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AN INVESTIGATION OF SUSTAINABILITY, PREFERENCE, AND PROFITABILITY IN DESIGN OPTIMIZATION

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ABSTRACT

Customer preferences for sustainable products are dependent upon the context in which the customer makes a purchase decision. This paper investigates a case study in which fifty-five percent of survey customers say they prefer recycled paper towels, but do not purchase them. These customers represent a profit opportunity for a firm. This paper explores the impact of investing capital in activating pro-environmental preferences on a firm's profitability and greenhouse gas (GHG) emissions through a multi-objective optimization study. A product optimization is designed to include models of carbon dioxide emissions, manufacturing costs, customer preference, and technical performance. Because the optimization includes a tradeoff between recycled paper and performance, a model of customer preferences, and a market of competing products, the maximum GHG reduction occurs at less than 100% recycled paper. Also, the tradeoff between GHG reductions and profit is not dictated by the configuration of the product, but instead by its price. These results demonstrate the importance of including customer preferences with engineering performance in design optimization. Investment in the activation of pro-environmental preferences is high at all points on the Pareto optimal frontier, suggesting that further engineering design research into the activation of pro-environmental product preferences is warranted.

1 INTRODUCTION

The "usual suspects" in product optimization models that consider both engineering design and environmental sustainability include manufacturing costs, material costs, engineering constraints and environmental impact metrics. Recently, another set of parameters has been appearing: the representation of customer preference [see 1, 2]. The inclusion of a model and its associated parameters describing customer preference allows researchers to explore how to configure products such that they are not only sustainable in the traditional sense, but also desirable in the market, which is an important but neglected aspect of sustainability. A design optimized under traditional sustainable criteria that customers do not purchase is of no value. Customer preferences are typically represented with utility, a measure of customer preference for a given product built from configurations of possible product attributes [see 3, 4]. In some utility models, researchers use existing market data to estimate utility. In other models, researchers conduct a customer survey on potential product configurations, and use the results of this survey to estimate utility. While the former is based on existing product design, the latter (or a hybrid approach) can use a design of experiment approach to disentangle various contributors to preference and can be used to assess novel product designs [3].

This approach has led to important theoretical advances in the design and optimization of products [see 5, 6, 7] but parameterizing customer preference as utility does have limitations. For example, utility estimates from market data are

limited to incremental extrapolations on currently available products and may not accurately predict preference for an innovative product [3]. Estimates from surveys can be limited by the ability of respondents to imagine and assess their preference for a product choice scenario, and the complexities of hypothetical decision-making [3,8]. This paper discusses one such complexity that is particularly important for sustainable design: the frequent expression of pro-environmental preferences (PEP) on surveys that do not translate into pro-environmental behavior (PEB) in real choice situations. While there are a number of possible explanations for this behavior [9], this article adopts the premise that PEP elicited from surveys can translate into a real-world market behavior, under the right conditions.

Research in psychology provides evidence that customer decisions, including pro-environmental behavior, can be influenced by the specific context of the choice situation, including factors such as purchase incentive programs, design features that encourage sustainable thinking, environmental labeling, and sustainability education [see 9, 10, 11, 12]. A firm that invests in developing these context factors may boost demand for sustainable products.

This paper explores the tradeoff between maximizing profit and greenhouse gas (GHG) reductions for firm decisions on product design, price, and investment in PEB. The design of paper towels is used as a case study. The effects of recycled paper pulp (RPP) on the product attributes of strength, softness, and absorbency are captured in an empirical engineering model. Section 2 and 3 give a background on relevant literature in behavioral psychology and engineering design. Section 4 describes the methodology, including a description of the case study, the optimization formulation, and the models of customer preference and product performance. Results are described in Section 5, and Section 6 presents conclusions.

2 BACKGROUND

2.1 Construction of preference for sustainable products

In engineering design, customer preferences are often treated as existing a priori to the product decision at hand, such as a customer purchase decision or use decision. This treatment is evident in metaphors for the study of preferences such as “capturing preferences,” “eliciting preferences” or “finding user needs.” For some products, this a priori portrayal of preference is valid; for example, a customer will always need a drinking glass to hold liquid without dripping. However, more nuanced and complex product preferences can change with the context of a product decision due to construction of preference, a theory from behavioral psychology that asserts that individuals construct preferences on a case-by-case basis when called to make a decision [13]. Preference will be different in different decision contexts if the context leads to a different construction process.

Context effects are particularly prevalent in public-good decisions, such as those regarding sustainability [see 14, 15, 16]. For example, duck hunters were asked to put a monetary value to the destruction of a duck habitat. When the decision was framed as a payment the hunters would make for restoring the habitat, the hunters were willing to pay \$247. When the same decision was framed as a loss, a compensation for the destroyed habitat, duck hunters requested \$1044. Both decisions represent the utility of the duck habitat, but the values are different depending on the context of the decision [17]. These different values could lead to different policy implications. Similarly, customer preference for sustainable product attributes varies across decision context. Social desirability bias, a propensity for people to answer a survey in a manner congruent to “good” social behavior, leads to inflation of willing-to-pay measures and also possibly exaggerated self-report of PEB [18]. Social desirability bias provides an easy explanation for the PEPs displayed in survey results, but there are many explanations for these PEPs. For example, embedding effects occur because it is difficult to nest willingness-to-pay measures for a group of purchases. For example, it may be that a respondent states that they are willing to pay a fifty-cent premium for a recyclable yoghurt container, when in fact they are willing only to add fifty cents to their weekly shopping bill to purchase sustainable goods [14].

MacDonald [9] presents a review of cognitive concepts that affect preference for sustainable products, some of which will affect construction of preference in survey responses. For example, there is evidence that customers do not trust the quality of sustainable products. In a hypothetical survey environment, where the customers have no reason to question the motives of the researchers, they may be more likely to trust information given on the quality of the sustainable product, and therefore express higher preferences for it. Product trust and sense of responsibility can be addressed by a product in the real-world. To address trust, a design could use emotional design or product semantics [see 19, 20, 21] to evoke feelings of trust in the customer. A firm could combine this with an excellent warranty program, and a customer advocacy marketing approach [22] to recreate the sense of trust in product quality that respondents felt in the survey.

2.2 Customer preferences for sustainable products in engineering design

Customer demand or preferences have been included in engineering models of product sustainability in a number of ways. Utility theory has been blended with product design optimization to study the design impacts of environmental regulation [1, 23]. A systems and interdisciplinary perspective, including customer considerations, is recommended for both addressing and teaching sustainability in the literature [see 23, 24, 25, 26]. [2] develops metrics to measure the alignment of profitability and sustainability objectives, similar to the one discussed in this paper, in enterprise-level product design. [27] studies a system dynamics model of electric vehicle adoption,

in which customer preference changes over time through the effects of word of mouth, customer experiences, and marketing.

2.3 Variability in customer preference addressed in engineering design

Engineering design has previously addressed construction of preference as customer preference inconsistency via modeling random error, [7]. This approach is important to account for the uncertainty inherent in the fitting of respondent data to a preference model. But it is also important to account for factors that influence the decision context that firms have control over. This is demonstrated in the analysis below and in [27], in which customer preference changes systematically and dynamically based on firm and governmental decisions, allowing for the diffusion of alternative fuel vehicle adoption.

3 METHODOLOGY

3.1 Case Study Overview

In this article's case study, a hypothetical firm considers the design of a sustainable paper towel for household purchase. The firm has decided to address sustainability through the reduction of GHG emissions by including recycled paper pulp (RPP) in their product. It wishes to determine a mixture of virgin and recycled paper pulp that will garner a favorable combination of profitability and sustainability. The firm must consider the fact that this mixture will affect important towel properties: strength, softness, and absorbency, as discussed in Section 3.6. These properties will, in turn, affect the towel's market share. The firm looks to introduce a successful sustainable product that provides a healthy profit, as well as decreasing the amount of GHG emissions from manufacturing.

The firm is having difficulty measuring customer preference for RPP in towels. A customer survey, discussed in Section 3.4, suggests that there is a group of customers with a strong preference for towels that include some level of RPP, but these customers are not current purchasers of existing RPP towels. These individuals make good targets for future sustainable product customers because they already have pro-environmental attitudes, and only lack the corresponding behaviors; it is more difficult to change both attitudes and behaviors than it is to change only behaviors. The firm is considering undertaking the design and market launch of a towel that can "activate" these pro-environmental preferences (PEP) in this customer group and lead the customer to purchase. It may require a sizeable investment to do so, as modeled in the Section 3.5. The firm can manipulate the level of RPP in the towels, the price it charges, and the size of the investment in PEB.

The optimization of the firm's product, represented with the subscript k , has two objective functions: maximize GHG reductions, $-\Delta_{\text{GHG}}$, (equivalent to minimizing GHG emissions) and maximize firm profit, Π_k , as shown in Eq. 1.

$$\begin{aligned} & \max \{ \Pi_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k), -\Delta_{\text{GHG}}(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) \} \quad (1) \\ & \text{subject to: } \mathbf{0} \leq \mathbf{r}_k \leq \mathbf{1}, \$1.29 \leq \mathbf{p}_k \leq \$4.59, \mathbf{d}_k \geq \mathbf{0} \end{aligned}$$

The trade-off between these two objectives will be explored as a Pareto set [2] of results in Section 4. There are three variables, as mentioned above: the percentage of recycled paper pulp (RPP) in the towels, r_k , the retail price of a package of towels, p_k , and the amount of research and development (R&D) money spent on activating PEP, d_k . The model to be optimized includes representations of GHG emissions, revenue, manufacturing costs, pulp costs, customer preferences, and R&D costs. An overview of the model is presented in Fig. 1.

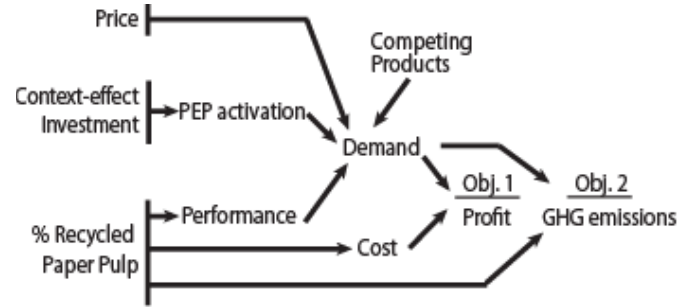


Figure 1. OVERVIEW OF OPTIMIZATION MODEL

3.2 Profit Maximization

Maximizing profit is the first objective in the dual-objective optimization problem. Profit is calculated as revenue, R_k , minus costs, C_k , for a single year (Eq. 2). Maximum profit will occur at the configuration of retail price, recycled paper content, and R&D spending that causes the largest difference between revenue and cost.

$$\Pi_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) = R_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) - C_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) \quad (2)$$

Revenue is the wholesale price of a package of towels (fifty-five percent of the retail price), μp_k , multiplied by the choice share of the product, S_k , and the market size, M , see Eq. 3. The retail price, a variable in the optimization, affects the percent choice share of the product via the customer preference model, and thus the number of packages of towels sold, MS_k (see Section 3.4).

$$R_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) = M\mu p_k S_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) \quad (3)$$

Costs consist of the manufacturing cost and the investment in PEB. The manufacturing cost is the production cost per towel package plus the cost of pulp (virgin and recycled) per package, both multiplied by the number of packages sold, see Eq. 4. Mark Lewis, an industry expert and Programs Operation Manager at University of Washington's School of Forest Resources, was consulted with to arrive at estimated production costs of \$0.50 per package of towels (7.9 square meters). There

is not a noticeable difference in the manufacturing costs with virgin or recycled pulps (assuming use of Northern Bleached Softwood Kraft for virgin pulp and De-Inked Pulp for recycled). The cost of recycled pulp is \$0.27 per pound, and the cost of virgin pulp is \$0.36 per pound, arrived at by combining Dr. Lewis' estimates with industry forecasts [28]. The cost of investing in PEB is discussed below in Section 3.5.

$$C_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) = S_k(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k)((1 - r_k)(\$0.36) + r_k(\$0.27) + \$0.50)M + \mathbf{d}_k \quad (4)$$

A market size of 1.1 billion packages of towels sold per year was calculated from sales data of \$2.2 billion in consumer paper towel sales in the United States in 2006 (excluding Wal-Mart) [29], and an estimated average price of \$2.00 per towel package. This market size is assumed to be fixed. Because the market size excludes Wal-Mart sales, it is conservative. A towel package, which may have one roll or several, is assumed to contain 7.9 square meters of towel, 1 lb of pulp, and approximately 150 sheets. The competing towel brands in the market are: Bounty, Brawny, Scott, Viva, Sparkle, Seventh Generation, and Green Forest. The strength, softness, absorbency, and price of these towel brands was measured at the University of Washington and is included in the appendix. Seventh Generation and Green Forest are 100% RPP, the rest are made from 100% virgin pulp.

3.3 GHG Reductions

The second objective is to maximize GHG reductions, $-\Delta_{\text{GHG}}$, resulting from the entry of the new firm. This objective is calculated by first determining the GHG emissions of all competing brands before the firm enters the market and subtracting them from the GHG emissions of the market after the firm enters. This approach was chosen because it accounts for the following factors: 1) the GHG emissions resulting from the firm's product, 2) emission reductions caused by customers switching from higher-emitting products to the firm's product, 3) emission increases from customers switching from lower-emitting products to the firm's product, and 4) reductions from customers switching from a higher-emitting competitor to a lower-emitting competitor due to investment in PEB.

Publically available data inventorying product lifecycle GHG emissions for virgin or recycled paper towels made in North America is sparse, but there are two studies of note. Franklin Associates Ltd., a lifecycle assessment (LCA) consulting firm, conducted a LCA for paper towels made from 100% virgin pulp and paper towels made from 100% recycled pulp [30]. Their results indicate that the recycled paper towels reduce lifecycle GHG emissions by 64% compared to the virgin paper towels. The second study, performed by Kimberly-Clark with review from Environmental Resource Management, compared lifecycle emissions for paper towels made from 40% recycled fiber and for virgin paper towels [31]. These results indicate that the recycled paper towels increase GHG emissions by 26% compared to the virgin paper towels. The authors chose

to use the Franklin Associates study for our analysis, believing that this study is potentially less biased given that Kimberly-Clark manufacturers Scott paper towels, which are made from 100% virgin pulp. Implications of using the Kimberly-Clark LCA results will be investigated in future work.

In order to represent GHG emissions as a continuous function of recycled paper content, a linear interpolation is taken between the two extremes of 18.415 kg CO₂-eq emissions per lb of 100% virgin pulp and 11.685 kg CO₂-eq emissions per lb of 100% recycled fiber. This linear representation is done largely for simplicity and lack of richer data. Still, because a large amount of GHG emissions come from lifecycle stages before the pulp streams are mixed—for example, in collecting, treating, and deinking the recycled pulp, and pulping the virgin pulp [30,31]—a linear representation may be a sufficient approximation.

Eqs. 5-7 describe the model of GHG reductions. GHG^0 are the GHG emissions produced by the towel market, 1.1 billion packages, without the firm's product (choice share divided amongst competitors). GHG are the emissions produced by the market with firm k in the market.

$$-\Delta_{\text{GHG}}(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) = -(GHG(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) - GHG^0) \quad (5)$$

$$GHG^0 = M \left(18.415 - 11.685 \sum_{j \neq k} r_j S_j^0 \right) \quad (6)$$

$$GHG(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k) = M(18.415 - 11.685 \sum_j r_j S_j(\mathbf{p}_k, \mathbf{r}_k, \mathbf{d}_k)) \quad (7)$$

More sophisticated approaches to modeling product sustainability would consider implications that are beyond the scope of this article [see 23, 24, 32]. For example, increasing the recycled paper content in paper towels may cause people to feel they can use them more liberally, therefore increase the number of towels used, thus partially or wholly negating the positive effects of including recycled paper content.

3.4 Customer Preference Model

The parameters of a latent class discrete choice model were estimated from the results of a survey on paper towels. The attributes (with levels/configurations in parentheses) included in the survey were: strength (1 out of 3, 2 out of 3, 3 out of 3), softness (1,2,3 out of 3), absorbency (1, 2, 3, out of 3), recycled paper content (0%, 30%, 60%, 100%), and price (\$1.29, \$2.39, \$3.49, \$4.59). The survey framework, analysis and results were previously reported in [33] and are explained extensively in [34]. The survey sample size of 217 individuals and the implementation of an out-of-the-box statistical analysis package are akin to a mini-study that a company would do to see if further investment of funds in a more sophisticated study is warranted.

Customer preferences are represented using a latent class model, which is a mixed-logit model with a discrete

distribution of preferences [3]. The probability that a survey respondent n chooses product j is calculated as:

$$P_{nj} = \sum_{l=1}^L g_{n,l}^0 \left(\frac{e^{v_{j,l}}}{\sum_i e^{v_{i,l}}} \right) \quad (8)$$

$$v_{j,l} = \sum_{\zeta,\omega} \beta_{\zeta\omega l} x_{j,\zeta\omega} \quad (9)$$

where v is the measurable portion of estimated utility; l is a latent class; L is the total number of latent classes; $\beta_{\zeta\omega l}$ is a part-worth given a latent class l for a product attribute ζ configured at level ω ; and $x_{j,\zeta\omega}$ is an index variable. The parameter $g_{n,l}$ represents the probability that respondent, n , is in latent class l . Choice share is estimated by averaging choice shares specific to the latent classes:

$$S_k(p_k, r_k, d_k) = \sum_{l=1}^L \left(\frac{\sum_{n=1}^N g_{n,l}(d_k)}{N} \right) \left(\frac{e^{v_{k,l}(p_k, r_k, d_k)}}{e^{v_{k,l}(p_k, r_k, d_k)} + \sum_{i \neq k} e^{v_{i,l}}} \right) \quad (10)$$

$$S_j(p_k, r_k, d_k) = \sum_{l=1}^L \left(\frac{\sum_{n=1}^N g_{n,l}(d_k)}{N} \right) \left(\frac{e^{v_{j,l}}}{e^{v_{k,l}(p_k, r_k, d_k)} + \sum_{i \neq k} e^{v_{i,l}}} \right) \quad (11)$$

There are many available models for describing heterogeneity in customer preference [see 3, 4]. This particular approach allows the model to account for heterogeneity of customer preferences while also distinguishing discretely between customers with pro-environmental preferences PEP and those without, which is integral to the implementation of the overall optimization model. The initial indication that there was a distinct group of customers with PEP came from responses to the customer survey. Near the end of the survey, respondents were asked to write-in a price for a paper towel that had no recycled paper content. If the respondent indicated that they would not purchase this towel for any price, no matter how low, they were asked to write-in an explanation as to why. Sixty respondents wrote-in responses for their refusal to buy that were driven by environmental concerns or the lack of recycled paper content, thus exhibiting strong PEP. The latent class framework allowed for the identification of a set of preferences for this group of respondents, as forty-eight of them are assigned to the same latent class, the Traders, discussed further below. The authors make an important distinction between a pro-environment attitude and a pro-environment preference. This study uses the machinery of decision models (willingness to pay, choice, data analyses that decompose utility into additive components, etc) so we are in the traditional domain of preference. Preferences as operationalized by choice are influenced by attitudes, but this study did not directly measure pro-environment attitudes.

Three different groups of customers (latent classes) exhibiting different “types” of customer preferences were

identified. The preferences for the three groups are detailed in Fig. 2: the higher the utility for a configuration, the stronger the preference. The y-axis scale is not absolute—negative numbers do not necessarily imply a non preferred attribute, and the utility values cannot be directly compared between the classes [4]. The three latent classes estimated are given names: “Savers,” the class concerned mostly with price; “PickUps,” the class concerned mostly with absorbency; and “Traders,” the class that trades-off between attributes.

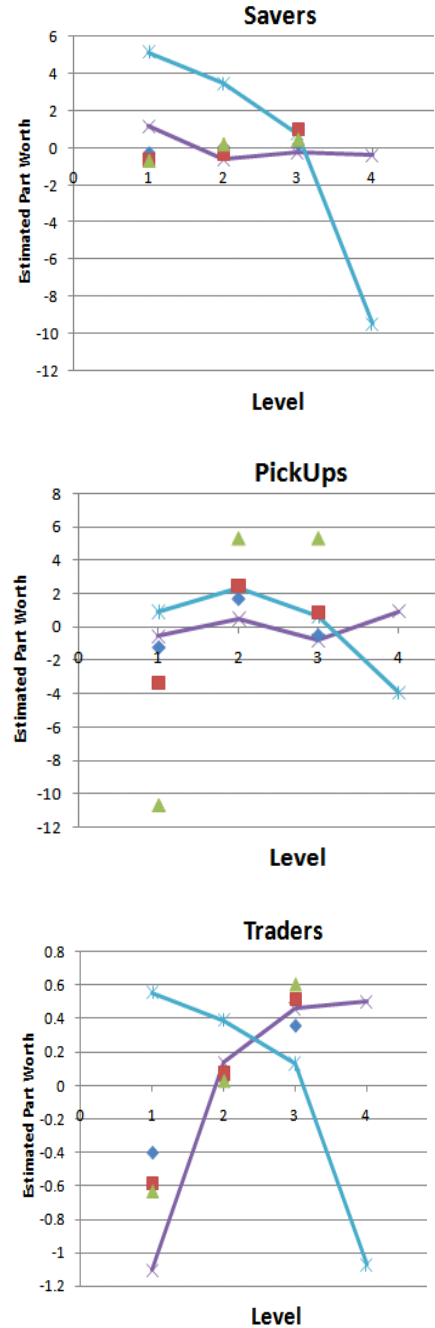


Figure 2. PART-WORTH UTILITIES OF PAPER TOWEL ATTRIBUTES FOR THREE LATENT CLASSES

The Traders exhibit PEP for recycled paper. Their estimated preferences indicate they were interested in buying towels with recycled paper content. However, only 2 out of the 127 respondents in this group purchased towels with recycled content the last time they went shopping. In another part of the survey, many Traders incorrectly reported the recycled paper content of the towels they purchased.

In the product design optimization model, strength, softness, and absorbency are represented as discrete utility categories, while levels for recycled paper content and price are modeled continuously; see the appendix for further model details. The discrete categories for product performance were created because the survey collected responses for three levels of each attribute with a corresponding quantitative description, *i.e.* “A rating of 1 out of 3 can absorb a 2.5 inch water spill (About the same size around as a tomato slice).” Modeling these preferences as continuous by interpolating between the utility values of the discrete categories may provide increased insight into product design and R&D investment strategy. This is left for future work as it will require linking qualitative customer measurements to quantitative values, for example via customer trials[see 35, 36].

It should be noted that an entirely different approach to incorporating the Trader’s preferences in the customer preference model is available. A hybrid model that combines revealed preference (existing market data) with stated preference (survey data) could model the discrepancy between the Trader’s PEP and their actual market behavior. In fact, this approach has been used for estimating preferences for “green” toilet paper [18]. There are some logistical problems with using this hybrid approach here, such as the fact that hardly any towels with recycled paper content are purchased – they accounted for approximately 2% of annual towel sales in 2005 [29]. The low sales volume is not only due to preference, but also to externalities such as low availability. There is also not a variety of attribute configurations available in these recycled towels—there are only two brands and neither have less than 100% recycled paper content. These types of conditions make a hybrid model difficult to fit [3]. But most importantly, it is not our intent to eliminate the discrepancy between survey and market preferences, but instead view it as an insight as to how the Traders could behave in the market, given an appropriate purchase context.

3.5 Activating Pro-environmental Preferences

In the latent class model of preference presented in above Section 3.4, each survey respondent was assigned a probability of behaving as a Trader, PickUp, or Saver. However, as previously discussed, the Traders did not buy towels with recycled paper content in the “real world,” despite the fact that their preference for recycled paper content was strong in the survey. There is thus an opportunity to activate the PEP that the Traders exhibited in the survey in the real world. The objective of activating these preferences is to create demand for recycled paper towels, to both reduce GHG emissions, and, potentially,

increase firm profit. The behavioral psychology field uses “activate” in conjunction with theoretical concepts like attitudes, stereotypes, and beliefs because of the underlying mechanism imposed by models of these concepts. We use the term “activate preferences” to refer to manipulations (such as marketing strategies, addition of product features, etc) that would help increase consistency between preference and actual behavior.

The firm hypothesizes that the PEP the Traders exhibited in the survey are attainable in the “real world” when the product decision is placed in the proper context. The ability to define and control this context is dependent on the amount of money the firm spends to research the activation of PEP. For example, the firm can design a marketing campaign that points out that other towels contain 0% recycled paper content, a fact that is not well understood according to the survey results [34]. They can address the possible fears of the inadequate technical performance of recycled paper towels, via an educational campaign or free trials. The firm can also provide purchase incentives to motivate a change in behavior. The firm can invest in developing design features, such as embossed recycled symbols, that may trigger sustainable product preferences at point of purchase [37, 38]. They can redesign product packaging to do the same. Previous studies on changing customer purchases to sustainable products suggest that a combination of approaches is best [see 9, 10, 11, 12]. For a discussion of such measures taken to promote the switch from incandescent to compact fluorescent light bulbs, a similar product, see [10, 12].

All of these approaches to activating PEP will require capital investment. In this model, PEB (pro-environmental behavior) R&D dollars spent is a variable in the optimization that impacts the Traders’ preferences. We introduce a parameter, κ , which represents the amount of survey PEP that translates to PEB in the marketplace. When the firm invests nothing in PEB R&D, κ is zero; and hence, the Traders do not exhibit preference for recycled paper towels in their purchasing decisions. Instead, they have an equal probability of behaving as a PickUp or a Saver. As more money is spent on R&D, κ increases, eventually to the maximum of one, where Traders behave consistently with their survey PEP.

The relationship between R&D investment and κ is represented as an s-curve, shown in Eq. 12-13 and Fig. 3. This accounts for the realistic costs of effective programs (below a certain level spending, little can be studied and there can only be little change in preference) and diminishing returns on programs that are extremely costly. Eq. 13 links κ to the latent class probabilities, $g_{n,l}$, used to compute choice shares.

The general relationship between investment and κ is based on industry estimates of such R&D investments, but the specific equation used in this paper was chosen to be illustrative. The relationship between R&D investment and the resulting activation of PEB is represented here without uncertainty, which is noted as a large assumption. Adding uncertainty to the relationship between R&D investment and

resulting PEB activation would improve the representation of the realities of such R&D efforts.

$$\kappa(d_k) = 0.99 - \frac{0.99}{1 + \exp\left\{\frac{d_k}{10^6} - 3\right\}} \quad (12)$$

$$g_{n,l}(d_k) = \begin{cases} \left(\frac{1 - \kappa(d_k)}{2}\right) g_{n,Traders}^0 + g_{n,l}^0 & l = Savers, PickUps \\ \kappa(d_k) g_{n,Traders}^0 & l = Traders \end{cases} \quad (13)$$

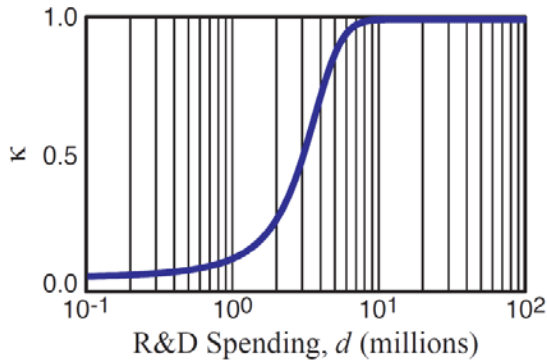


Figure 3. RELATIONSHIP BETWEEN INVESTMENT IN PEB AND PEP

In industry, the form of Eq. 12 and parameters such as κ can be determined through a variety of means. The manner of determination is dependent on the R&D efforts under consideration and the industry information available. Some industries look for a particular rate of return on R&D investment, and this could be used as a start point for the formulation of this curve. More detailed approaches exist, for example, web retail stores can run marketing and promotional experiments real-time and track purchases and clicks to see which ideas to pursue further. Google and Amazon frequently run experimental modifications to their websites to understand the financial impact of design modifications. Marketing research companies exist that have access to a membership-base who both take surveys and regularly report purchases back to the research company, and/or allow purchases to be tracked. In-store, small-scale promotional and product design launches are commonly used as a means of tracking response to new product features and incentives.

3.6 Technical Performance Model

The strength, softness, and absorbency of paper towels are determined by the manufacturing process, pulp binding additives, and, most importantly, the paper pulp used. For a detailed review of the relationship between these determinants and the final product, refer to [34]. This model holds manufacturing process and additives constant, and focuses on pulp selection. The two pulps assumed to be available are Northern Bleached Softwood Kraft, what this article refers to as

virgin pulp, and De-Inked Pulp, what this article refers to as recycled pulp. Recycled pulp is less flexible than virgin pulp; and flexibility allows for a strong, yet soft towel. Also, recycled pulp contains a larger percentage of *finer*, small pieces of fiber, than virgin pulp. Finer are created by the processing used to create recycled pulp, and have different characteristics than pulp fibers: they have a high surface area that improves drainage of water from pulp, they increase bonding between fibers (to a point), and improve opacity. They also can increase strength, but therefore also increase stiffness and decrease softness. Too many finer also lead to a weakly bonded fiber network.

As detailed in [34], technical performance of tissue products such as paper towel is daunting to model computationally. Competition amongst firms is fierce, and modeling work is carefully guarded and unpublished. Therefore, we built an empirical model to determine the relationship between pulp consistency (percentage of virgin and recycled pulps) and technical performance. The University of Washington College for Forest Resources Pulp and Paper Laboratory manufactured commercial-grade paper towels with different recycled/virgin ratios on their pilot paper towel machine, shown in Fig. 4. The university also performed the necessary testing to measure strength, softness, and absorbency of the towels. The spline curves described in the appendix describe the levels of softness, strength, and absorbency of towel for any given RPP from 0 (0%) to 1 (100%). The engineering model includes assumptions and data not detailed here; they are offered in [34] and in the appendix.

At 25% RPP, absorbency and softness improve dramatically. There is no publically available research to directly corroborate our finding of the synergistic effect of 25% recycled paper content on absorbency and softness properties. It is likely this combination of the two types of pulp, virgin and recycled, receives the benefits of plenty of flexible virgin pulp fibers with a favorable mixture of stiff recycled fibers and just enough finer to improve capillary intake and surface softness without yet sacrificing bulk and pliability.



Figure 4. PAPER TOWEL PILOT MACHINES: TOWEL FORMER ON RIGHT, TOWEL DRYER ON LEFT

4 ANALYSIS AND RESULTS

The mesh adaptive direct search optimization method was implemented in MATLABTM to identify the Pareto optimal set of solutions for maximum profitability and GHG reductions [39, 2]. The multi-objective optimization was performed by

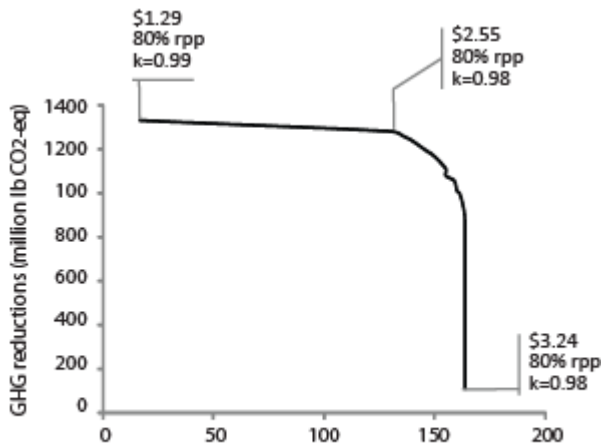


Figure 5. PARETO FRONTIER: TRADE-OFF BETWEEN REDUCTION IN EMISSIONS AND PROFITABILITY. NOTE: GHG REDUCTIONS REPRESENTED AS POSTIVE (THE-MORE-THE-BETTER).

maximizing profit, subject to a constraint on GHG reductions. The optimization was performed for many levels of the constraint to obtain the Pareto frontier, illustrated in Fig. 5.

The results indicate that for profits greater than \$160 million, the firm can make incrementally large GHG reductions for only small profit sacrifices. Profit losses across this region are approximately \$2 million although they are imperceptible in the figure. Between profits of approximately \$20 to \$130 million, the firm sacrifices incrementally large profits for relatively small GHG reductions. The results also indicate that the predominant tradeoff between profit and GHG reductions is determined by the price of the towels. The amount spent on R&D investment in PEB remains around \$8.3 million and the RPP remains at 80% along the frontier. Price is constrained at a lower bound of \$1.29 when GHG reductions are maximized and nearly doubles to \$3.24 at the profit maximum.

If this model focused on optimizing the GHG reductions of the firm's product without including market considerations such as competitor behavior or customer demand, RPP at the optimum GHG reductions would be 100%. However, a 100% RPP towel has worse technical performance, lower strength, than an 80% RPP towel. This drop in performance causes customers to choose other paper towels that have higher GHG emissions. Because of this, the optimal towel for reducing total market GHG emissions has 80% RPP.

The Pareto frontier suggests that R&D spending should be at amounts that approach attaining the highest possible value of $\kappa = 1$. For both firm profitability and GHG reductions, it is best that the Traders behave according to the PEP they exhibited in the survey. It is in the firm's best interest to activate the Trader's PEP, as well as reduce GHG. To offer an initial inquiry into the sensitivity of the model to κ , Fig. 6 presents a sensitivity analysis on the parameters that define the relationship between R&D spending and κ . The bimodal

distributions on profit are a consequence of the technical performance model of the paper towels: there is a 'sweet spot' for RPP. In the figure, the middle row of graphs corresponds the κ used in the case study. These graphs appear to show that profit increases monotonically with increasing R&D investment in PEB (*d*)—this is not the case, the optimum occurs at approximately \$8.3 million in R&D investment. In fact, all cases have an interior optimum. As κ approaches 1, there is no benefit to increased R&D investment. Therefore, if R&D spending increases enough, profits will start to decrease. The nearness to the upper bound of κ , the sensitivity of the solutions to the parameterization of κ , and the illustrative selection of Eq. 12, suggests that further investigation into the proper form and calibration of Eq. 12 is warranted. Future research will explore this relationship in depth.

5 CONCLUSION

This paper demonstrates a method that explicitly accounts for investment in influencing preferences for environmental product attributes. This is integrated into a product optimization model that includes profit, GHG reductions, and engineering performance.

Under the assumptions of the current framework, the optimization finds that the trade-off between GHG reductions and profit is not dictated by the environmental sustainability configuration of the product, namely the recycled paper content of the towels. Instead, it is price that dictates the trade-off. The Pareto frontier calls for the "activation" of pro-environmental preferences (PEP) in a group of customers that have the highest propensity to exhibit these preferences.

At all points along the frontier, the paper towel has 80% recycled paper pulp, which means it is not the most sustainable towel to make according to our sustainable metric, GHG emissions. A 100% recycled paper towel is a more sustainable product in a traditional sense, but it overlooks an important aspect of sustainability: customers must want to buy the product. In this case study, customers prefer a towel that is stronger than the strength of a 100% recycled paper towel. A towel with eighty percent recycled paper pulp is a compromise between technical performance, manufacturing costs (RPP is significantly less expensive than virgin pulp), customer demand, and GHG reductions. That is why it is important to include customer preferences and competing products in sustainability optimizations: the most sustainable product does not always lead to the largest reductions in GHG at the market level once sales (the behavior of the market) are taken into account.

If this case study occurred at a company, the results would not be enough to warrant the launch of a new R&D effort into promoting PEB, but the results do recommend further investigation into such a launch. A larger number of subjects should be included in the customer preference model, which may include more parameters as necessary; and the empirical technical performance model should be refined based on additional studies with consumer-grade towels.

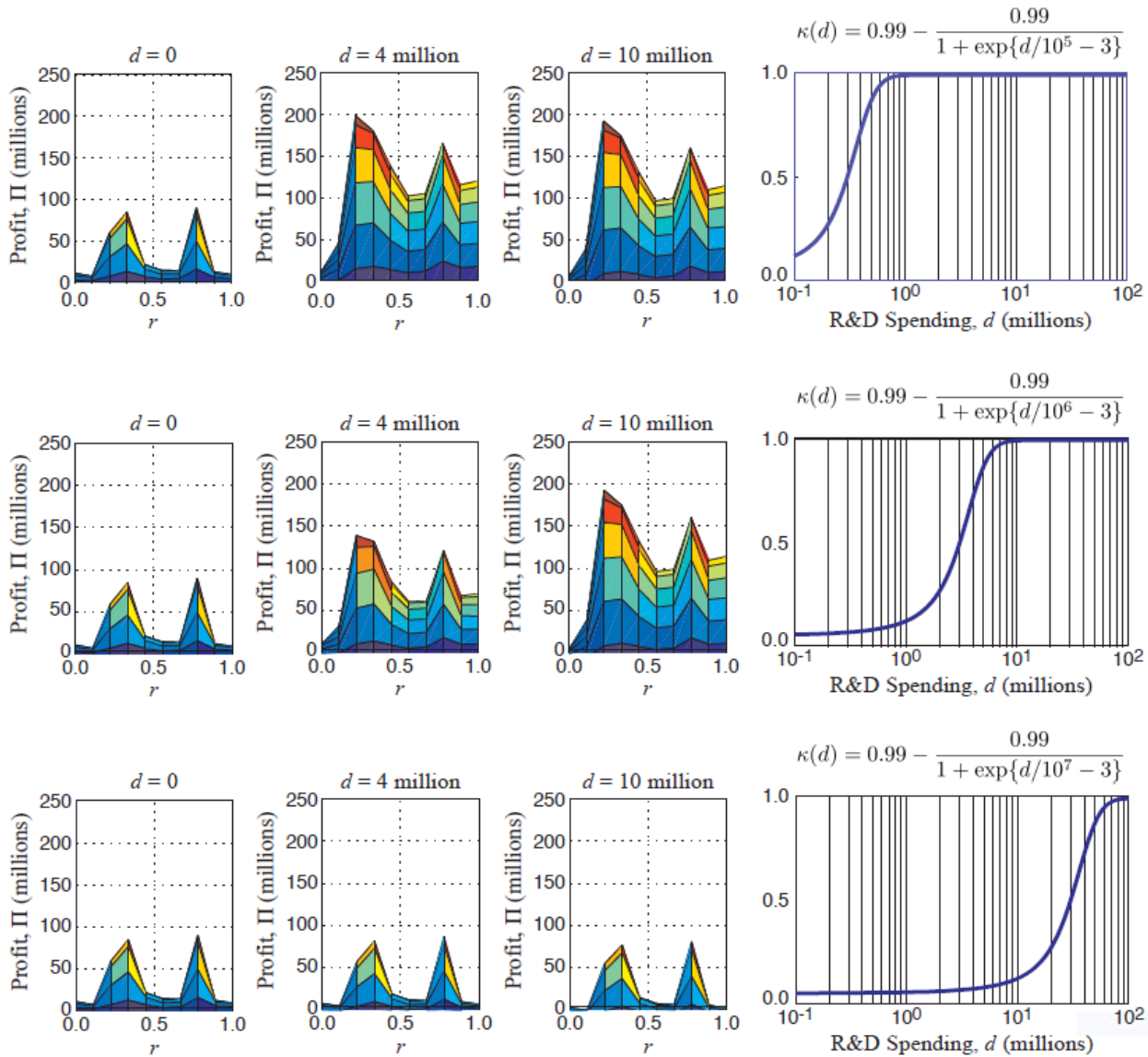


Figure 6. SENSITIVITY OF PROFIT TO κ

Although it is beyond the scope of our analysis, the company will need carefully consider the differing models on the GHG emissions of recycled vs. virgin paper. A competing company may use this contradictory model to call into question the reduced environmental impact of the towels, and accuse the company of “greenwashing.” For the purposes of this study, it is adequate to represent GHGs with the Franklin model [30], but it is important that the company a) consider alternative models and b) look further than recycled paper content alone for GHG reductions and environmental impact in general. A more sophisticated study and manufacturing implementation is appropriate and will help to improve customer trust of environmental claims.

There are a number of opportunities to extend the work, regarding the investment of R&D dollars. First, in this model, R&D can only be invested to influence the customer preferences. In reality, this money could also be spent on improving technical performance or the inherent sustainability of the product. A model that allows for this would give further insight to the tradeoffs between sustainability and technical performance—usually, this tradeoff occurs within the product. In this case, the tradeoff is expanded to include not just improvement on sustainability metrics of the product, but also sustainability preferences of the customer base.

Second, the nature of the relationship between R&D spending preference activation should be explored. Different

forms for Eq. 12 should be investigated to better understand the relationship. Uncertainty should be considered.

Finally, the findings of the study strongly suggest that further research is needed on the exact nature of the types of firm activities that could promote PEB in their customers. [9] provides insights from the social sciences, paired with existing design methods, and suggests the types of activities that may influence PEB. Case studies of increasing PEB, such as the promotion of CFL light bulb use, can be studied to offer insights as well. Future research on sustainable design should focus not only on improving the sustainability metrics of products, but also the customer preferences for the products and how to change preferences and behavior.

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APPENDIX

Nomenclature

a	Index referring to absorbency
d_k	Firm R&D investment in PEB
i	Towel property index, $i = a, t, s$
$g_{n,l}^0$	Probability respondent n is a member of Latent Class l prior to activation of PEP
$g_{n,l}$	Probability respondent n is a member of Latent Class l after activation of PEP
h_l	Polynomial used to calculate price part-worth for latent class l
k	Firm's product index
j	Competing product index
J	All products in market
l	Latent class index $l = PickUps, Traders, Savers$
M	Market Size
m_l	Polynomial used to calculate recycled paper content part-worth for latent class l
n	Index for survey respondents
N	Total number of survey respondents
p_k	Price of firm's towel
$q_i(r_k)$	Spline that calculates towel property i for a given r_k
r	Recycled paper content index
r_k	Recycled paper content of firm's towel (variable in optimizations)
S_k	Choice share of firm's product
S_j	Choice share of competitor j 's paper towel after firm's entry
S_j^0	Choice share of competitor j 's paper towel prior to firm's entry
s	Index referring to softness
t	Index referring to tear strength
v_j	Measurable utility of product j
ω	Attribute level
$x_{j\zeta\omega}$	Dummy variable that takes the value 1 for attributes/level $\zeta\omega$ in product j
z	Index of knot δ
$\alpha_{i,\delta_{i,z}}$	Cubic coefficient of spline $q_i'(r_k)$ applied to values of $(r_k - \delta_{i,z})$ between $\delta_{i,z}$ and $\delta_{i,z+1}$
$\beta_{\zeta\omega}$	Part-worth utility of attribute ζ , level ω
$\gamma_{i,\delta_{i,z}}$	Quadratic coefficient of spline $q_i'(r_k)$ applied to values of $(r_k - \delta_{i,z})$ between $\delta_{i,z}$ and $\delta_{i,z+1}$
$-\Delta_{\text{GHG}}$	Reduction in GHG emissions due to firm's entry
$\delta_{i,z}$	Knot in spline for towel property i
ζ	Product attribute
θ_i	Vector of cut-offs for towel property classifications
κ	The amount of survey PEP that translates to PEB in the marketplace
$\lambda_{i,\delta_{i,z}}$	Linear coefficient of spline $q_i'(r_k)$ applied to values of $(r_k - \delta_{i,z})$ between $\delta_{i,z}$ and $\delta_{i,z+1}$
μ	Retail markup (percentage)
$\tau_{i,\delta_{i,z}}$	Intercept of spline $q_i'(r_k)$ applied in between $\delta_{i,z}$ and $\delta_{i,z+1}$

Engineering Model

The towel's absorbency a , strength t , and softness s for recycled paper content r are given by the spline functions $q_i(r)$

$$\text{For } \delta_{i,z} < r \leq \delta_{i,z+1}: \quad \mathbf{q}_i(\mathbf{r}) = \alpha_{i,\delta_{i,z}}(\mathbf{r} - \delta_{i,z})^3 + \gamma_{i,\delta_{i,z}}(\mathbf{r} - \delta_{i,z})^2 + \lambda_{i,\delta_{i,z}}(\mathbf{r} - \delta_{i,z}) + \tau_{i,\delta_{i,z}} \quad (14)$$

where $(\delta_{i,z}, \delta_{i,z+1}, \delta_{i,z+2}, \dots)$ are the spline knots for towel property $i = a, t, s$; and $(\alpha_{i,\delta_{i,z}}, \gamma_{i,\delta_{i,z}}, \lambda_{i,\delta_{i,z}}, \tau_{i,\delta_{i,z}})$ are the spline coefficients from Tab. 1-3 below. These splines were created by fitting curves to empirical measurements of paper samples made at the

University of Washington with RPP of 0%, 12.5%, 25%, 37.5%, 50%, 75%, and 100%, and adjusting these curves to the measurement range of towels seen in the market. The units are derived from the unit of the testing procedures; additional details on the testing are given by MacDonald [12].

For the firm's towel product, the part-worths $\beta_{a,k,l}$, $\beta_{t,k,l}$ and $\beta_{s,k,l}$ are discrete values determined using $q_i(r)$ from Eq. 14 and the cut-offs θ_i for absorbency, strength, and softness described in Tab. 4. For example:

$$\begin{aligned} \text{If } \theta_{t,1} \leq q_t(r) < \theta_{t,2}: \\ \beta_{t,k,l}(q_t(r)) = \beta_{t,1,l} \end{aligned} \quad (15)$$

$\beta_{a,1,l}$ is the part-worth for absorbency, 1 out of 3 (below average), for latent class l . These part-worths are shown in Fig. 2, and also detailed in [36] along with the part-worths $\beta_{a,j,l}$, $\beta_{t,j,l}$ and $\beta_{s,j,l}$ for the competitors.

Table 1. SPLINE KNOTS AND COEFFICIENTS FOR STRENGTH (mN)

z	Knot δ_t	Intercept τ_t	Linear λ_t	Quadratic γ_t	Cubic α_t
1	0	369.9003	708.3274	0.0000	-21726.1991
2	0.125	416.0072	-310.0882	-8147.3247	63563.5524
3	0.25	374.0918	632.6222	15689.0075	-88741.4064
4	0.375	524.9873	395.1206	-17589.0199	63918.7891
5	0.5	424.3903	-1005.9411	6380.5260	-12377.8908
6	0.75	378.2833	-136.5326	-2902.8921	3870.5228
7	1	223.1963	-862.2557	0.0000	0.0000

Table 2. SPLINE KNOTS AND COEFFICIENTS FOR ABSORBENCY (s)

z	Knot δ_a	Intercept τ_a	Linear λ_a	Quadratic γ_a	Cubic α_a
1	7.1421	14.3561	0.0000	-927.9298	7.1421
2	7.8128	3.4820	-173.9868	676.1634	7.8128
3	7.5158	-10.3426	-47.2062	-5383.4957	7.5158
4	5.3707	-79.3312	-1056.6116	22588.5413	5.3707
5	1.7999	53.3018	3178.7399	-1158332.8904	1.7999
6	1.8552	56.1843	-296.2588	283.3423	1.8552
7	4.0565	26.9176	-250.7824	623.4396	4.0565
8	4.8742	5.4372	-150.7203	646.7140	4.8742
9	4.8328	-5.1367	-46.9227	460.3066	4.8328
10	4.4941	-6.2049	26.9565	-75.7437	4.4941
11	4.2277	-3.9710	14.7996	5.3994	4.2277
12	4.0585	-2.3411	15.6662	-16.3428	4.0585
13	3.9756	-0.8051	13.0432	-10.5170	3.9756
14	3.9682	0.5002	11.3552	-12.0780	3.9682
15	4.0256	1.6115	9.4167	-11.6598	4.0256
16	4.1370	2.5190	7.5453	-11.7718	4.1370
17	4.2916	3.2252	5.6559	-11.7418	4.2916
18	4.4785	3.7296	3.7714	-11.7498	4.4785
19	4.6870	4.0323	1.8855	-11.7478	4.6870
20	4.9064	4.1331	0.0000	0.0000	4.9064

Table 3. SPLINE KNOTS AND COEFFICIENTS FOR SOFTNESS (g)

z	Knot δ_s	Intercept τ_s	Linear λ_s	Quadratic γ_s	Cubic α_s
1	0	7.1972	-21.1316	0.0000	-215.6093
2	0.125	4.1347	-31.2383	-80.8535	1441.9482
3	0.25	1.7829	16.1397	459.8771	-1984.1731
4	0.375	7.1106	38.1008	-284.1878	651.6091
5	0.5	8.7054	-2.4019	-39.8344	35.4183
6	0.75	6.1687	-15.6782	-13.2706	17.6942
7	1	1.6962	-18.9959	0.0000	0.0000

Table 4. CUT-OFFS FOR STRENGTH, SOFTNESS, AND ABSORBENCY CLASSIFICATIONS

Attribute	$\theta_{i,1}$	$\theta_{i,2}$
Absorbency a	5.48	2.73
Strength t	362.97	472.00
Softness s	5.821	4.900