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Topology optimization of thermally actuated compliant mechanisms considering time-transient effect☆

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Abstract

Thermal transducers, which can be regarded as thermally actuated compliant mechanisms, have found a wide range of applications, due to easy accessibility of heat source, and due to their good controllability and reliability. During the past decade topology optimization techniques have been developed as efficient tools to design distributed compliant mechanisms and accepted by engineers. In this research, based on topology optimization, time-transient effect of heat transfer is proposed to produce the localized thermal actuation, using only simple forms of boundary heating. Non-uniform temperature distribution can be achieved by controlling the heating time before steady state is reached. Two numerical examples are presented. The resulting technique can be applied to design a novel type of integral attachment mechanisms, which benefits the design for disassembly.

Keywords: Topology optimization; Compliant mechanism; Time-transient effect; Thermal actuation

1. Introduction

A compliant mechanism [1] is the mechanism that relies on its own elastic deformation to transfer or transform motion or force. Common compliant mechanisms function under the application of force at certain location (input) and generate desired force or deflection at another location (output). Thermally actuated compliant mechanisms are those compliant mechanisms, onto which thermal loading is applied as input instead of force. This actuation is simply based on thermal expansion of common materials while being heated. In practice, thermal loading can be easily applied and

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achievable temperature can be easily monitored. Furthermore thermal expansion induced deformation does not associate with as much mechanical stresses as force induced deformation, thus would reduce the chance of material failure and fatigue.

The most well known type of thermally actuated compliant mechanisms is the so-called “electro-thermal-compliant” (ETC) micro-actuators used in micro-electrical-mechanical system (MEMS). Unlike other types of actuators, ETC does not require sophisticated actuation source or material with special property. The mechanisms are simply made of conductive materials that are actuated through joule heating effect by an electrical current, instead of piezoelectric effect in PZT actuators or phase change effect in shape memory alloy (SMA) actuators. Early thermal actuators are mostly bimorph actuators made of two materials with dissimilar thermal expansion coefficients [2,3]. Recently, idea of embedded actuation in one single material has been introduced to design ETCs [4]. One example is the Guckel actuator [5], which is a folded U-shaped beam with a pair of wide and narrow arms. When current passes between the two anchors, larger current density in the narrow beam causes a larger thermal expansion than that in the wide beam thus forms the pseudo-bimorph type of thermal actuator. Another successful design is the so-called bent-beam electro-thermal actuator [6], which comprises of V-shaped beams anchored at both ends. Each individual beam is heated through electric current and the thermal expansion caused by joule heating pushes the apex outward. There are also many other designs and applications reported in the literature [7–11]. In addition, design methods have been developed involving topology optimization techniques by several different research groups [7,12,38]. The basic idea is to distribute material in a certain design domain and form a compliant structure to be heated and deform in a desirable manner. Both uniform temperature change and steady-state electro-thermal loading have been considered as thermal actuation.

In this research, we introduce the time-transient effects to thermal actuator design. It has been noticed that the topology optimization for transient problems has been studied by Turteltaub [13], where material distribution is optimized to achieve a temperature distribution that is closest to a target field function. However, in this actuator design problem the objective is related to thermal–mechanical phenomena, therefore the optimization problem is much more complicated.

Unlike in ETC cases, where non-uniform temperature distribution is caused by non-uniform joule heating through non-uniform current density, this work proposes to utilize the non-uniform temperature distribution due to time transient response of heat transfer. In other words, during a certain period of time of heat transfer process, significant temperature difference exists between different parts of the mechanisms. In this case the thermal loading condition can simply be a boundary heating. It is necessary to understand that this proposed approach is not generally applicable to micro-systems, due to the short steady-state time constant associated with micro-scale systems. Based on the topology optimization technique for compliant mechanisms, the time transient heat transfer analysis is integrated with thermal–mechanical analysis to formulate the design optimization problem. With this proposed design synthesis, the mechanism layout is generated by conducting the heat to the portion of mechanisms to be thermally expanded in favor of the desired mechanism motions. The easiness of time and temperature control makes the time-transient effect realizable in practice. Because no bi-material structural is needed, this type of design requires relatively simple manufacturing process. In macro-scale systems, mature actuation state does not depend on the steady state, which can be a save of actuation time and manufacturing cost. Numerical examples of a thermal actuator design and snap-fit mechanism design will be presented to demonstrate a specific application of the proposed design approach.

2. Background

2.1. Homogenization design method

Homogenization based topology optimization is the basis for the design technique proposed in this research. The topology optimization problem is formulated as a problem of finding the optimal distribution of materials in an extended fixed domain where some structural cost function is maximized. If the material densities considered are allowed with only value of 0 or 1 at an arbitrary point in the design domain, the mathematical problem is ill conditioned and the existence of a solution is unsure [14]. To relax the problem a microstructure proposed by Bendsøe and Kikuchi [15] is defined at each point of the domain, which is a unit cell with a rectangular hole inside (Fig. 1). The use of microstructure allows the intermediate materials rather than only void or full material in the final solution. The design variables are the dimensions α, β and the orientation θ of the micro-hole. In this sense the problem is to optimize the material distribution in a perforate domain with infinite, infinitesimally small microscope voids. The effective properties of the porous material, for instance, effective material stiffness coefficient matrix $\mathbf{E}^{\mathrm{H}}$ and effective thermal conductivity vector $\boldsymbol{\lambda}^{\mathrm{H}}$, are calculated by following the homogenization procedure [16].

There are also many other methods of topology optimization, such as solid isotropic material with penalization (SIMP) or power law method [17,18], controlled perimeter method [19] and newly emerged discrete and evolutionary method [20,21]. However, many applications have proved that HMD is still a valid and effect topology optimization technique. In this research, HMD is used with square holes ($\alpha = \beta = x$) microstructure because of the requirement of isotropic thermal conductivity to avoid extreme anisotropic thermal property.

2.2. Design of compliant mechanisms

Compliant mechanisms can be characterized into two categories: partially compliant mechanisms and fully compliant mechanisms [22]. The design synthesis of partial ("lumped") compliant mechanisms has been developed as undulating elastica approach [23], kinematics synthesis approach [24] and more recently pseudo-rigid body model approach [25], while the fully ("distributed") compliant mechanisms is design using continuum synthesis approach based on topology optimization for solid structures [26,27] and truss-like structures [28,29]. Different formulations of optimization objective

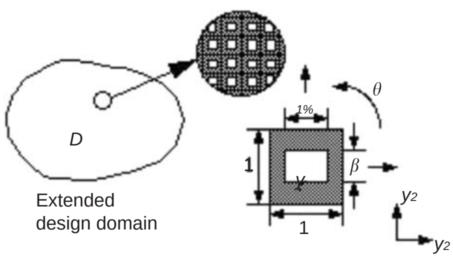

Fig. 1. Microstructure proposed in Homogenization Design Method.

have been used including output displacement, output work, mechanical advantages and multiple objective of energy.

For application of motion transfer/amplifier, compliant mechanisms are required to have flexibility to deform or transform motion. Large deformation is usually expected under functioning state. This consideration is included in pseudo-rigid body model with large deflection analysis of beams and other structural members using chain algorithms [30]. Topology optimization techniques have also been developed to reflect the importance of large displacements of solid structures [31,32] and extended for compliant mechanisms designs [33–35].

For force transfer/amplifier, compliant mechanisms can be also called as actuators. Due to the advantages of being lightweight, easy for assembly, free from joint bearing and wearing. Compliant mechanisms have been extensively used in MEMS. Micro-actuators are composed of a compliant mechanism plus an actuation source; therefore, the topology optimization techniques have been proposed to design piezoelectric actuators [36], thermal actuators [37,38] and other related multi-physics flextensional actuators [39].

In this research, the topology optimization problem is formulated in energy forms as in the previous research. The two design requirements, flexibility and stiffness, are formulated into energy format based on the principle of virtual work.

3. Topology optimization problem

3.1. Thermal actuation

In heat-activated compliant mechanisms, the deformation is caused by thermal expansion of material due to change of temperature distribution. The difference with thermal actuation is, instead of elastic strain energy, what the system stores is the thermal strain energy.

In this research we make use of the time transient effect of heat transfer to generate non-uniform temperature distribution, which means a certain time period, must be specified before the system reaches the steady state. Let t_f denote the actuation time specified, and its value is selected so that the design domain has a diversity of temperature distribution and significant temperature gradient. The strain–stress relation with temperature change can be written as

$$\boldsymbol{\varepsilon} = \mathbf{E}^{-1}\boldsymbol{\sigma} + \boldsymbol{\alpha}\Delta\phi(t_f), \tag{1}$$

where $\mathbf{E}$ is the material stiffness coefficient matrix and $\boldsymbol{\alpha}$ is thermal expansion coefficient in vector form and $\Delta\phi(t_f)$ denotes the change of temperature distribution $\phi(t)$ at actuation time t_f obtained from heat transfer equation [40]:

$$\rho c\dot{\phi} = \nabla \cdot (\lambda\nabla\phi)) + Q, \tag{2}$$

where ρ is material density, c is heat capacity coefficient, λ is heat conductivity coefficient and Q is heat source distribution. In addition, initial and boundary conditions are needed to achieve the solution. This equilibrium equation can be expressed in a integration form according to the principle

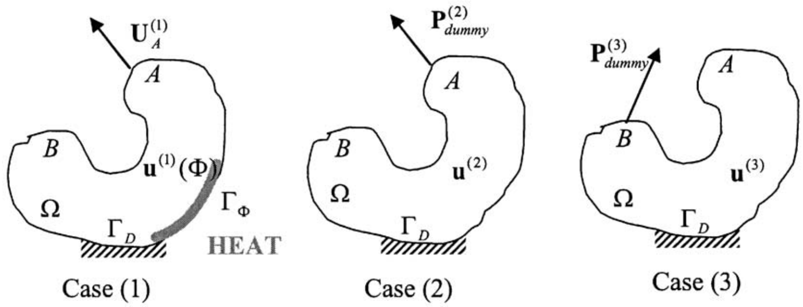

Fig. 2. Design requirements of compliant mechanism.

of virtual work:

$$\int_\Omega \boldsymbol{\varepsilon}^{\mathrm{T}}(\delta\mathbf{u})\boldsymbol{\sigma}\,\mathrm{d}\Omega = \int_\Omega \boldsymbol{\varepsilon}^{\mathrm{T}}(\delta\mathbf{u})\mathbf{E}\{\boldsymbol{\varepsilon}(\mathbf{u}) - \boldsymbol{\alpha}\Delta\phi\}\,\mathrm{d}\Omega = 0, \tag{3}$$

where $\mathbf{u}$ is the displacement vector, $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{\sigma}$ is stress tensor, $\mathbf{E}$ is the material property tensor. If strain is related to displacement as

$$\boldsymbol{\varepsilon} = \tfrac{1}{2}(\nabla\mathbf{u} + (\nabla\mathbf{u})^{\mathrm{T}}) = L\mathbf{u}, \tag{4}$$

where L denotes a linear differential operator. $\mathbf{f}_{\mathrm{t}} = \mathbf{E}\boldsymbol{\alpha}\Delta\phi L$ can be used to simplify Eq. (3) and can be regarded as an differential operator of equivalent thermal body force. The equation then can be written in a more general form:

$$\int_\Omega \boldsymbol{\varepsilon}^{\mathrm{T}}(\delta\mathbf{u})\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u})\,\mathrm{d}\Omega = \int_\Omega \mathbf{f}_{\mathrm{t}}^{\mathrm{T}}\delta\mathbf{u}\,\mathrm{d}\Omega. \tag{5}$$

3.2. Problem formulation

The topology optimization of compliant mechanisms with heat actuation shares the similar design criteria as the general compliant mechanisms. Two types of design criteria are considered to formulate an optimization problem: flexibility requirement and stiffness requirement. The flexibility requirement, also called mechanisms requirement means the designed compliant object must be deformed in a favorable manner to complete its functionality. Mathematically this requirement can be captured by using the concept of mutual mean compliance [41,42] based on the reciprocal theorem for linear elasticity.

Consider the loading cases shown in Fig. 2. If displacement field in case (1), $\mathbf{u}^{(1)}$, caused by temperature change is used as the virtual displacement for equilibrium equation under the dummy loading case (2), the principle of virtual work can be expressed as the following:

$$\int_\Omega \boldsymbol{\varepsilon}^{\mathrm{T}}(\mathbf{u}^{(1)})\boldsymbol{\sigma}^{(2)}\,\mathrm{d}\Omega = \int_\Omega \boldsymbol{\varepsilon}^{\mathrm{T}}(\mathbf{u}^{(1)})\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}^{(2)})\,\mathrm{d}\Omega = \mathbf{P}^{(2)^{\mathrm{T}}}\mathbf{u}_{\mathrm{A}}^{(1)}. \tag{6}$$

And if $\mathbf{u}^{(2)}$ is used as the virtual displacement for case (1), we have

$$\int_\Omega \boldsymbol{\varepsilon}^{\mathrm{T}}(\mathbf{u}^{(2)})\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}^{(1)})\,\mathrm{d}\Omega = \int_\Omega \mathbf{f}_{\mathrm{t}}^{\mathrm{T}}(\Delta\phi^{(1)})\mathbf{u}^{(2)}\,\mathrm{d}\Omega. \tag{7}$$

The bilinear form $a(\mathbf{u}^{(1)},\mathbf{u}^{(2)}) = \int_\Omega \boldsymbol{\varepsilon}(\mathbf{u}^{(1)})\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}^{(2)})\,\mathrm{d}\Omega$ defines the mutual mean energy between case (1) and case (2). If $P^{(2)}$ is a unit dummy load, this mutual mean energy equals to the displacement at A, $\mathbf{u}_\mathrm{A}^{(1)}$, in the desired direction due to temperature change.

The stiffness requirement provides the compliant mechanisms with the internal ability to resist an external loading. Mathematically it can be formulated as the well-know concept in structural optimization, compliance. Consider loading case (3) in Fig. 2, if an external force is to be loaded to the mechanisms, in the form of resistive force or functioning disturbance force (as discussed in later section), the stiffness required to sustain the loading is characterized by the displacement at the loading location. Using the same displacement field as the virtual displacement:

$$a(\mathbf{u}^{(3)},\mathbf{u}^{(3)}) = \int_\Omega \boldsymbol{\varepsilon}^\mathrm{T}(\mathbf{u}^{(3)})\mathbf{E}\boldsymbol{\varepsilon}(\mathbf{u}^{(3)})\,\mathrm{d}\Omega = \mathbf{P}^{(3)^\mathrm{T}}\mathbf{u}_\mathrm{B}^{(3)}. \tag{8}$$

The directions and locations of required flexibility, as well as those of required stiffness need to be determined according to individual design problem. General rules usually are applied as the following:

(1) For force transfer type mechanisms, the required flexibility is in the direction of desired output forces at the actuation location, while required stiffness is in the direction opposite to the output force at the same location.
(2) For motion transfer type of mechanisms, the required flexibility is in the direction of desired output displacement at the desired location, while the required stiffness needs not to necessarily the similar direction or location. It can be at the stiffness resistive to input motion (or thermal expansion), or any other locations and directions the specified mechanism requires.

The multi-objective optimization problem is formulated as

$$\begin{aligned}
\max_{\delta(\mathbf{x}),\theta(\mathbf{x})} \quad & f = \frac{a^\mathrm{H}(\mathbf{u}^{(1)},\mathbf{u}^{(2)})}{a^\mathrm{H}(\mathbf{u}^{(3)},\mathbf{u}^{(3)})} \\
\text{s.t.} \quad & a^\mathrm{H}(\mathbf{u}^{(1)},\mathbf{u}^{(1)}) = \int_\Omega \mathbf{f}_\mathrm{t}^\mathrm{H}(\Delta\phi^{(1)})\cdot\mathbf{u}^{(1)}\,\mathrm{d}\Omega, \\
& \Delta\phi^{(1)} = \phi(t_f) - \phi_0, \\
& \int_\Omega \nabla(\varphi)\cdot\boldsymbol{\lambda}^\mathrm{H}\cdot\nabla\phi = \int_\Omega \varphi\cdot\rho^\mathrm{H} c\dot{\phi}\,\mathrm{d}\Omega, \quad \forall\varphi, \\
& a^\mathrm{H}(\mathbf{u}^{(2)},\mathbf{u}^{(2)}) = L^{(2)}(\mathbf{u}^{(2)}), \\
& a^\mathrm{H}(\mathbf{u}^{(3)},\mathbf{u}^{(3)}) = L^{(3)}(\mathbf{u}^{(3)}), \\
& a^\mathrm{H}(\mathbf{u}^{(1)},\mathbf{u}^{(2)}) = \int_\Omega \mathbf{f}_\mathrm{t}^\mathrm{H}(\Delta\phi^{(1)})\cdot\mathbf{u}^{(2)}\,\mathrm{d}\Omega, \\
& V = \int_\Omega (1-\delta^2)\,\mathrm{d}\Omega \leqslant \bar{V}, \\
& 0 \leqslant \delta \leqslant \bar{\delta} < 1,
\end{aligned} \tag{9}$$

where $\bar{V}$ is the volume constraint and x is the size of the square micro-void in proposed microstructure, which is a measure of density.

4. Numerical implementation and optimization procedure

4.1. Finite element analysis

In HMD, the extended design domain is discretized into finite elements such that each element corresponds to an independent microstructure (design variable). No need for mesh generation at each iteration is one of the most important advantages of topology optimization.

Ignoring the coupling effect between thermal material properties and mechanical stresses, the evaluation procedure during the topology optimization process is a sequential thermal-mechanical analysis conducted via finite element simulation. The finite element equilibrium equations that need to be solved are [40,43]:

$$\mathbf{C}\dot{\boldsymbol{\Phi}} + \mathbf{K}_{\mathrm{t}}\boldsymbol{\Phi} = \mathbf{P}_{\mathrm{t}},$$
$$\mathbf{K}\mathbf{U}^{(i)} = \mathbf{F}^{(i)}, \quad i = 1, 2, 3, \tag{10}$$

where $\mathbf{C}$, $\mathbf{K}_{\mathrm{t}}$ and $\mathbf{P}_{\mathrm{t}}$ are heat capacity matrix, stiffness matrix and heat source vector, respectively, in time-transient heat transfer finite element analysis, while $\mathbf{F}^{(i)}$ is force vector in mechanical analysis for the three loading cases described earlier. In particular, $\mathbf{F}^{(1)} = \mathbf{F}_{\mathrm{t}}$ is the equivalent thermal force vector in thermal-stress analysis, calculated from temperature distribution result $\boldsymbol{\Phi}(t_f)$ as suggested in Eq. (5). For example, for general 2D problem:

$$\mathbf{F}_{\mathrm{t}} = \int_{\Omega} \mathbf{B}^{\mathrm{T}}\mathbf{E}\boldsymbol{\alpha}\Phi[1, 1, 0]^{\mathrm{T}}\,\mathrm{d}\Omega = \mathbf{A}\boldsymbol{\Phi}, \tag{11}$$

where $\mathbf{A}$ denotes the transformation matrix between nodal temperature and nodal equivalent thermal force, while $\mathbf{B}$ is the strain–displacement matrix calculated from relationship (2). It should also be noticed that the stiffness matrices are calculated from the homogenized material properties $\mathbf{E}^{\mathrm{H}}$ and $\boldsymbol{\lambda}^{\mathrm{H}}$. The backward finite difference is used as the time integration scheme for transient heat transfer analysis.

4.2. Sensitivity analysis

During topology optimization, the updating of design variable causes variations of material density and homogenized material properties. However, thermal expansion coefficient and specific heat parameter does not change because single material is used.

In discrete form, the bilinear forms of mutual mean compliance (6) and mean compliance (8) can be expressed as $\mathbf{U}^{(i)^{\mathrm{T}}}\mathbf{K}\mathbf{U}^{(j)}$. Taking derivatives of the finite element equilibrium equations with respect to design variable x, and using the symmetric properties of the stiffness matrix $\mathbf{K}$, the sensitivity of this energy form with respect to a design variable x is

$$\frac{\partial}{\partial x}(\mathbf{U}^{(i)^{\mathrm{T}}}\mathbf{K}\mathbf{U}^{(j)}) = -\mathbf{U}^{(i)^{\mathrm{T}}}\frac{\partial \mathbf{K}}{\partial x}\mathbf{U}^{(j)} + \frac{\partial \mathbf{F}^{(i)^{\mathrm{T}}}}{\partial x}\mathbf{U}^{(j)} + \mathbf{U}^{(i)^{\mathrm{T}}}\frac{\partial \mathbf{F}^{(j)}}{\partial x}. \tag{12}$$

According to definition of finite element matrices:

$$\frac{\partial \mathbf{K}}{\partial x} = \int_{\Omega} \mathbf{B}^{\mathrm{T}} \frac{\partial \mathbf{E}}{\partial x} \mathbf{B} \, \mathrm{d}\Omega,$$

$$\frac{\partial \mathbf{F}_{\mathrm{t}}}{\partial x} = \int_{\Omega} \mathbf{B}^{\mathrm{T}} \frac{\partial \mathbf{E}}{\partial x} \alpha \Phi [1,1,0]^{\mathrm{T}} \, \mathrm{d}\Omega + \int_{\Omega} \mathbf{B}^{\mathrm{T}} \mathbf{E} \alpha \frac{\partial \Phi}{\partial x} [1,1,0]^{\mathrm{T}} \, \mathrm{d}\Omega$$

$$= \frac{\partial \mathbf{A}}{\partial x} \boldsymbol{\Phi} + \mathbf{A} \frac{\partial \boldsymbol{\Phi}}{\partial x}. \tag{13}$$

From time transient heat transfer equation, the sensitivity of temperature field can be solved as

$$\mathbf{C} \frac{\partial \dot{\boldsymbol{\Phi}}}{\partial x} + \mathbf{K}_{\mathrm{t}} \frac{\partial \boldsymbol{\Phi}}{\partial x} = -\frac{\partial \mathbf{C}}{\partial x} \dot{\boldsymbol{\Phi}} - \frac{\partial \mathbf{K}_{\mathrm{t}}}{\partial x} \boldsymbol{\Phi} + \frac{\partial \mathbf{P}_{\mathrm{t}}}{\partial x}. \tag{14}$$

If the derivatives are calculated with direct method, i.e. through (14), such an equation needs to be solved for each design variable and sensitivity analysis is extensively time-consuming. However, an adjoint [44] vector $\boldsymbol{\Lambda}(t)$ can be introduced as

$$\int_0^{t_f} \boldsymbol{\Lambda}^{\mathrm{T}} \left(\mathbf{C} \frac{\partial \dot{\boldsymbol{\Phi}}}{\partial x} + \mathbf{K}_{\mathrm{t}} \frac{\partial \boldsymbol{\Phi}}{\partial x} \right) \mathrm{d}t = \int_0^{t_f} \boldsymbol{\Lambda}^{\mathrm{T}} \left(-\frac{\partial \mathbf{C}}{\partial x} \dot{\boldsymbol{\Phi}} - \frac{\partial \mathbf{K}_{\mathrm{t}}}{\partial x} \boldsymbol{\Phi} + \frac{\partial \mathbf{P}_{\mathrm{t}}}{\partial x} \right) \mathrm{d}t. \tag{15}$$

After integration by parts, it can be proved if $\boldsymbol{\Lambda}$ satisfies:

$$\boldsymbol{\Lambda}(t_f) = \mathbf{0},$$

$$-\mathbf{C}\dot{\boldsymbol{\Lambda}} + \mathbf{K}_{\mathrm{t}} \boldsymbol{\Lambda} = \delta(t_f) \mathbf{A} \mathbf{U}. \tag{16}$$

Therefore, the second term of sensitivity of mutual mean compliance $(\partial \mathbf{F}_{\mathrm{t}}/\partial x)\mathbf{U}^{(2)}$ can be obtained from:

$$\mathbf{A} \left. \frac{\partial \boldsymbol{\Phi}}{\partial x} \right|_{t_f} \mathbf{U}^{(2)} = \mathbf{U}^{(2)^{\mathrm{T}}} \mathbf{A} \left. \frac{\partial \boldsymbol{\Phi}}{\partial x} \right|_{t_f} = \int_0^{t_f} \boldsymbol{\Lambda}^{\mathrm{T}} \left(-\frac{\partial \mathbf{C}}{\partial x} \dot{\boldsymbol{\Phi}} - \frac{\partial \mathbf{K}_{\mathrm{t}}}{\partial x} \boldsymbol{\Phi} + \frac{\partial \mathbf{P}_{\mathrm{t}}}{\partial x} \right) \mathrm{d}t. \tag{17}$$

This sensitivity analysis only requires one time-transient analysis equation to be solved at each iteration. It is significantly timesaving compared to the direct method.

4.3. Optimization procedure

Fig. 3 shows a flow char of optimization procedure. Sequential linear programming (SLP) is used as an optimizer. One iteration consists of, in sequence, a time-transient heat transfer analysis, and a thermal–mechanical analysis and sensitivity analysis to both finite element procedures. The termination criteria are convergence of objective function or a maximum iteration time.

5. Examples

Two numerical examples are presented in this section. Only 2D problems are presented. The material properties used in these examples are chosen as be close to those of a common plastic

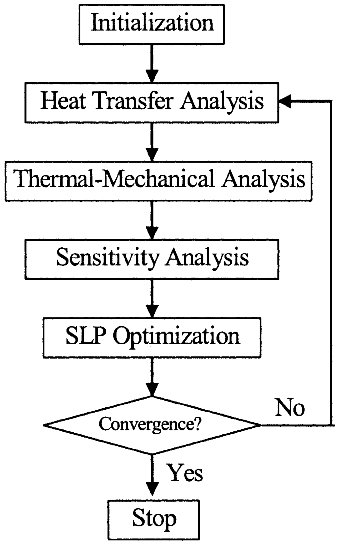

Fig. 3. Optimization procedures.

material polypropylene: density $\rho = 930\ \mathrm{kg/m^3}$, thermal conductivity $\lambda = 0.22\ \mathrm{J(s\,m\,K)^{-1}}$, specific heat $c = 1.9\mathrm{E}3\ \mathrm{J(kg\,K)^{-1}}$, Young's modulus $E = 1.1$ GPa at room temperature 20°C, Poison's ratio $v = 0.45$, thermal expansion coefficient $\alpha = 8.0\mathrm{E} - 5\ \mathrm{K^{-1}}$. Boundary heating applied is the constant temperature of 100°C. More consideration of realistic material properties and boundary conditions must be taken while dealing practical design needs. In order to save computational cost, for both examples, design domains are discretized using relatively coarse mesh (20×20).

The heating time t_f are chosen to cause sufficient temperature difference for bi-morph deformation. It cannot be either too short or too long. If it is too short, enough thermal expansion is not enough. It t_f is too long, the mechanism is over heated and global expansion may be achieved. Therefore, in order to decide on an appropriate heating time, several trial runs should be conducted.

5.1. A thermal actuator

This example is to illustrate the described topology optimization algorithm is efficient to generate non-intuitional designs. In the square design domain as shown in Fig. 4(a), a thermal actuator type of compliant mechanism is to be designed to generate an output force. Due to the specified boundary constraints, if the whole region is subjected to a uniform temperature change, the profile of the expanded design domain is shown in Fig. 4(b). However, with transient effect, the desired output can be generated.

The design domain is $10\ \mathrm{cm} \times 10\ \mathrm{cm}$. A heating time is chosen to be 600 s. The problem is formulated to maximize flexibility in the direction of desired output force and stiffness in the opposite direction as discussed in Section 3.2.

Optimal configuration in Fig. 5(a) shows that the optimization procedures have redistribute material in order to form the temperature field in favor of design requirements. Fig. 5(c) is the contour plot of the temperature distribution of the whole design domain along with the deformed pattern. From the

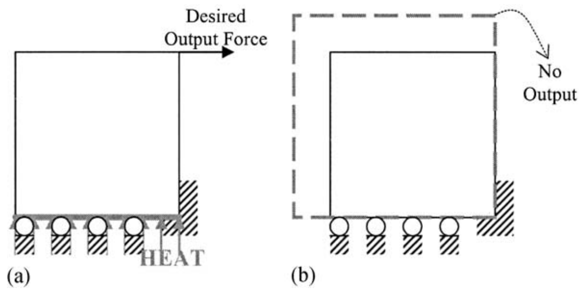

Fig. 4. Example 1 (a) design domain with boundary conditions; (b) deformation pattern of uniform temperature change.

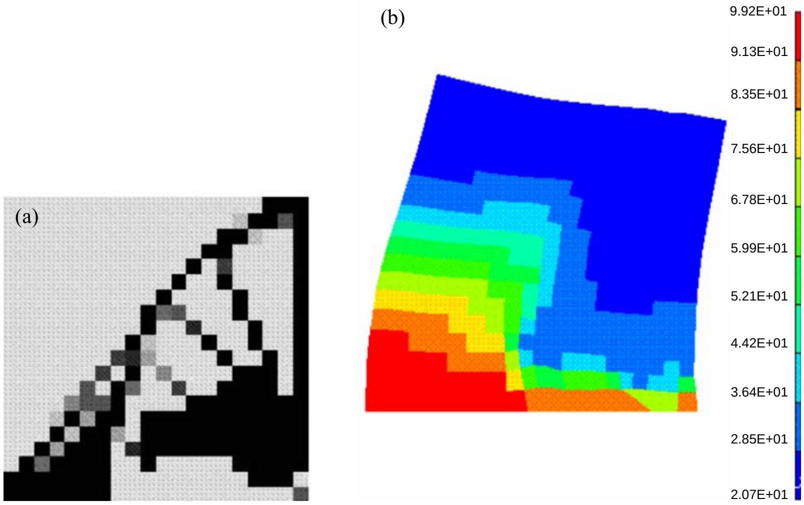

Fig. 5. Design results (a) optimal configuration; (b) deformation and temperature distribution.

deformation shown, an output force in the specified direction can be expected to verify the purpose of design.

5.2. Design of snap-fit mechanisms

To demonstrate a result of the proposed design technique, a numerical result related to practical application is presented in this section. Integral snap-fit attachments [45–48] have been widely used as substitutes for separate fasteners for the purpose of design for assembly (DFA) [49,50]. However, snap fits are not necessarily a favored choice fore design for disassembly since they are often difficult to disengage without inherent destruction of the components [51]. While some snap-fits are designed to be reversible (e.g., battery covers for cellular phones), they require the application of auxiliary forces in a direction different from the insertion deliration in order to unlatch the snapping features.

A snap-fit mechanism can be engaged by simply pushing the two counter parts together, while disengaging is a problematic process. Automatic application of disengaging force is challenging task.

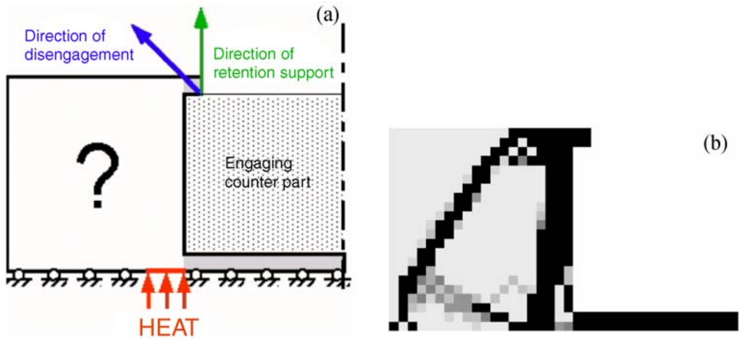

Fig. 6. Design domain and result (a) design domain; (b) topology optimization result.

Fig. 7. Verification of result.

Using heating as disengaging actuation can be an advantage. Furthermore, a uniform temperature change or steady-state temperature distribution takes time to obtain, and also tend to generate excessive thermal expansion, but a limited actuation time is necessary and realistic. Besides, the control of heating time is trivial in practice. Therefore, the design of snap-fit mechanisms can be formulated as topology optimization problem considering time-transient heat transfer. As discussed in Section 3, two design criteria are considered, as illustrated in Fig. 6(a) (only half of the symmetric geometry is considered). The flexibility requirement is that when the object is heated as specified, the induced deformation should be in favor of disengagement of the snap-fit. The stiffness requirement is to provide retention force, i.e. the undesired disengagement.

Topology optimization result of the design domain is shown in Fig. 6(b). Although there are some intermediate density values visible in the result, the basic layout can be extracted and easily understood by intuition. This configuration is post-processed by extracting basic geometric elements revealed by topology optimization. The simplified model is verified using commercial software ABAQUS. Both deformation pattern and temperature distribution are shown in Fig. 7. The external heating is applied, the vertical bar becomes heated quickly and expands more than the slanted bar, which causes the hangover part to be lifted in the upper and outer direction. At the same time, the expansion of bottom horizontal bar further push the locking members aside. Even though the engaging counter part is also expected to be heated and expand, the expected lower temperature distribution make the thermal expansion less signification. Furthermore, the triangular shape of the structure provides large stiffness for retention support.

When a height of 10 cm and a heating time of 500 s are considered for this snap mechanism, the outward deflection at the tip of hangover is about 0.2 mm. It is in the same order of magnitude as the achieved deflection in earlier example.

6. Discussions and conclusions

In this research, topology optimization synthesis for design heat-activated compliant mechanisms considering time-transient effect has been proposed. Examples have shown that novel compliant mechanisms of macro-scale can be designed.

Since relatively coarse mesh are used in the above examples, the optimized density results may not be absolutely clear-structured. However, in this research topology optimization results are only intended to serve as inspiration to practical designs. Therefore, as long as the key elements of the new design can be revealed in topology optimization configurations, effective post-processing measures are essential to the formation of final designs, as demonstrated in Example 2.

The design proposed in this paper has its certain advantages over the existing type thermal actuators:

(1) There is no restriction to material properties as long as the material has an appreciated thermal expansion coefficient. This type of thermally actuated mechanisms can be made from any type of material.
(2) Usually when a mechanism is in function, it would have a physical contact with another system components. To avoid high temperature at the external contact portion, Yin et al. [12] proposed to use two different materials. However, in design synthesis proposed in this research, the desired output portion is usually placed away from the heating location, and high-temperature change is automatically avoided.
(3) Compared to the embedded electro-thermal actuation idea, it is believed that the presented designs are less sensitive to post design modification. This is because density change only affects temperature distribution in this thermal conductivity based design synthesis, while in ETC it also affects electric current density which determines the heat source. Therefore even minor modification to ETC design can vary temperature distribution significantly due to the multiple effects.
(4) While designing relatively large-scale mechanisms, this design synthesis saves time by allowing system to function during an actuation time instead of waiting for steady state.
(5) Since the transient thermal actuation requires specific heating time and location, the incurrence of undesired actuation caused by environments can be avoided.

There are also some limitations associated with time-transient effect. For instance, for micro-scale system, the steady-state constant can be very small, which is not ideal for utilizing time-transient temperature distribution. This will limit the majority of applications of time-transient thermal actuators to the macro-scale systems. Secondly, as mentioned earlier, the achieved desired displacement is about 0.2–0.5%, which is not sufficient for real application. This is associated with the nature of material thermal property. However, improvements can be achieved using more flexible boundary and loading conditions in combined with improved optimization scheme.

Since thermal actuation has attracted more and more attention in practical applications, proposed design proposed and presented results in this paper are expected to be constructive towards novel designs of mechanical systems.

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