Practical Advances in Optimal System Design: What Method Should be Used When?

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Overview

1. Overview of single-level methods for decomposition based optimization

2. Demonstrate IDF finds optima hidden to traditional methods

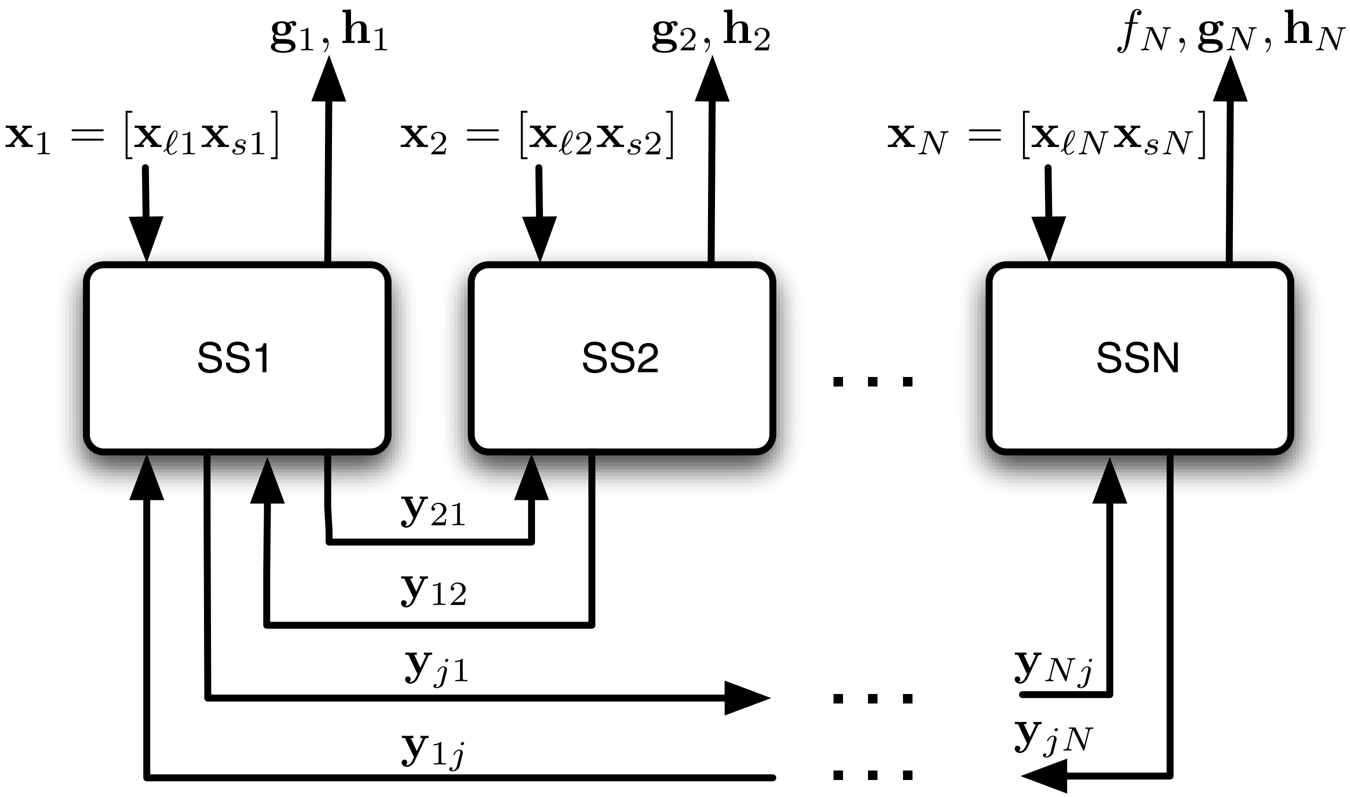
3. Introduce a thermoelastic design problem

4. Investigate the performance of MDF and IDF on problems with varying levels of coupling strength

Decomposition Based Optimization

- Some systems must be approached in a decomposed manner
  - Distributed analysis sometimes required (design groups/analysis tools)
  - Single, complete analysis may be infeasible
  - Interactions between system members must be considered
- System Analysis: seek to find a consistent analysis solution
- System Design: seek to find a feasible and optimal design

Partitioned Analysis

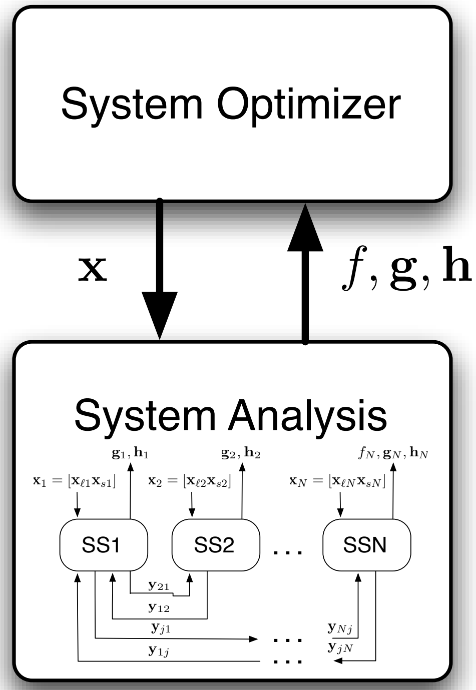


Partitioned Design: Multidisciplinary Feasible (MDF) Approach

$$\min_{\mathbf{x}} \quad f(\mathbf{x})$$

$$\text{subject to} \quad \mathbf{g}(\mathbf{x}) = [\mathbf{g_1}, \mathbf{g_2}, \ldots, \mathbf{g_N}] \leq \mathbf{0}$$

$$\mathbf{h}(\mathbf{x}) = [\mathbf{h_1}, \mathbf{h_2}, \ldots, \mathbf{h_N}] = \mathbf{0}$$



- Can find system optimum
- Analysis may converge slowly, or not at all
- Sequential process

Partitioned Design: Individual Disciplinary Feasible (IDF) Approach

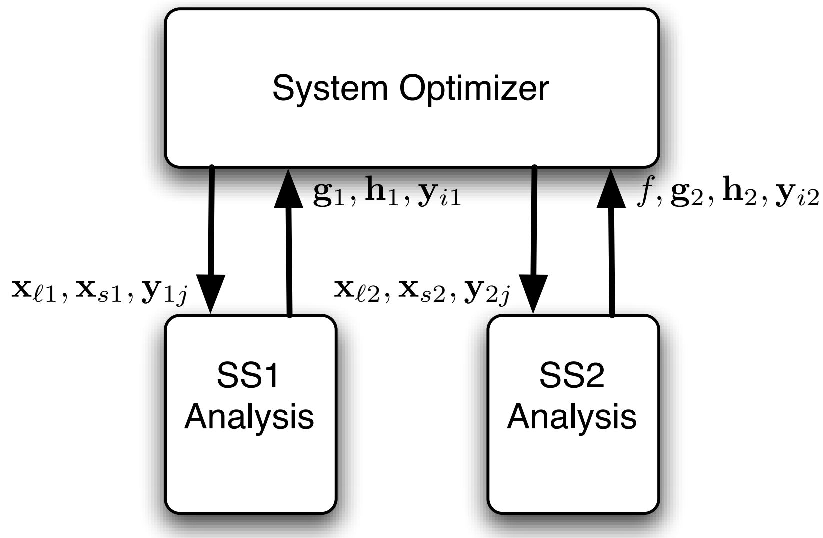
System optimizer chooses design variable values $\mathbf{x}$ *and* coupling variable values $\mathbf{y}$, i.e., simultaneous analysis and design (SAND).

- Auxiliary equality constraints enforce system consistency
- Parallel process

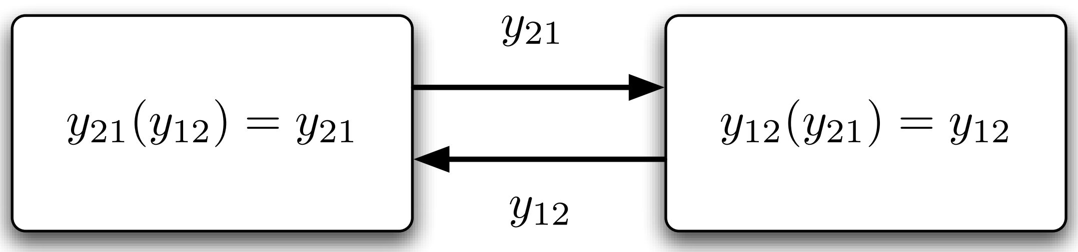
IDF Formulation and Architecture

$$\min_{\mathbf{x},\mathbf{y}} \quad f(\mathbf{x}, \mathbf{y})$$

subject to
$$\mathbf{g}(\mathbf{x}, \mathbf{y}) = [\mathbf{g_1}, \mathbf{g_2}, \ldots, \mathbf{g_N}] \leq \mathbf{0}$$
$$\mathbf{h}(\mathbf{x}, \mathbf{y}) = [\mathbf{h_1}, \mathbf{h_2}, \ldots, \mathbf{h_N}] = \mathbf{0}$$
$$\mathbf{h_{aux}}(\mathbf{x}, \mathbf{y}) = \mathbf{y}(\mathbf{x}, \mathbf{y}) - \mathbf{y} = \mathbf{0}$$



System Analysis: Fixed Point Iteration



(Step 0) choose initial guess y_{12}^0, set $i = 0$

(Step 1) $i = i + 1$

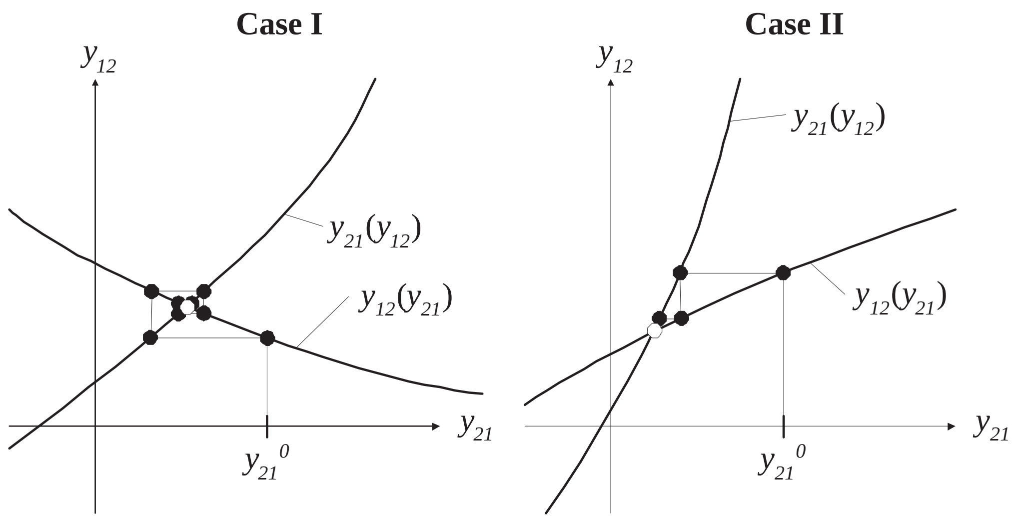
(Step 2) $y_{21}^i = y_{21}(y_{12}^{i-1})$

(Step 3) $y_{12}^i = y_{12}(y_{21}^i)$

(Step 4) if $\|\mathbf{y}^i - \mathbf{y}^{i-1}\| < \varepsilon$ stop, otherwise go to **(Step 1)**

Fixed Point Iteration Convergence

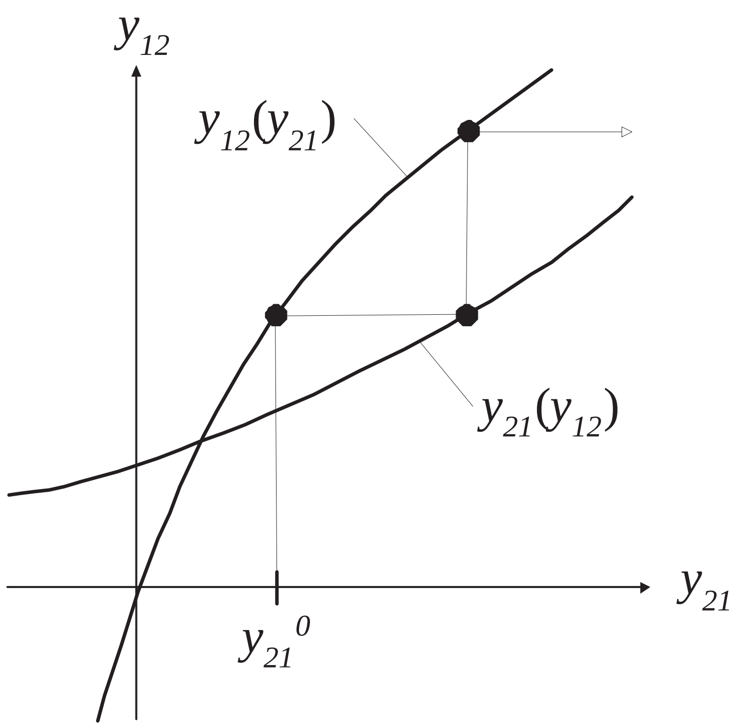
Developed proof of new convergence condition form



$$\left|\frac{\partial y_{21}(y_{21})}{\partial y_{21}}\right| > \left|\frac{\partial y_{12}(y_{21})}{\partial y_{21}}\right| \Leftrightarrow \left|\frac{\partial y_{12}(y_{12})}{\partial y_{12}}\right| > \left|\frac{\partial y_{21}(y_{12})}{\partial y_{12}}\right|$$

$$\mathbf{y_p} = \mathbf{y}(\mathbf{y_p})$$

Fixed Point Iteration Divergence

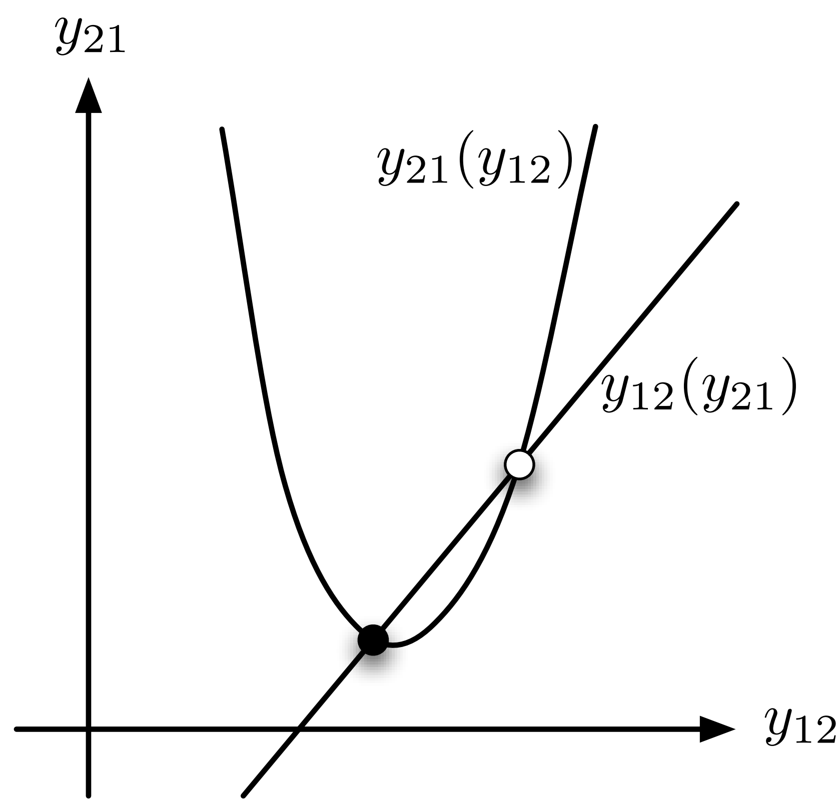


Multiple Fixed Points

FPI:

Unknown if a repelling fixed point would have led to a better solution.

- ● Attractive fixed point
- ○ Repelling fixed point



Hidden Optimum Example

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) = y_{12}^2 - 100y_{21} + 0.1\mathbf{x}'\mathbf{x}$$

where

$$y_{21}(y_{12}, x_1) = a(x_1)(y_{12} - b)^2$$

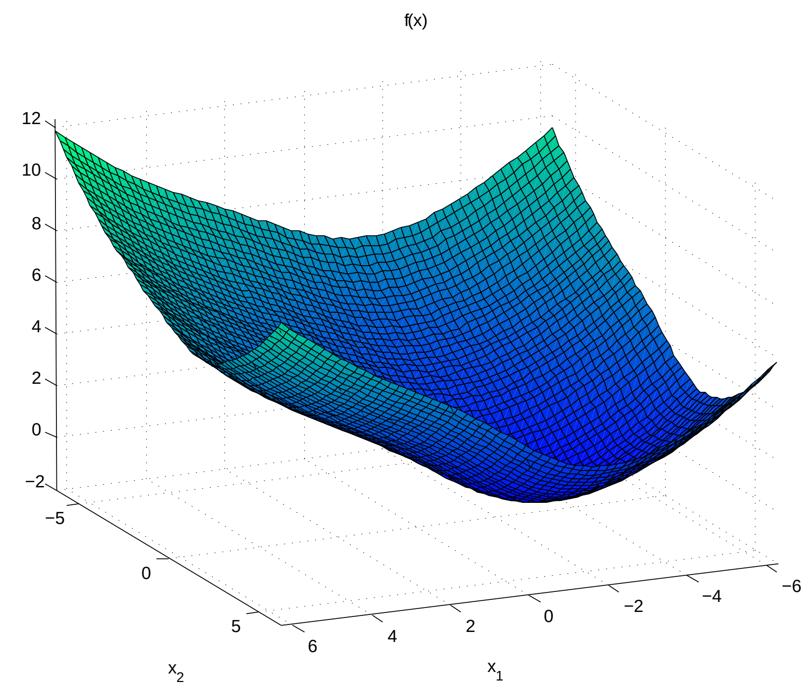
$$y_{12}(y_{21}, x_2) = c(x_2)y_{21} + d$$

$$a(x_1) = \frac{0.25}{1 + e^{x_1}} + .5$$

$$b = 3$$

$$c(x_2) = -\left(\frac{1}{1 + e^{x_2}} + .5\right)$$

$$d = 3.5$$



Solution Results

MDF solution: -0.244

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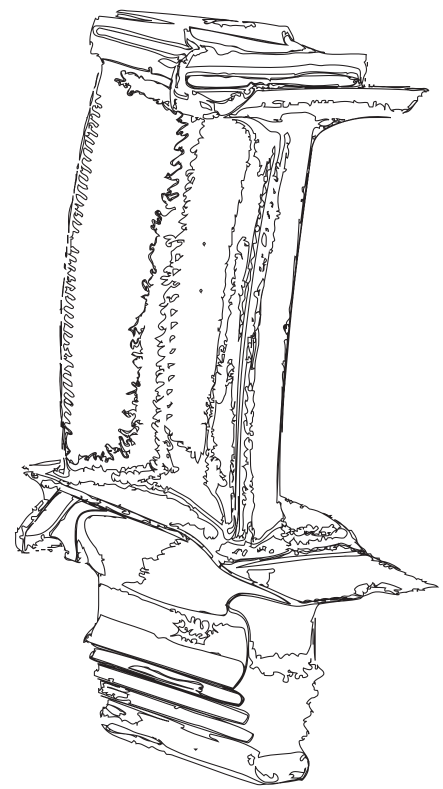
MDF solution: -0.244

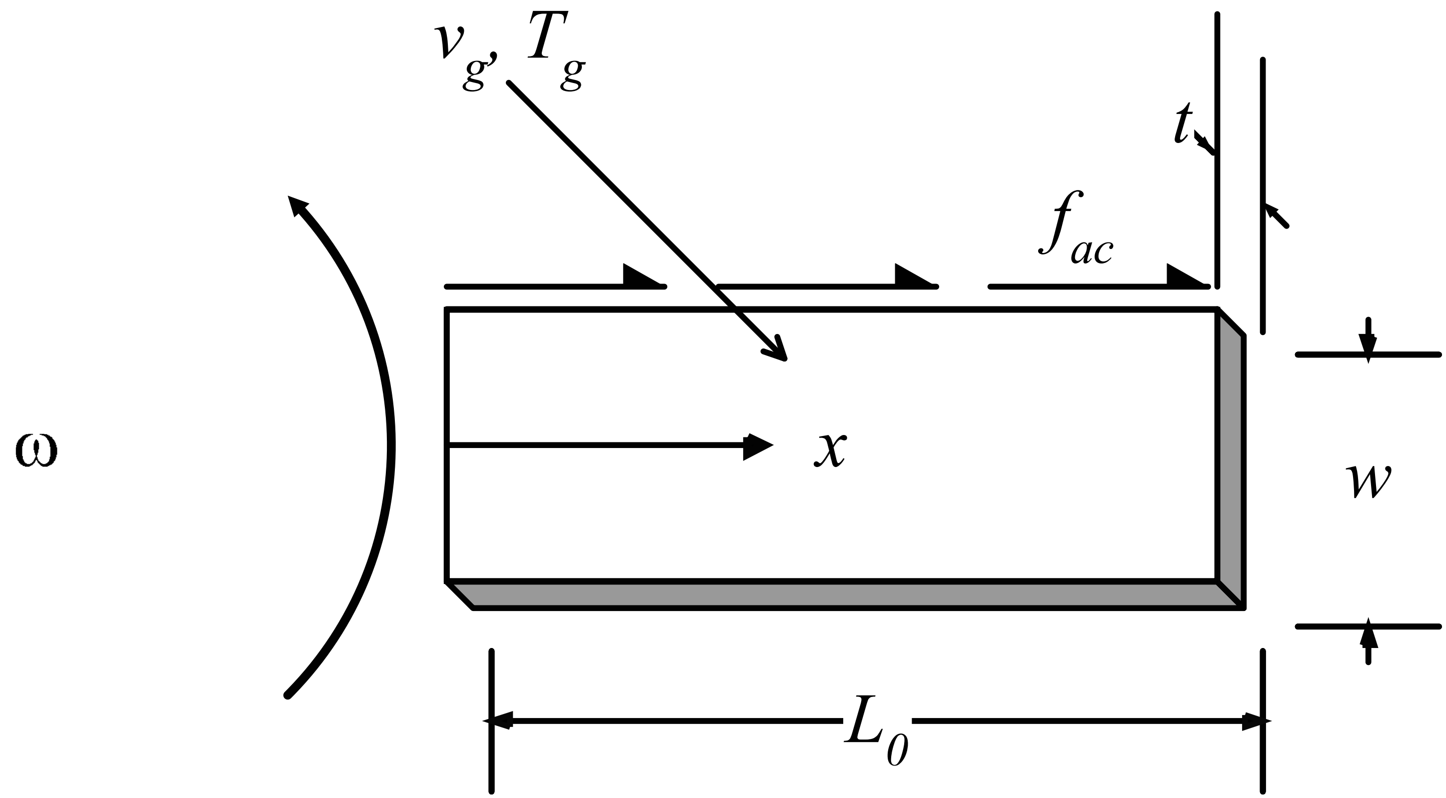
IDF solution: $\mathbf{-975.7}$

New Example Problem—Thermoelastic Turbine Blade Design

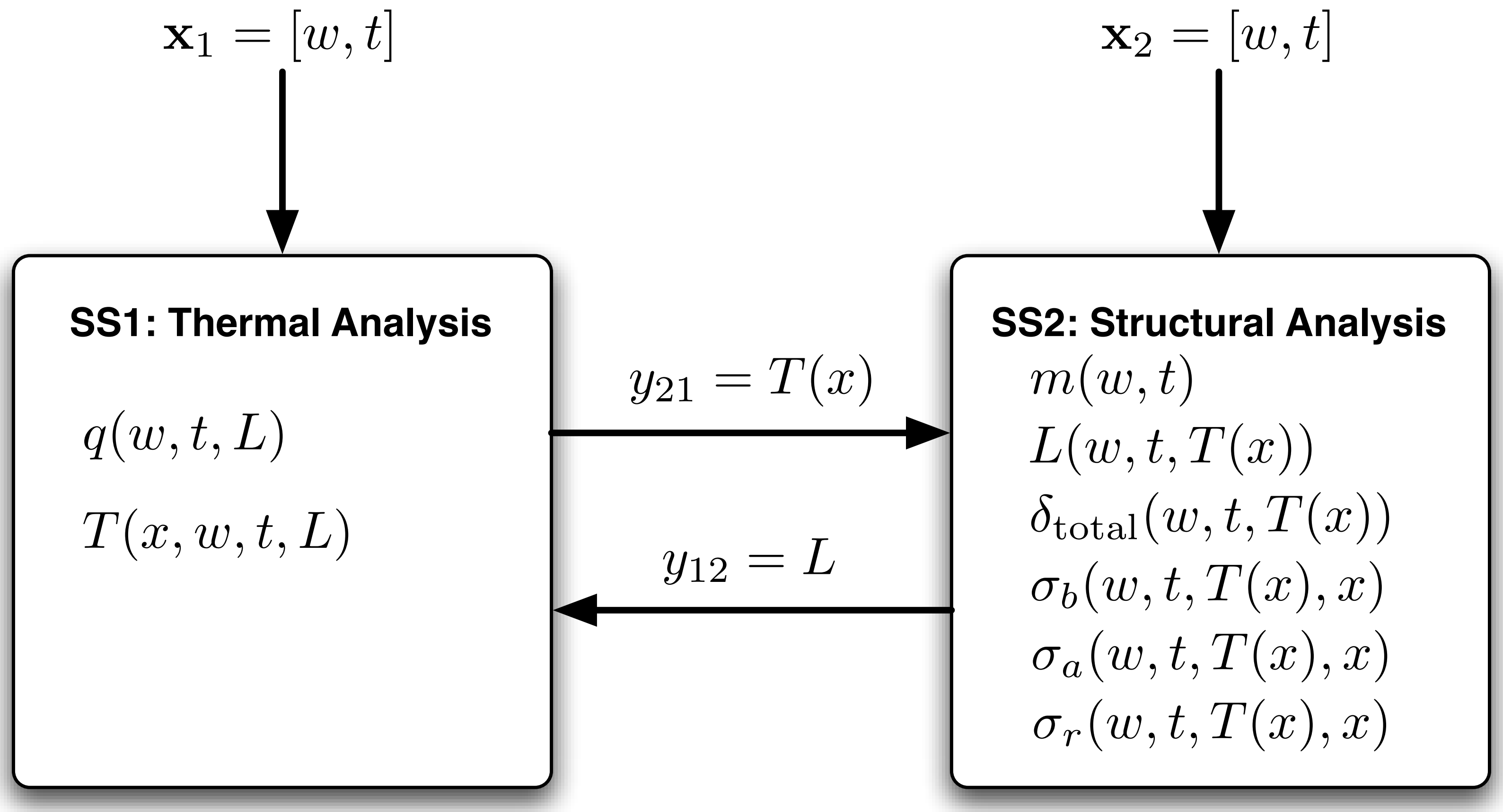
Developed in order to aid studies on coupling strength variation



Thermoelastic Turbine Blade Design



Coupled Turbine Blade Analysis



MDF Formulation

$$
\begin{aligned}
\min_{\mathbf{x}=[\mathbf{w},\mathbf{t}]} \quad & q \\
\text{subject to} \quad & g_1(\mathbf{x}) = T_{max} - T_{melt} \leq 0 \\
& g_2(\mathbf{x}) = \delta_{total} - \delta_{allow} \leq 0 \\
& g_3(\mathbf{x}, x) = \sigma_a(x) - \sigma_r(T(x)) \leq 0 \\
& g_4(\mathbf{x}, x) = \sigma_b(x) - \sigma_r(T(x)) \leq 0 \\
& g_5(\mathbf{x}, x) = m - m_{max} \leq 0 \\
& 0 \leq x \leq L_0 + \delta_{total}
\end{aligned}
$$

$$[w_*,\ t_*] = [0.0131,\ 0.0075]$$

IDF Formulation

$$\min_{\mathbf{x}=[\mathbf{w},\mathbf{t}],T(x),L} \quad q$$

subject to

$$g_1(\mathbf{x}) = T_{max} - T_{melt} \leq 0$$

$$g_2(\mathbf{x}) = \delta_{total} - \delta_{allow} \leq 0$$

$$g_3(\mathbf{x}, x) = \sigma_a(x) - \sigma_r(T(x)) \leq 0$$

$$g_4(\mathbf{x}, x) = \sigma_b(x) - \sigma_r(T(x)) \leq 0$$

$$g_5(\mathbf{x}, x) = m - m_{max} \leq 0$$

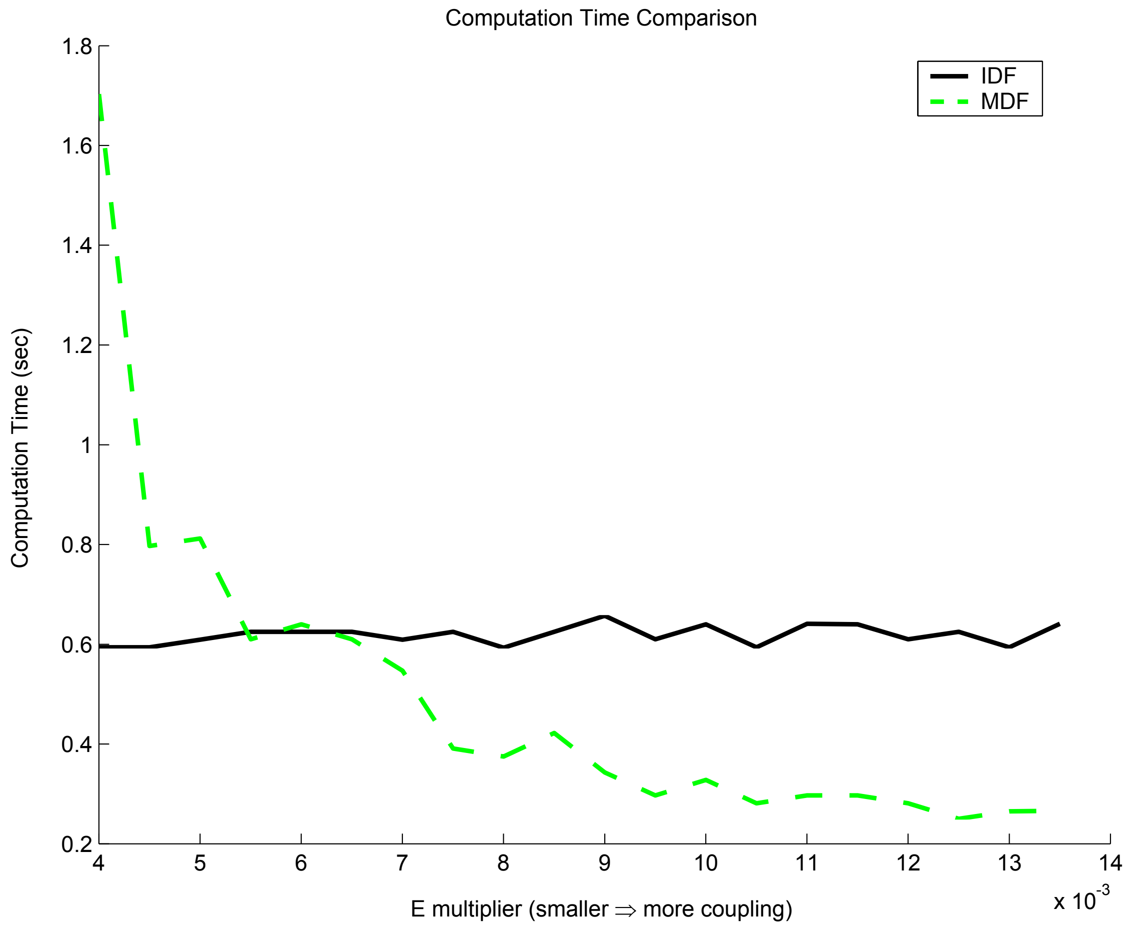
$$g_6(\mathbf{x}, x) = T(x) - T(\mathbf{x}, x) = 0$$

$$g_7(\mathbf{x}, x) = L - L(\mathbf{x}) = 0$$

$$0 \leq x \leq L_0 + \delta_{total}$$

$$[w_*,\ t_*] = [0.0128,\ 0.0074]$$

MDF and IDF Comparison



Conclusion

- Prediction of Cramer *et al.* that IDF is more computationally efficient confirmed, in the case of strong coupling
- Dependence of method performance on coupling strength exposed
- Demonstrated that IDF can find optima hidden to MDF

Future Work:

- Investigate thresholds of performance advantages
- Study method behavior on problems with multiple analysis solutions
- Perform these studies on IDF and other methods, including multilevel, that utilize optimization for analysis tasks

Thanks for Your Attention!

Sample Analysis Results