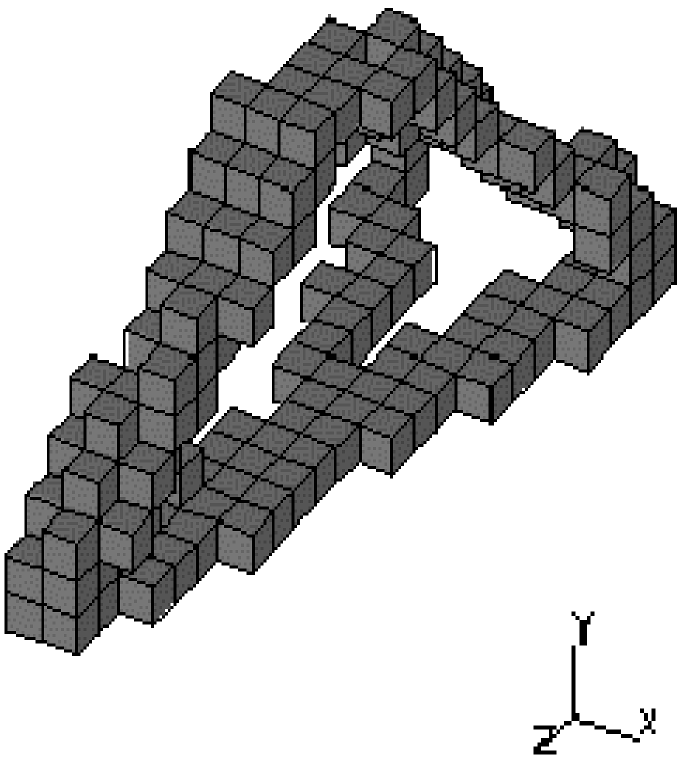
Applied Force

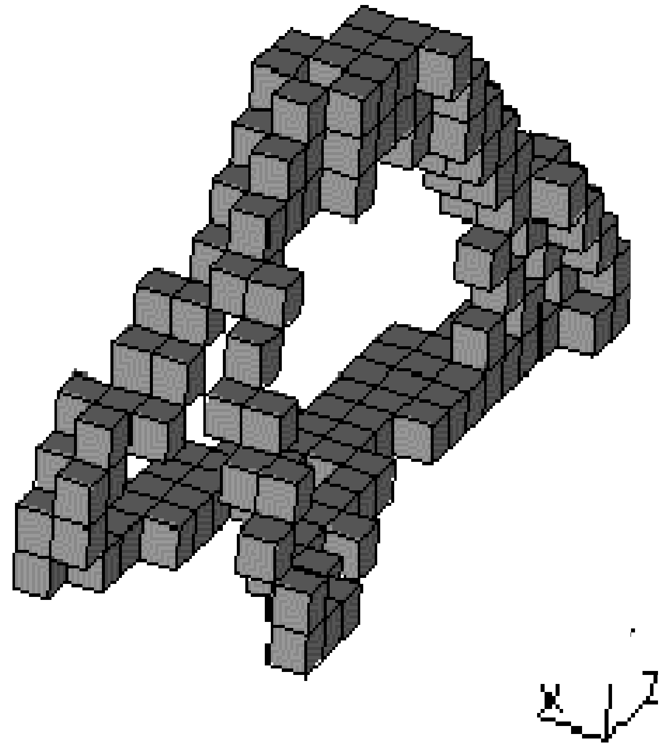
Flexibility



Design Domain

Optimal Configuration



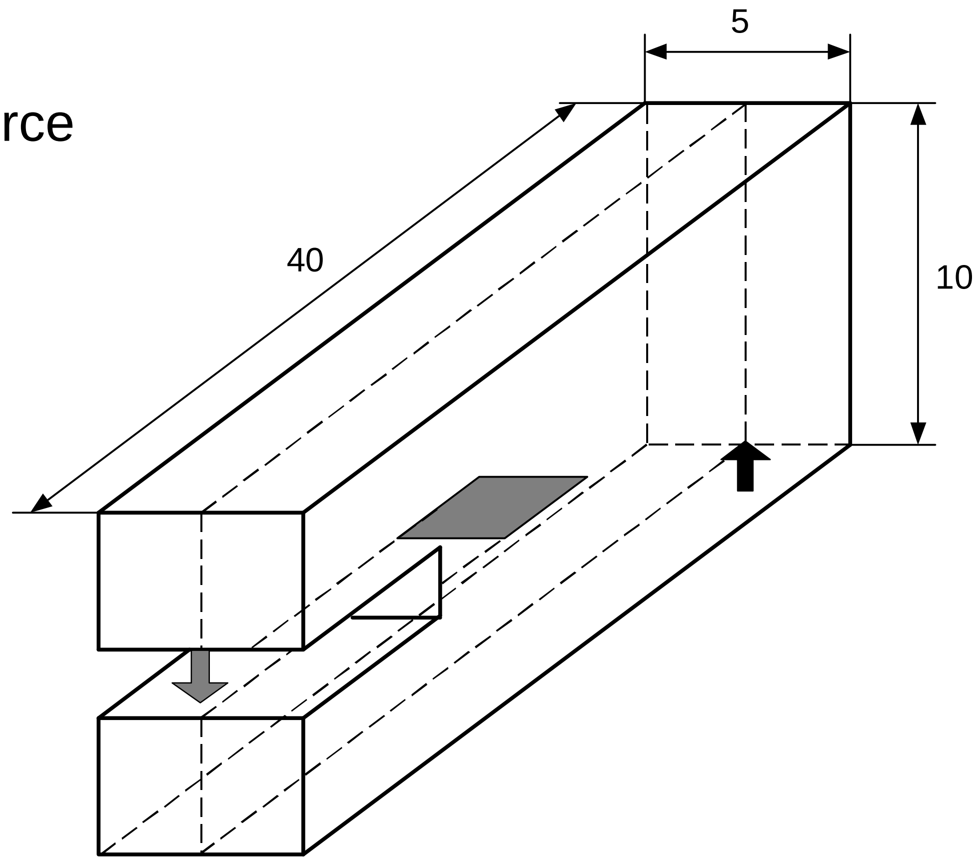


Total Volume =10%

Design of Another Gripper

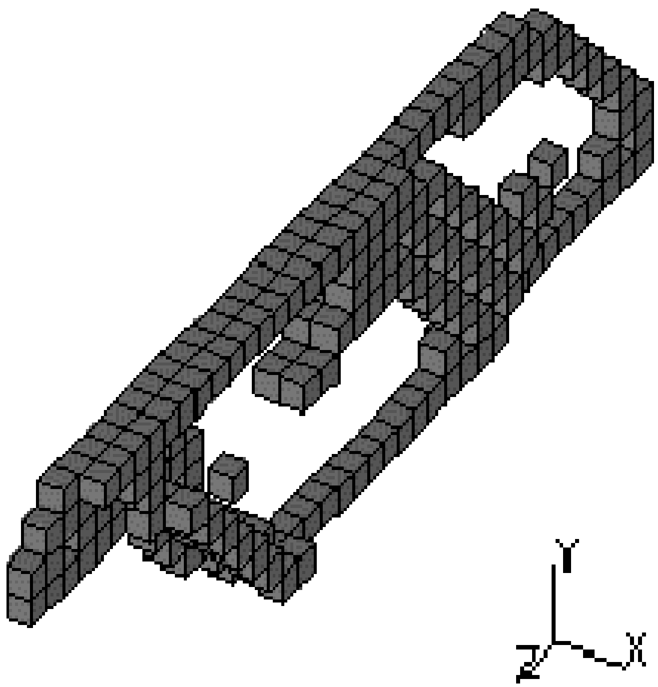
Applied Force

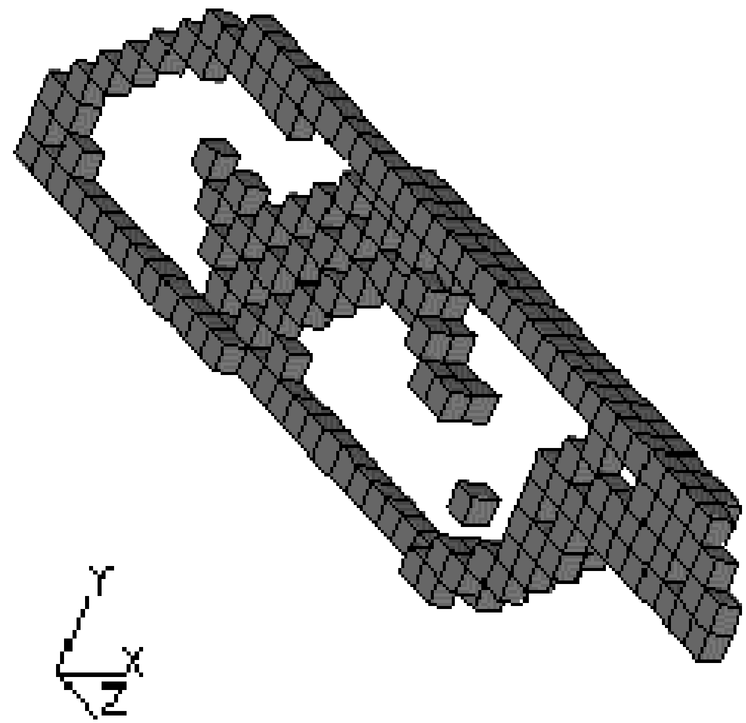
Flexibility



Design Domain

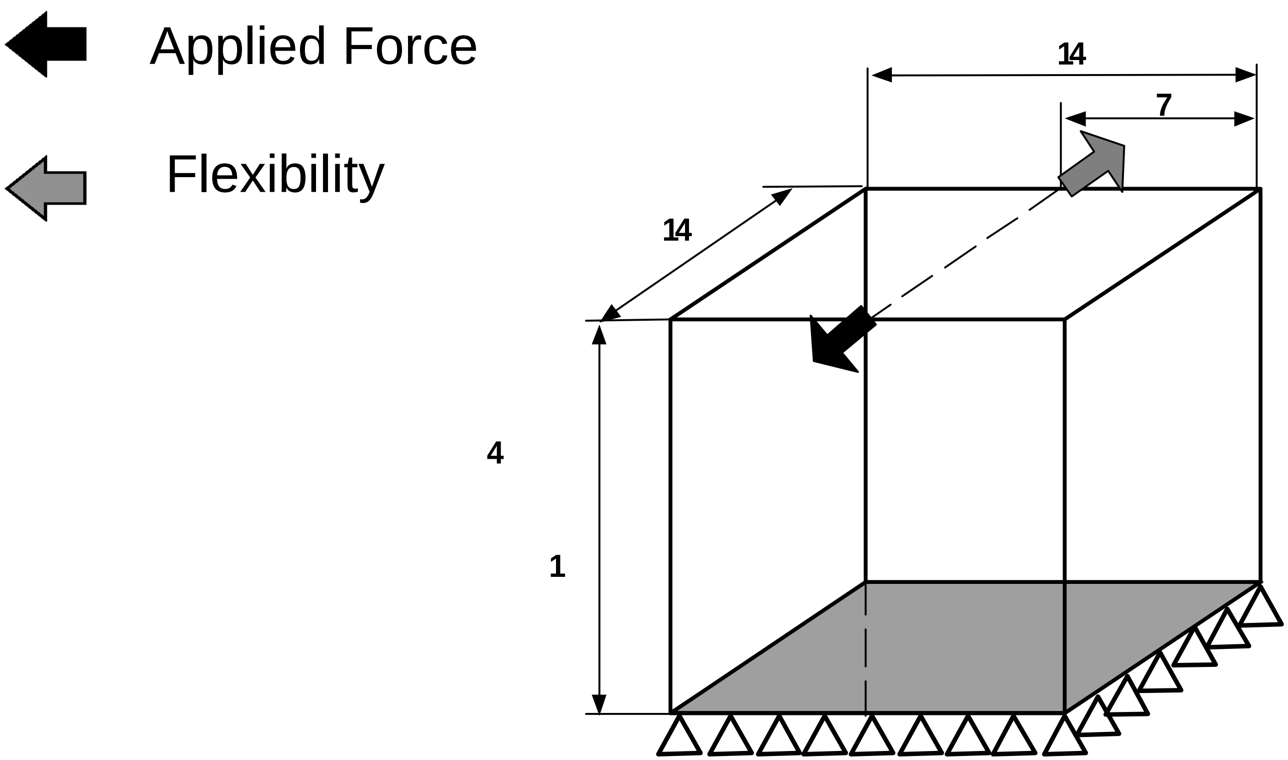
Optimal Configuration





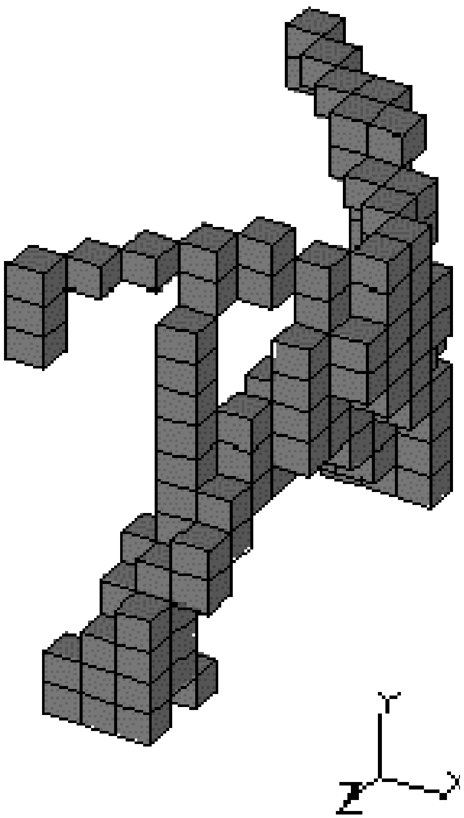
Total Volume =10%

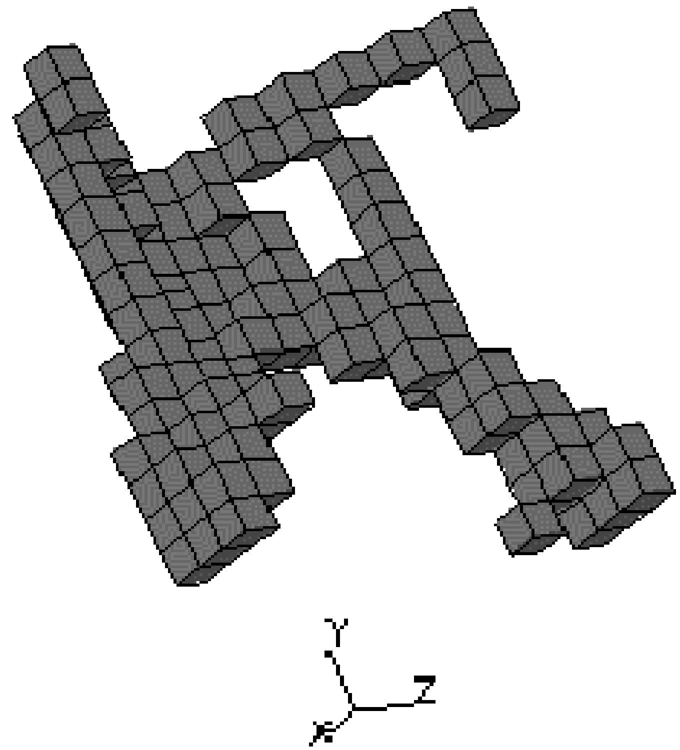
Design of Tensile Model



Design Domain

Optimal Configurations





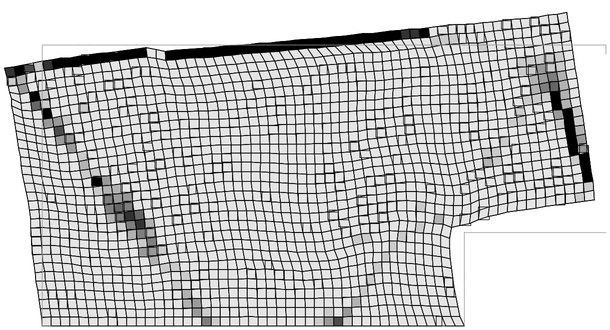
Total Volume =10%

Complaint Mechanism Design with a Displacement Constraint

Multi-Objective Function (a)

Phase 1

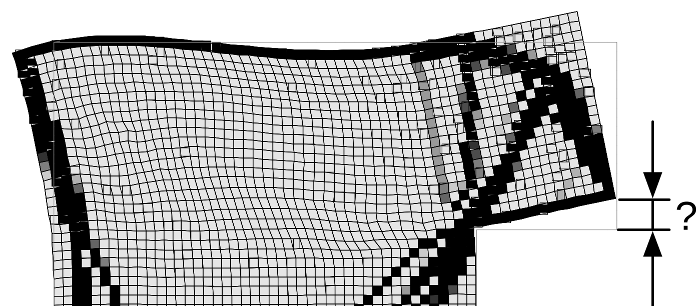
$$Max[Flexibility]$$



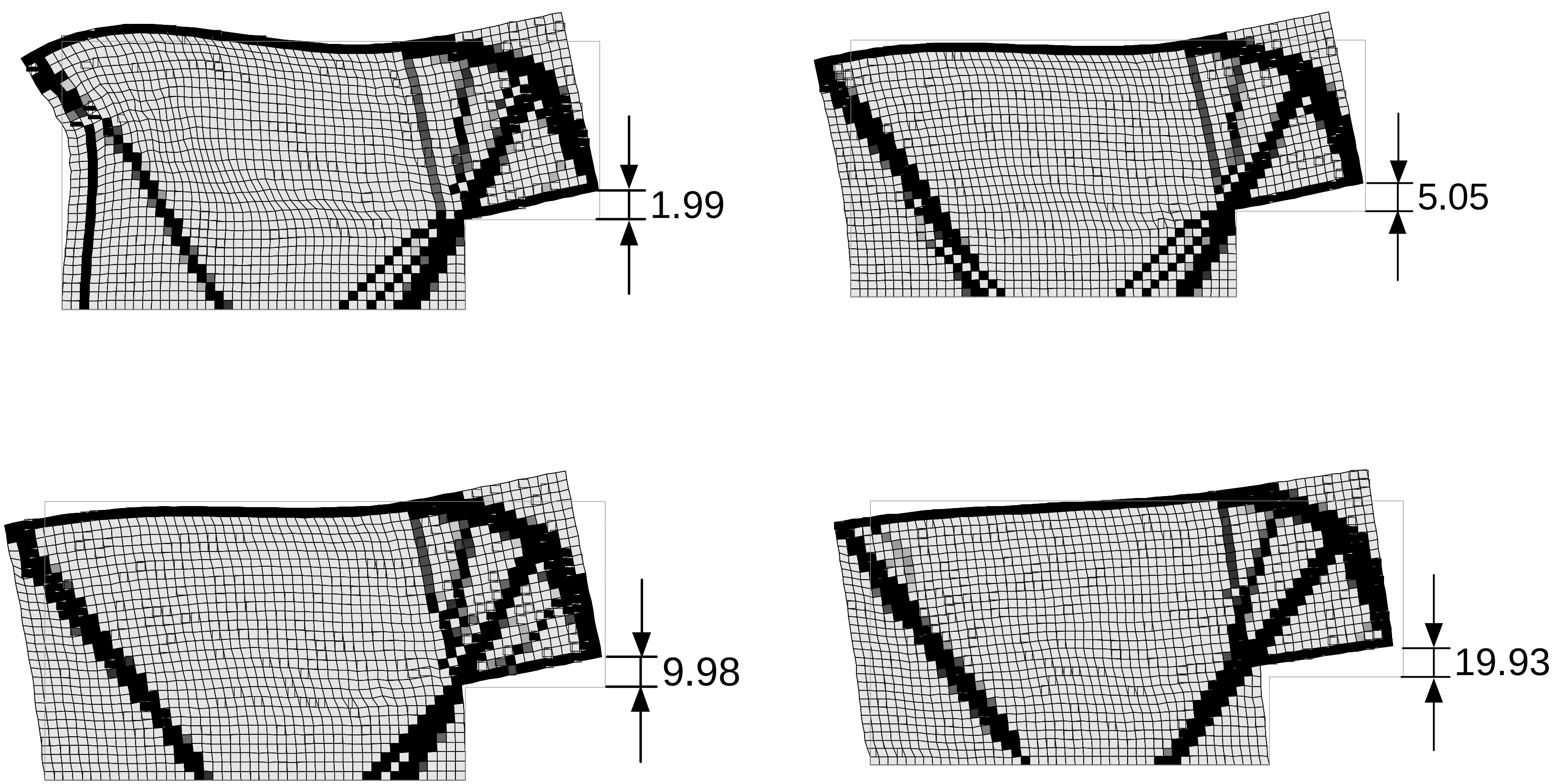
Phase 2

$$Max[Stiffness]$$

$$St.\ Flexibility \geq Prescribed\ \ Flexibility$$



Optimal Configurations (a)

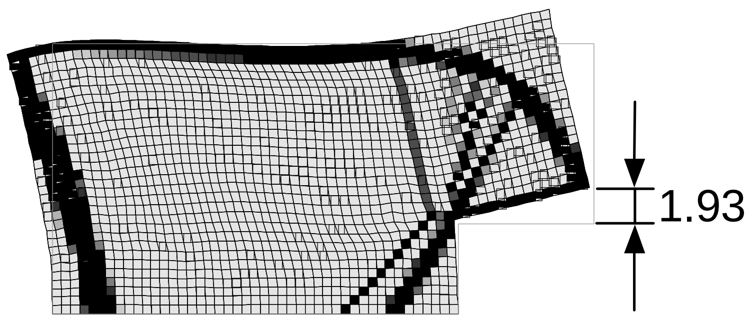


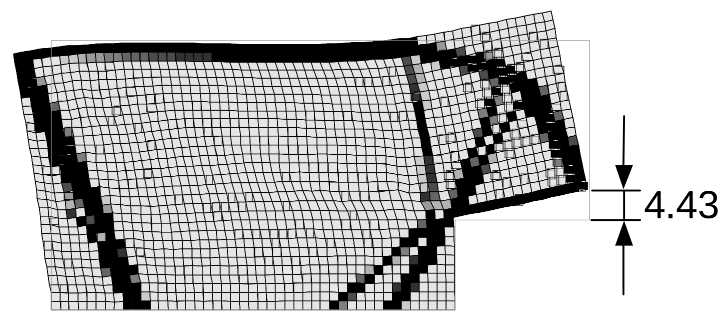
Multi-Objective Function (b)

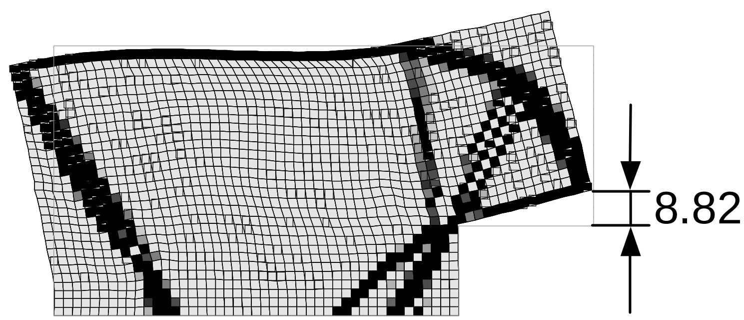
Exterior penalty function method

$$Min\left\{\begin{array}{l} -\log\left(Stiffness\right) \\ +\mu\left[\log\left(Flexibility\right)-\log\left(Prescribed\ Flexibility\right)\right]^{2} \end{array}\right\}$$

Optimal Configurations (b)







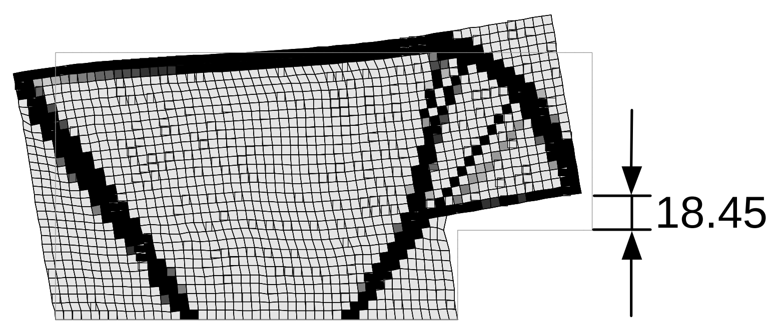
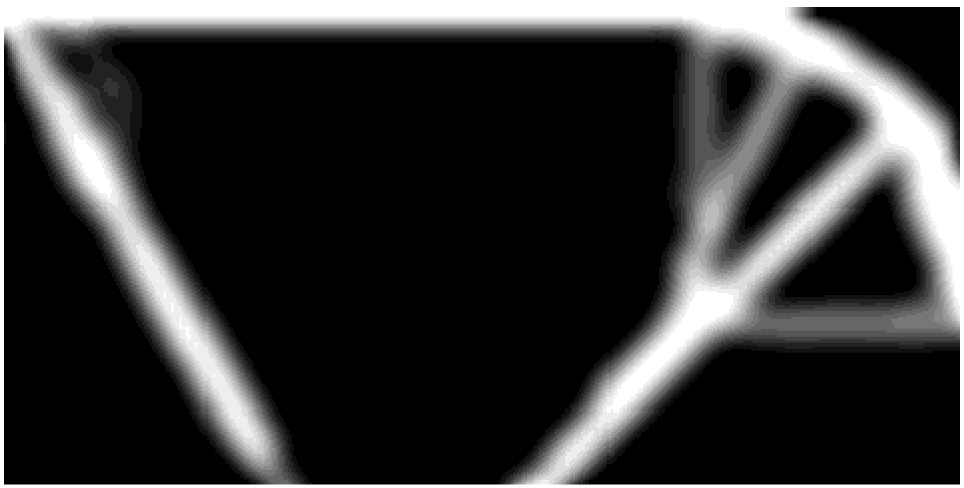


Image Based Design

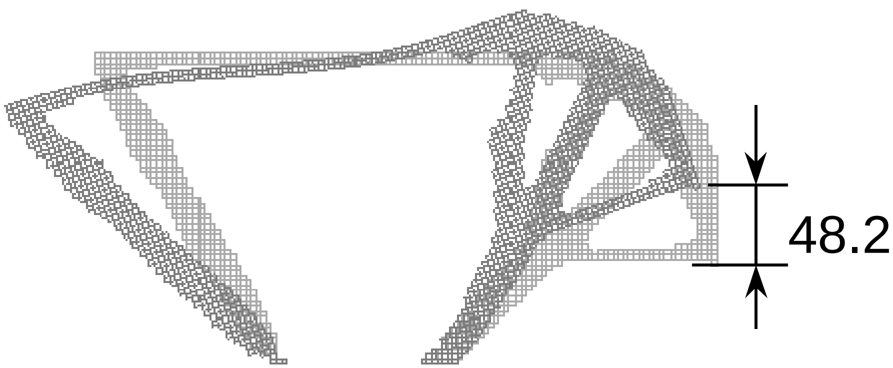
Optimal Topology ⇨ Practical Structure

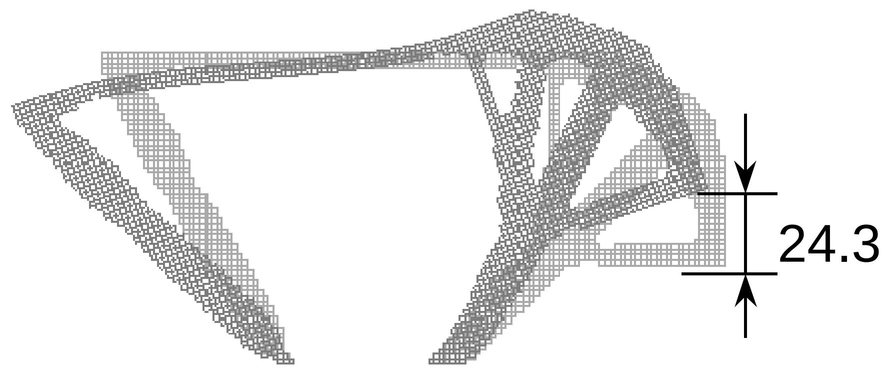
Image processing



Verification of Displacement

Digital Mesh (Voxelcon)





Threshold=100

Threshold=80

Gray scale (0~256)

Material Micro-structure Design

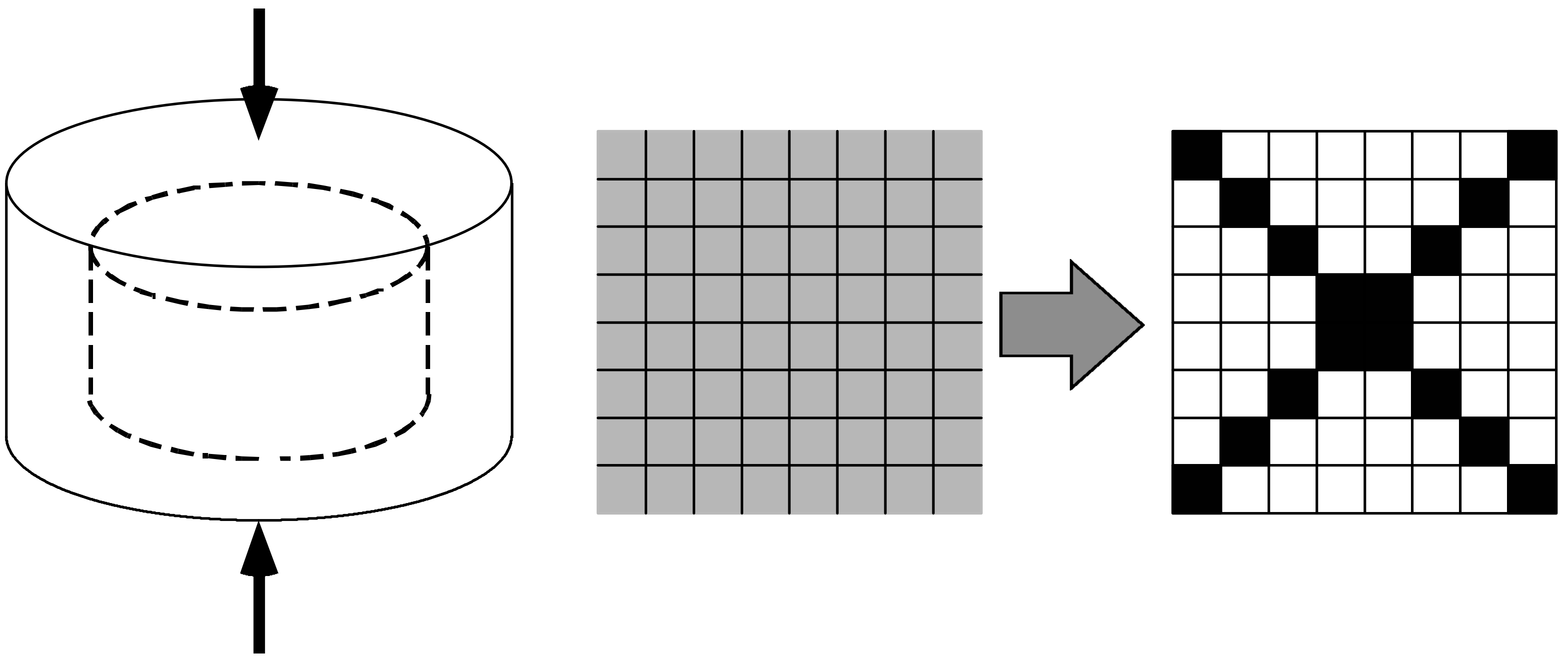
Jun Fonseca : Brazil

and

O. Sigmund : Denmark

Material Design

- Design negative Poisson's ratio by the homogenization design method



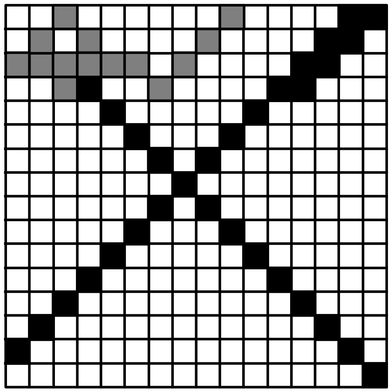
Solution Method 1

1. Specify the Material Constants Desired

$$\boldsymbol{D}^{-1} = \begin{bmatrix} \frac{1}{E_1} & \frac{\nu_{12}}{E_1} & 0 \\ \frac{\nu_{21}}{E_2} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \qquad \mathrm{a} = \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix}$$

2. Define the Unit Cell for Microstructural Design

Design Variable = Holes



Solution Method 2

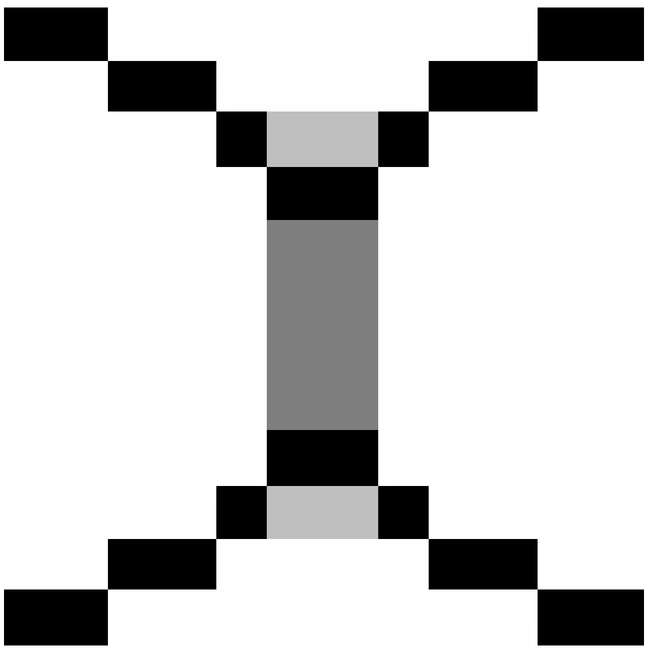
3. Apply the Homogenization Method to find the material constants for a design

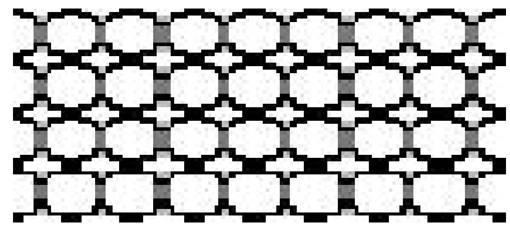
4. Solve the Inverse Problem Based on the Least Squares Method

$$\min_{\text{design}} \frac{1}{2}\left\| \boldsymbol{D}_d - \boldsymbol{D} \right\|^2$$

with symmetry and periodic conditions

Bendsoe's Material

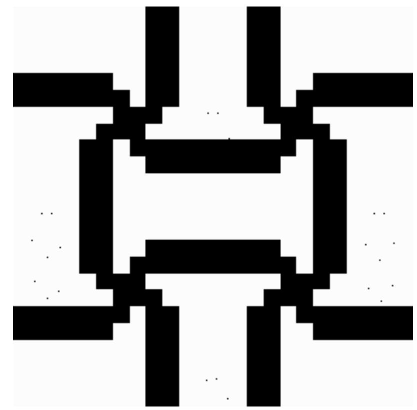


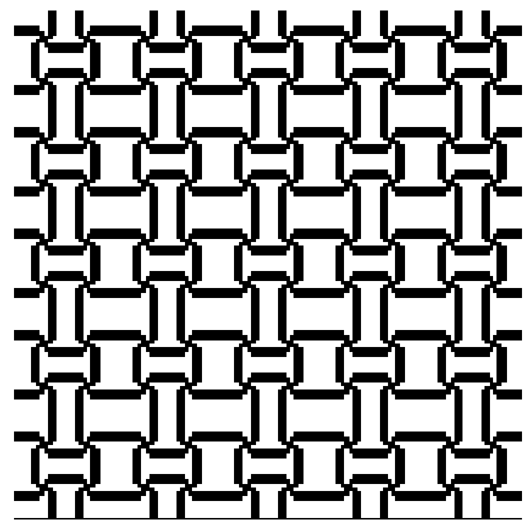


$$\boldsymbol{D} = \begin{bmatrix} 1 & 0.48 & 0 \\ 0.48 & 0.25 & 0 \\ 0 & 0 & 0.02 \end{bmatrix}$$

Negative Poisson's Ratio

- *Plane stress*
- *Cubic Material (3 independent elastic constants)*

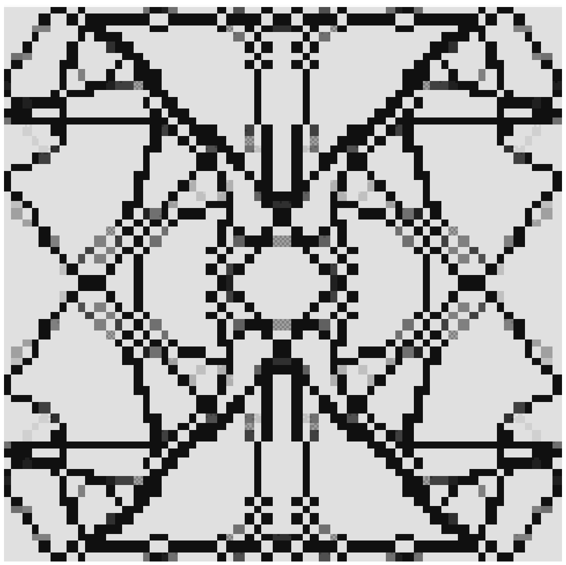




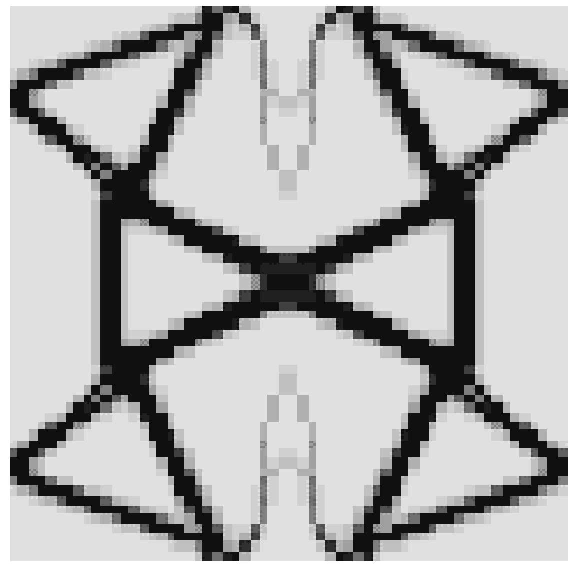
$$\mathbf{E} = 0.3434\,\mathbf{E}_0 \begin{bmatrix} 1 & -.66 & 0 \\ -.66 & 1 & 0 \\ 0 & 0 & .02 \end{bmatrix}$$

Application of a Filtering

isotropic negative Poisson's ratio microstructures



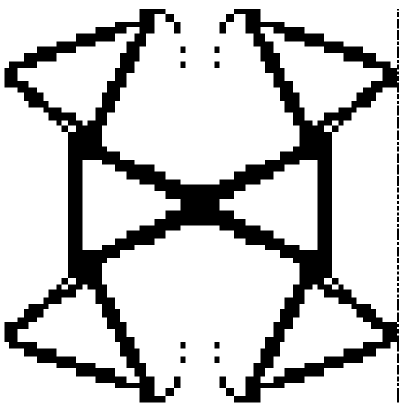
non filtered 60x60

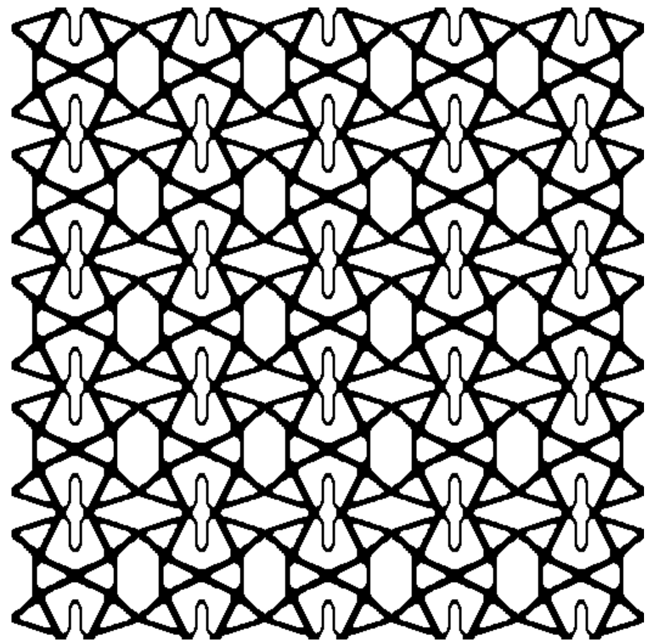


filtered 60x60

Easier Interpretation of the topology without changing of the properties

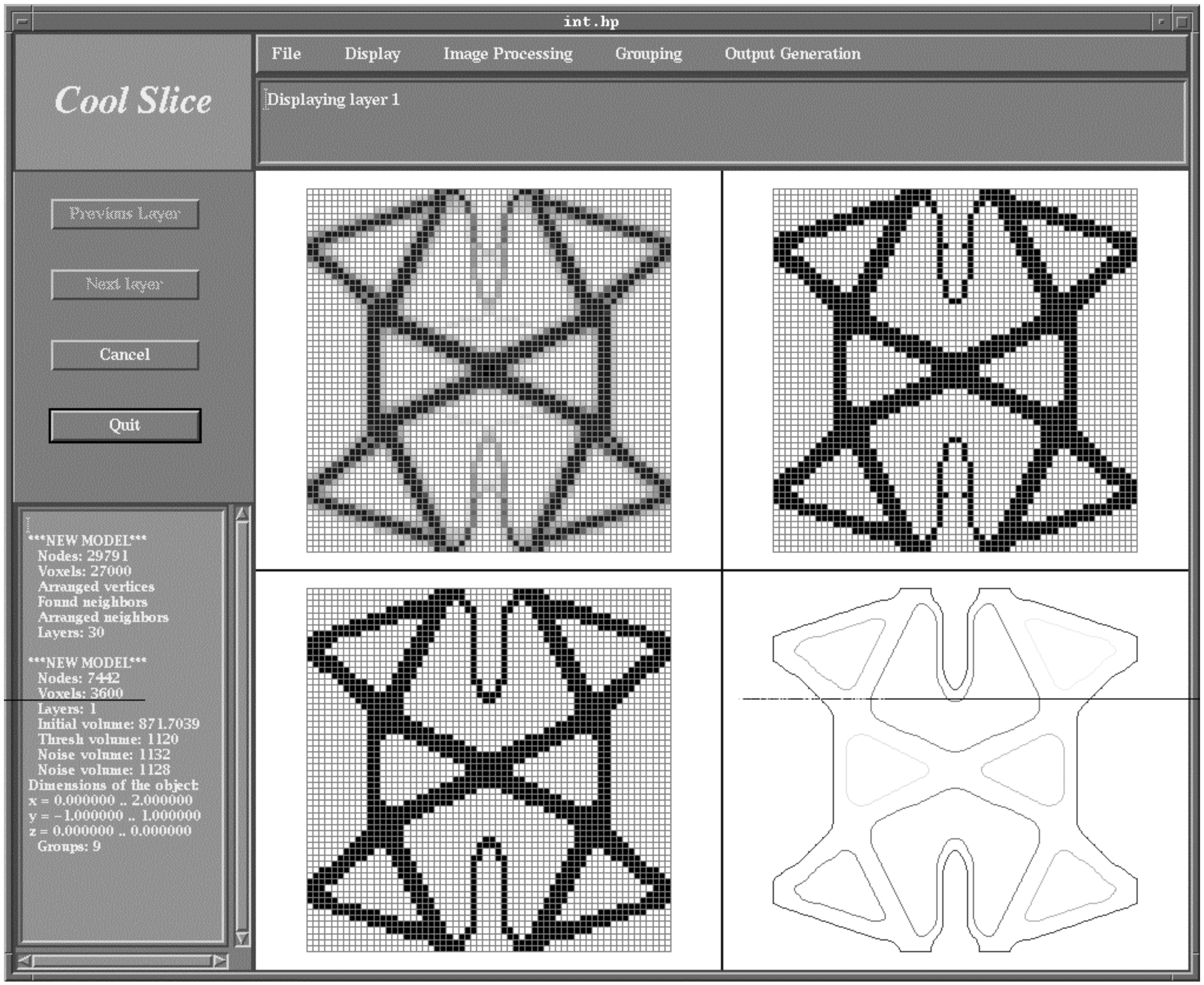
Isotropic Material Design



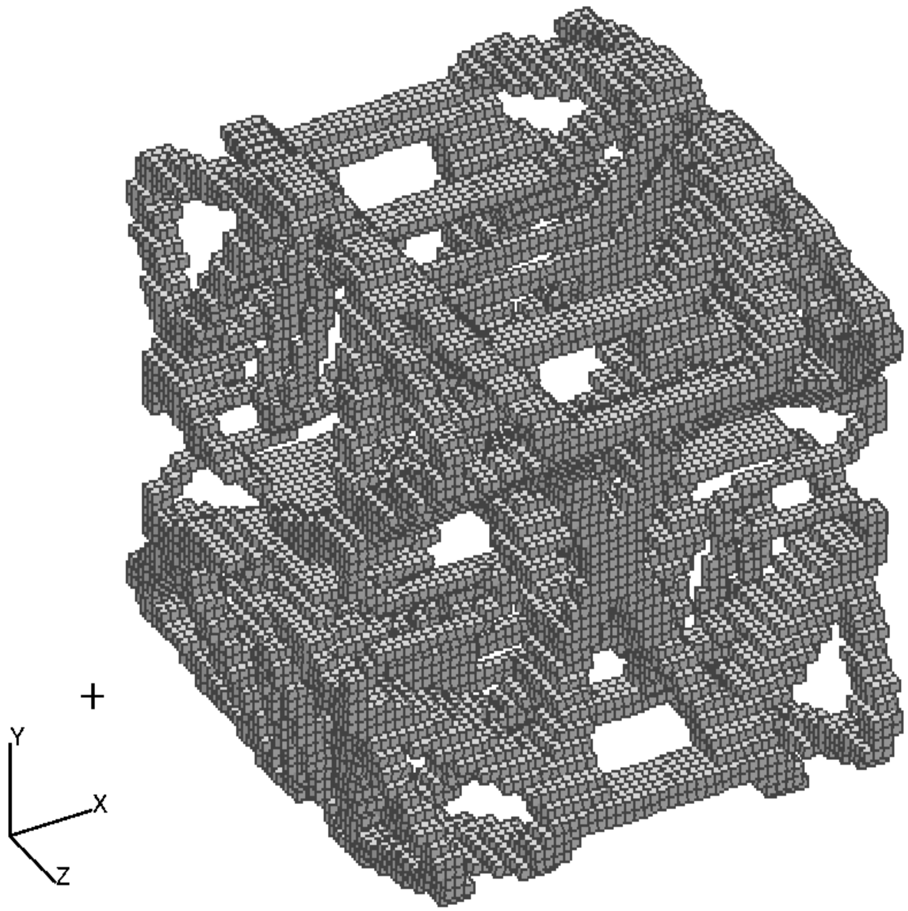


■*Poisson's ratio -0.5*

Jun Fonseca & Anne Marsan



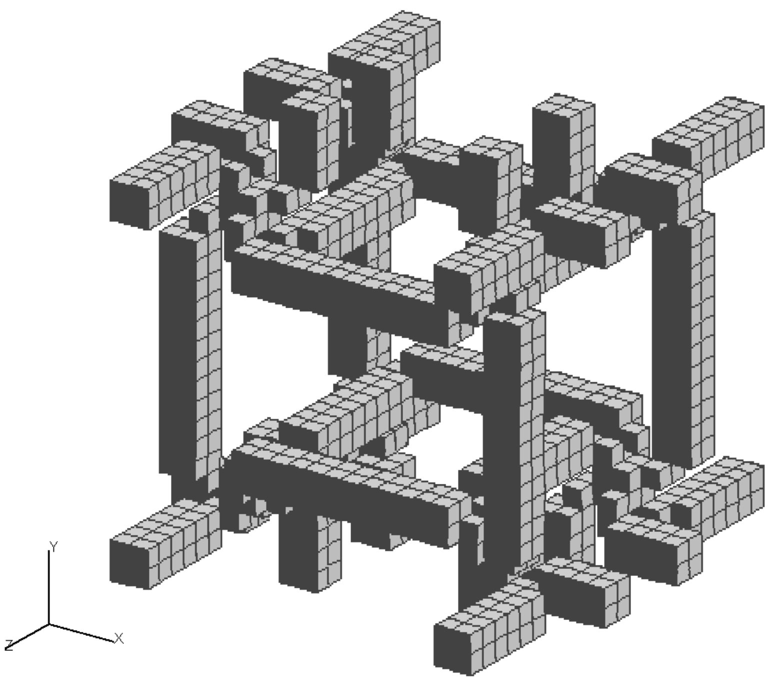
Three Dimensional Design



- *Tensor Product*
- *Orthotropic*
- *Two negative and one positive Poisson's ratios*

$$\mathbf{E}=\begin{bmatrix} 1 & -0.513 & 0.276 & 0 & 0 & 0 \\ -0.513 & 0.656 & -0.341 & 0 & 0 & 0 \\ 0.276 & -0.341 & 0.713 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.254 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.284 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.286 \end{bmatrix}$$

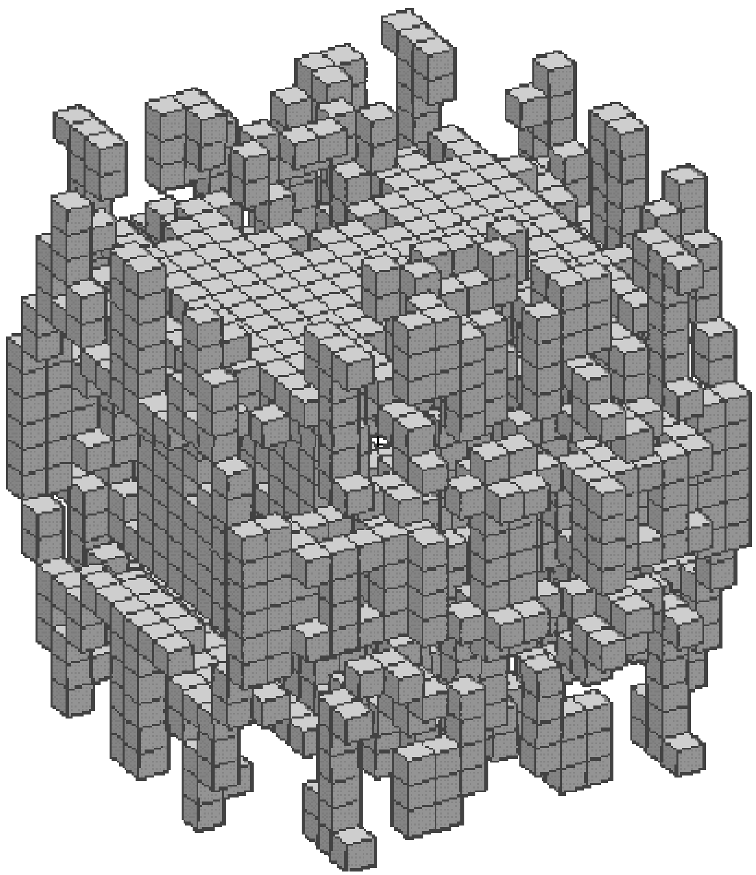
3D Microstructure



- *Cubic symmetry (3 independent elastic coefficients)*
- *Negative Poisson's ratio*

$$E = 10^{-4} E_0 \begin{bmatrix} 1.0 & -.28 & -.28 & 0 & 0 & 0 \\ -.28 & 1.0 & -.28 & 0 & 0 & 0 \\ -.28 & -.28 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & .03 & 0 & 0 \\ 0 & 0 & 0 & 0 & .03 & 0 \\ 0 & 0 & 0 & 0 & 0 & .03 \end{bmatrix}$$

Another 3D Microstructure



- *Full 3D design*
- *smaller mesh: 20x20x20*
- *isotropic with negative Poisson's* n=*1/4*

Future Material Design

Jose M. Gedes : Portugal

and

J.E. Taylor : USA

Direct Use of Elasticity Matrix

$$\max_{\boldsymbol{E}} \quad \min_{\boldsymbol{v}} \frac{1}{2} a(\boldsymbol{v}, \boldsymbol{v}) - f(\boldsymbol{v})$$

subject to

$$\int_D \rho \, dD \leq W_0$$

More design variables, more optimum

Still More Research on Homogenization Design

Continuous Development and Enhancement of OPTISHAPE

Practice Demands

many new feature

of

OPTISHAPE

Request from Practice

- Automatic surface recognition
  - Applying the image based modeling method developed by Minako Sekiguchi,we can define a smoothed three dimensional body
  - Then convert to STL format
- Automatic Mesh Generation for Detailed FE analysis
  - Applying VOXELCON to extract Wire Frame Model, then go to a CAD system & FE soft