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COMBINED PLANT AND CONTROLLER DESIGN USING DECOMPOSITION-BASED DESIGN OPTIMIZATION AND THE MINIMUM PRINCIPLE

James T. Allison, Sam Nazari
The MathWorks
Natick, MA 01721
Email: {james.allison,sam.nazari}@mathworks.com

ABSTRACT

An often cited motivation for using decomposition-based optimization methods to solve engineering system design problems is the ability to apply discipline-specific optimization techniques. For example, structural optimization methods have been employed within a more general system design optimization framework. We propose an extension of this principle to a new domain: control design. The simultaneous design of a physical system and its controller is addressed here using a decomposition-based approach. An optimization subproblem is defined for both the physical system (i.e., plant) design and the control system design. The plant subproblem is solved using a general optimization algorithm, while the controls subproblem is solved using a new approach based on optimal control theory. The optimal control solution, which is derived using the the Minimum Principle of Pontryagin (PMP), accounts for coupling between plant and controller design by managing additional variables and penalty terms required for system coordination. Augmented Lagrangian Coordination is used to solve the system design problem, and is demonstrated using a circuit design problem.

1 Introduction

Numerous methods have been developed for solving engineering system design problems that have been partitioned into smaller subsystem design subproblems. These decomposition-based design optimization methods, or decomposition methods, solve subproblems iteratively, guiding the system toward a consistent and optimal system design solution using a coordination algorithm. Advantages of decomposition methods include computational parallelism, exploitation of problem sparsity, and solution of increasingly complex design problems [1, 2].

Another often cited benefit of decomposition methods is the opportunity to employ specialized optimization algorithms to solve individual subproblems [3, 4]. These algorithms exploit problem structure to solve them more efficiently. Incorporating specialized algorithms within a decomposition-based optimization framework has been demonstrated for systems involving structural design [5, 6]. Another discipline with well-developed theory and optimization techniques is optimal control [7]. While multidisciplinary approaches exist that account for interactions between the design of a physical system (i.e., the plant) and its controller [8], these all employ either a sequential or nested design optimization process rather than the distributed approach of decomposition methods. An alternate strategy is applied here; a new optimal control solution is derived that accounts directly for interactions with other aspects of an engineering system, and fits naturally within existing decomposition methods. This type of system design approach enables specialists, such as controls engineers, to focus deeply on developing analysis and design techniques for their discipline, but within a formal interaction management framework that eases integration with other aspects of system design.

1.1 Decomposition-Based Design Optimization

Decomposition methods apply to system design problems that have been partitioned into multiple design subproblems.

Each subproblem is formulated as an optimization problem, and is solved independently of the other subproblems, enabling coarse-grained parallelism. Subproblem independence is temporary; non-trivial partitioned systems exhibit interaction, or linking, between subproblems. Subproblems may be linked in two ways: through shared design variables, or through coupling variables. At the system-wide solution, subproblems must be consistent with respect to these links. Consider the following undecomposed optimization problem:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}, \mathbf{y}_{\mathrm{p}}(\mathbf{x})) \\ \text{subject to} \quad & \mathbf{g}(\mathbf{x}, \mathbf{y}_{\mathrm{p}}(\mathbf{x})) \leq \mathbf{0} \\ & \mathbf{h}(\mathbf{x}, \mathbf{y}_{\mathrm{p}}(\mathbf{x})) = \mathbf{0}, \end{aligned} \tag{1}$$

Solutions to Eqn. (1), $\mathbf{x}^*$, minimize the objective function $f(\mathbf{x}, \mathbf{y}_{\mathrm{p}}(\mathbf{x}))$, while satisfying the inequality and equality design constraints, $\mathbf{g}(\mathbf{x}, \mathbf{y}_{\mathrm{p}}(\mathbf{x})) \leq \mathbf{0}$ and $\mathbf{h}(\mathbf{x}, \mathbf{y}_{\mathrm{p}}(\mathbf{x})) = \mathbf{0}$, respectively. Calculating the objective and constraint functions for a given design vector $\mathbf{x}$ may involve the solution of a coupled system of equations. In engineering design these equations often take the form of numerical simulations, a collection of which can be viewed as a system of analysis functions: $\mathbf{a}(\mathbf{x}, \mathbf{y})$. The objective and constraint function values are outputs of a subset of analysis functions. Analysis function outputs required as input to other analysis functions are coupling variables: $\mathbf{y}$. If feedback coupling exists, a set of consistent coupling variables $\mathbf{y}_{\mathbf{p}}(\mathbf{x})$ must be found for a given $\mathbf{x}$ using an iterative algorithm. A coupling variable vector is consistent if it satisfies $\mathbf{y} - \mathbf{S}\mathbf{a}(\mathbf{x}, \mathbf{y}) = \mathbf{0}$, where $\mathbf{S}$ is a selection matrix that extracts the components of $\mathbf{a}(\mathbf{x}, \mathbf{y})$ that correspond to $\mathbf{y}$.

Equation (1) is known as the All-in-One (AiO) or Multidisciplinary Feasible (MDF) formulation [9]. MDF may lead to solution difficulty when the dimension of $\mathbf{x}$ is large, or when the underlying system of analysis functions is strongly coupled [10]. Decomposition methods can address these issues by partitioning the system design problem into smaller optimization subproblems. How a system is partitioned influences the success of a decomposition method implementation [11]; if partitions cut across many or strongly-coupled links, a decomposition method may be inefficient, whereas thoughtful partition choices can exploit problem sparsity by minimizing links between subproblems. Decomposition methods also facilitate application of specialized optimization algorithms to appropriate portions of a system; in this article we address the extension of optimal control techniques for use in decomposition methods.

If an analysis function in one subproblem requires as input an output from an analysis function from another subproblem, the associated coupling variable 'links' the two subproblems. The second way subproblems may be linked is through shared design variables $\mathbf{x}_{\mathbf{s}}$, which are design variables required as input by more than one subproblem. Coupling variables between subproblems and shared design variables together form the linking variable vector $\mathbf{z} = [\mathbf{y}, \mathbf{x}_{\mathbf{s}}]$. Decomposition methods employ local copies of linking variables in appropriate subproblems. These copies must agree (i.e., are consistent) at convergence of the decomposition method. System consistency typically is enforced with equality constraints on these copies, or with penalty functions. Local linking variable copies are requisite for independent subproblem solution and application of specialized subproblem optimization methods. A coordination algorithm directs the repeated solution of subproblems, and guides the system toward a state of system consistency and optimality.

The specific decomposition method used here is Augmented Lagrangian Coordination (ALC) [12, 13], which is a generalization of Analytical Target Cascading (ATC) [14], based on augmented Lagrangian decomposition methods [15, 16]. Both ATC and ALC have convergence proofs under standard assumptions. The most general ALC formulation allows for linking functions, in addition to linking variables. Many system design problems, including co-design problems, are formulated naturally without linking functions (i.e., quasi-separable).

After a system design problem, in the form of Eqn. (1), with linking variables and an additively separable objective function (i.e., $f(\mathbf{x}) = \sum_i f_i(\mathbf{x})$), is partitioned, local copies of linking variables are made for each subproblem; $\mathbf{z}_i$ are the local copies for subproblem i, and the consistency constraint $\mathbf{c}_i(\mathbf{x}_i, \mathbf{z}_i, \hat{\mathbf{z}}_i) = \mathbf{0}$ ensures consistency between subproblem i and subproblems it is linked to. In the consistency constraints, $\mathbf{x}_i$ are the design variables required by subproblem i, and $\hat{\mathbf{z}}_i$ are the local copies of $\mathbf{z}_i$ from other subproblems; consistency constraints are satisfied when $\mathbf{z}_i = \hat{\mathbf{S}}_i \hat{\mathbf{z}}_i$, where $\hat{\mathbf{S}}_i$ is a selection matrix that maps components of $\hat{\mathbf{z}}_i$ to corresponding elements of $\mathbf{z}_i$; this notation allows for linking variables that connect more than one subproblem. Consistency constraints are relaxed using an augmented Lagrangian penalty function [17]. This enables independent subproblem solution, while ensuring consistency at ALC convergence:

$$\phi_i(\mathbf{c}_i(\mathbf{x}_i, \mathbf{z}_i, \hat{\mathbf{z}}_i), \mathbf{v}_i, \mathbf{w}_i) = \mathbf{v}_i \mathbf{c}_i(\mathbf{x}_i, \mathbf{z}_i, \hat{\mathbf{z}}_i)^{\mathrm{T}} + \|\mathbf{w}_i \circ \mathbf{c}_i(\mathbf{x}_i, \mathbf{z}_i, \hat{\mathbf{z}}_i)\|_2^2, \tag{2}$$

The ALC coordination algorithm manages $\mathbf{v}_i$ and $\mathbf{w}_i$, the linear and quadratic penalty weights, respectively; $\circ$ indicates the Hadamard product (i.e., element-by-element multiplication). The linear weights converge to the Lagrange multipliers for the consistency constraints. Note that each $\mathbf{z}_i$ does not need to contain copies of all components of $\mathbf{z}$; problem sparsity can be exploited by including only those components that link subproblem i with the rest of the system. The quasi-separable ALC formulation for the i-th subproblem is:

$$\begin{aligned} \min_{\mathbf{x}_i, \mathbf{z}_i} \quad & f_i(\mathbf{x}_i, \mathbf{z}_i) + \phi_i(\mathbf{c}_i(\mathbf{x}_i, \mathbf{z}_i, \hat{\mathbf{z}}_i), \mathbf{v}_i, \mathbf{w}_i) \\ \text{subject to} \quad & \mathbf{g}_i(\mathbf{x}_i, \mathbf{z}_i) \leq \mathbf{0} \\ & \mathbf{h}_i(\mathbf{x}_i, \mathbf{z}_i) = \mathbf{0} \\ & \tilde{\mathbf{h}}_i(\mathbf{x}_i, \mathbf{z}_i) = \mathbf{0}, \end{aligned} \tag{3}$$

If subproblem i has any analysis functions that pass coupling variables to other subproblems, auxiliary equality constraints $\tilde{\mathbf{h}}_i(\mathbf{x}_i, \mathbf{z}_i)$ are required to treat the associated components of $\mathbf{z}_i$ as independent optimization variables. Alternatively, these constraints may be eliminated through substitution if coupling and shared design variables are treated separately [18], which will be demonstrated in the next section. Note that $\hat{\mathbf{z}}_i$ and the penalty weights are held fixed during the solution of subproblem i.

The ALC coordination algorithm consists of an outer loop and an inner loop. For a given set of penalty weights, the ALC inner loop iteratively solves all subproblems until a fixed point for the linking variable copies is found. A fixed point does not guarantee system consistency; the ALC outer loop updates the penalty weights after solving the inner loop, guiding the system toward consistency and optimality. The basic update formula is:

$$\mathbf{v}^{k+1} = \mathbf{v}^k + 2\mathbf{w}^k \circ \mathbf{w}^k \circ \mathbf{c}^{k*}$$

$$\mathbf{w}^{k+1} = \beta \mathbf{w}^k.$$

The outer loop iteration number is k; the system penalty weight vectors $\mathbf{v}$ and $\mathbf{w}$ are formed via concatenation of subproblem penalty weight vectors; $\mathbf{c}^{k*}$ is the system consistency constraint vector at the end of inner loop k. At outer loop convergence, assuming the original problem given in Eqn. (1) is feasible and meets ALC convergence requirements, the consistency constraints are satisfied, and the resulting solution is system-optimal.

1.2 Optimal Control and Co-design

In the 1960's, modern control theory was the focal point of the control and estimation community [19]. The ground breaking work of Pontryagin [20] became available in English for the first time, igniting increased interest in the theory of optimal control. Many useful results were published and the classic texts of Athans [7], Bryson [21] and Kirk [22] were widely read. Two major contributions by Kalman [23, 24] paved the way for the Linear Quadratic Gaussian (LQG) framework of control and estimation. The Linear Quadratic Regulator (LQR) technique, which is the control methodology at the heart of LQG, has become a popular approach primarily because of its ease of application.

Although LQR is an effective control design technique, further improvements in system performance require a more holistic approach. The conventional control system design process is sequential; the physical system, or plant, is designed first to meet specified requirements (Fig. 1). The control system is then designed, without an opportunity to readjust plant design [19]. During the plant design phase engineers may give consideration to control system performance, but this does not account fully for plant-controller design interaction [6, 25–27]. In practice this is managed using elaborate design iterations entailing *rapid prototypes* [28] instead of a comprehensive system design framework.

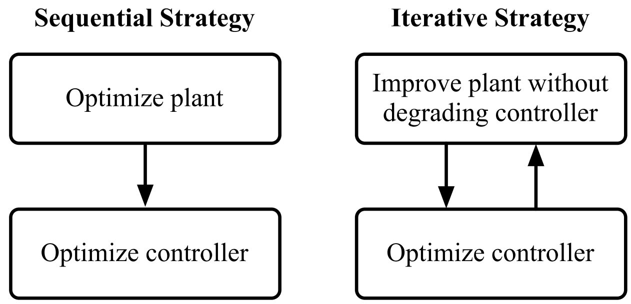

Figure 1: Sequential and Iterative Solution Strategies.

A design approach that tackles plant and control design simultaneously can account fully for controller-plant design coupling, yielding optimal systems. Sequential strategies often result in suboptimal designs [29–31]. In some cases a simultaneous design approach can identify optimal solutions where sequential approaches fail even to deliver a feasible design.

Several techniques for combined plant and control design (co-design) have been examined empirically [25, 29, 32, 33]. One approach is to simply repeat the sequential design process, iterating until convergence on a system design (Fig. 1). This process is similar to the block-coordinate descent algorithm, which may not converge to the system optimum in some cases [34].

A nested approach has been proven to identify a system optimal solution [29]. The outer loop seeks to optimize the overall system performance by varying the plant design. For every candidate plant design tested by the outer loop, the inner loop computes the optimal control for the given plant design.

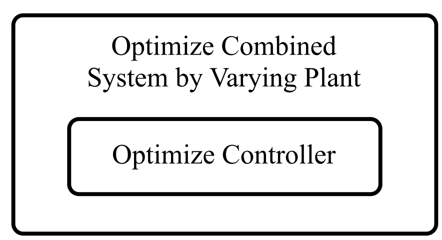

Figure 2: Nested solution strategy

This article presents a new approach to co-design. Rather than nesting control design within plant design, these two design problems are solved using a decomposition method, which coordinates their solutions such that a consistent and optimal system design is produced. The primary contributions here are the formulation of the co-design subproblem, and the extension of optimal control theory to the solution of this subproblem. A decomposition approach to co-design is then illustrated using a circuit design example.

2 ALC Codesign Formulation

The general co-design problem can be formulated as:

$$\begin{aligned} \min_{\mathbf{x_p},\mathbf{x_c}} \quad & f_s(\mathbf{x_p},\mathbf{x_c}) \\ \text{subject to} \quad & \mathbf{g_p}(\mathbf{x_p},\mathbf{x_c}) \leq \mathbf{0} \\ & \mathbf{g_c}(\mathbf{x_p},\mathbf{x_c}) \leq \mathbf{0}, \end{aligned} \tag{4}$$

where $\mathbf{x_p}$ is the vector of plant design variables (such as geometric specifications), $\mathbf{x_c}$ is the vector of control design variables (such as control gains $\mathbf{K}$ or control input $u(t)$), $f_s(\mathbf{x_p},\mathbf{x_c})$ is a metric of overall system performance, and $\mathbf{g_p}(\mathbf{x_p},\mathbf{x_c})$ and $\mathbf{g_c}(\mathbf{x_p},\mathbf{x_c})$ are plant and control design constraints, respectively.

Often dynamic performance of a system depends on physical properties of the plant, which depend on the plant design; this relationship is expressed here as $\mathbf{y} = \mathbf{a}_1(\mathbf{x_p})$, where $\mathbf{y}$ is a vector of physical properties computed with the analysis function $\mathbf{a}_1(\mathbf{x_p})$. Here we assume the plant design constraints do not depend on $\mathbf{x_c}$. Equation (4) now can be rewritten as:

$$\begin{aligned} \min_{\mathbf{x_p},\mathbf{x_c}} \quad & f_s(\mathbf{a}_1(\mathbf{x_p}),\mathbf{x_c}) \\ \text{subject to} \quad & \mathbf{g_p}(\mathbf{x_p}) \leq \mathbf{0} \\ & \mathbf{g_c}(\mathbf{a}_1(\mathbf{x_p}),\mathbf{x_c}) \leq \mathbf{0}. \end{aligned} \tag{5}$$

In the nested solution approach (introduced in the previous section), the outer loop seeks to optimize the system objective with respect to $\mathbf{x_p}$ only, and an inner loop seeks to optimize the system objective with respect to $\mathbf{x_c}$ only, for a fixed $\mathbf{x_p}$.

$$\begin{aligned} \min_{\mathbf{x_p}} \quad & f_s^*(\mathbf{a}_1(\mathbf{x_p})) \\ \text{subject to} \quad & \mathbf{g_p}(\mathbf{x_p}) \leq \mathbf{0} \\ \text{where} \quad & f_s^*(\mathbf{a}_1(\mathbf{x_p})) = \min_{\mathbf{x_c}} f_s(\mathbf{a}_1(\mathbf{x_p}),\mathbf{x_c}) \\ & \quad \text{subject to} \quad \mathbf{g_c}(\mathbf{a}_1(\mathbf{x_p}),\mathbf{x_c}) \leq \mathbf{0}. \end{aligned} \tag{6}$$

Equations (5) and (6) are mathematically equivalent [29]. The outer loop may be solved using a general-purpose optimization algorithm, such as sequential quadratic programming [35], and the inner loop may be solved with optimal control techniques (a significant benefit). Nested solutions, however, are not amenable to coarse-grained parallelism, can be computationally intensive, and are hard to generalize to systems with more than two subproblems. Rather than adapt solution approach to fit existing optimal control techniques, we propose a method that extends optimal control theory to solve an optimal control subproblem within a more general system design framework. This enables parallelism, solution of systems with more than two subproblems, and opens the door to combining other specialized optimization algorithms for other disciplines with optimal control in a system design optimization implementation.

The first step in the proposed approach is to partition the co-design problem into plant and control design subproblems, as illustrated in Fig. 3. The class of co-design problems considered here has no shared design variables, but are linked by a coupling variable computed in the plant subproblem and passed to the control subproblem. For the control subproblem **P2** to be independent of the plant subproblem **P1** during the ALC solution process, a local copy of the coupling (linking) variable $\mathbf{y}$ is created for each subproblem: $\mathbf{z}_1 = \mathbf{a}_1(\mathbf{x_p})$ is the local copy of $\mathbf{y}$ for **P1**, and $\mathbf{z}_2$ is the local copy for **P2**. The ALC subproblems are:

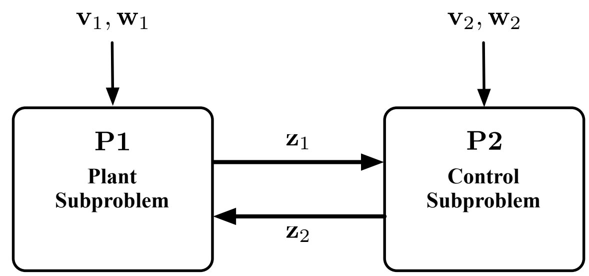

Figure 3: Relationship between plant and control design subproblems

P1: Plant Design

$$\begin{aligned} \min_{\mathbf{x_p}} \quad & \phi_1(\mathbf{z}_1,\mathbf{z}_2,\mathbf{v}_1,\mathbf{w}_1) \\ \text{subject to} \quad & \mathbf{g_p}(\mathbf{x_p}) \leq \mathbf{0} \\ \text{where} \quad & \mathbf{z}_1 = \mathbf{a}_1(\mathbf{x_p}) \end{aligned} \tag{7}$$

P2: Control Design

$$\begin{aligned} \min_{\mathbf{x_c},\mathbf{z}_2} \quad & f_s(\mathbf{z}_2,\mathbf{x_c}) + \phi_2(\mathbf{z}_1,\mathbf{z}_2,\mathbf{v}_2,\mathbf{w}_2) \\ \text{subject to} \quad & \mathbf{g_c}(\mathbf{z}_2,\mathbf{x_c}) \leq \mathbf{0}. \end{aligned} \tag{8}$$

The values of $\mathbf{z}_2$, $\mathbf{v}_1$, and $\mathbf{w}_1$ are fixed during the solution of **P1**, and $\mathbf{z}_1$, $\mathbf{v}_2$, and $\mathbf{w}_2$ are fixed during the solution of **P2**. For notational simplicity, $\mathbf{z}_2$ and $\mathbf{z}_1$ have been used in place of $\hat{\mathbf{z}}_i$, described in Eqn. (3), for **P1** and **P2**, respectively. The augmented Lagrangian penalty function is used to relax the consistency constraint $\mathbf{z}_1 = \mathbf{z}_2$ in each subproblem, and the penalty weights are updated in the outer loop of the ALC coordination algorithm. Also note that the auxiliary constraints for coupling variables is eliminated from **P1**; the output of $\mathbf{a}_1(\mathbf{x_p})$ is substituted directly for $\mathbf{z}_1$, and consequently $\mathbf{z}_1$ does not appear in the set of optimization variables for **P1**; this ALC simplification is described in [18].

In this decomposition approach to co-design, **P2** determines how the control system would like the plant to behave, embodied by the value of $\mathbf{z}_2$ determined by the control subproblem. This target value of $\mathbf{z}_2$ is then passed to **P1** by the ALC coordination inner loop, and **P1** seeks to find a feasible plant design $\mathbf{x_p}$ that produces plant characteristics $\mathbf{z}_1$ that are as close as possible to the target $\mathbf{z}_2$. Normally there is a conflict between control and plant design; plant characteristics that are ideal for the control typically violate plant design constraints $\mathbf{g_p}(\mathbf{x_p})$. The ALC coordination algorithm can be viewed as a negotiation process that brings these conflicting requirements into agreement, satisfying the consistency constraint $\mathbf{z}_1 = \mathbf{z}_2$ at convergence.

P1 can be solved using general optimization algorithms. It is desirable to solve **P2** using optimal control techniques. However, the linking variable and penalty function have no time variance, precluding direct application of established optimal control techniques to the solution of **P2**. The following section presents an extension of optimal control theory for solving **P2**.

The ALC co-design formulation may be extended to other classes of co-design problems, such as those with shared design variables and feedback coupling. This generalization is a topic for future work.

3 Control Subproblem Solution

This section develops the solution to the control subproblem given in Eqn. (8). A review of the relevant optimal control theory is presented, followed by a derivation of a direct solution to the control subproblem.

3.1 Optimal Control

The objective of the control subproblem **P2** is to compute the optimal response trajectory of the dynamic system, while accounting for interactions with the plant subproblem. The solution to optimal control without plant interaction is presented here as background. Figure 4 illustrates the system block diagram of a linear, time invariant, causal system. This model defines the relationships between the inputs and outputs of a system over time. $\boldsymbol{\xi}(t)$ and $\dot{\boldsymbol{\xi}}(t)$ are n-dimensional column vectors denoting the plant state and the plant state time rate of change, respectively. $\boldsymbol{\xi_e}$ is an l-dimensional column vector representing the exogenous states of the system. $\mathbf{u}(t)$ is an m-dimensional column vector representing the plant control signals. $\mathbf{y}(t)$ is the r-dimensional output vector. The plant coefficient matrix $\mathbf{A}$ is an $n \times n$ dimensional matrix. The control signal coefficients are embodied in $\mathbf{B}$, which is an $n \times m$ matrix. The plant observation coefficient matrix, $\mathbf{C}$ is an $r \times n$ matrix. If there is direct feedthrough in the linear system then the corresponding coefficients of the feedthrough signals are embodied in $\mathbf{D}$, which is an $n \times m$ matrix. Finally, the plant exogenous coefficients are captured by $\mathbf{E}$, which is an $l \times l$ square matrix.

If state-feedback control is used, the control input is given by $\mathbf{u}(t) = -\mathbf{K}\boldsymbol{\xi}(t)$, where $\mathbf{K}$ is the gain matrix, and is also the control design variable. The objective in optimal control is to find $\mathbf{K}$ such that a cost function $J(\mathbf{K})$ is minimized to J^*. The solution to the optimal control problem advanced by Pontryagin et al. produced an open loop control signal and the corresponding state trajectory for a specified initial state [36]. The solution is embedded in a two-point boundary value (TPBV) problem. For closed-loop feedback control, it is desirable to have an expression for the present optimal control $\mathbf{u}^*(t)$ as a function of the

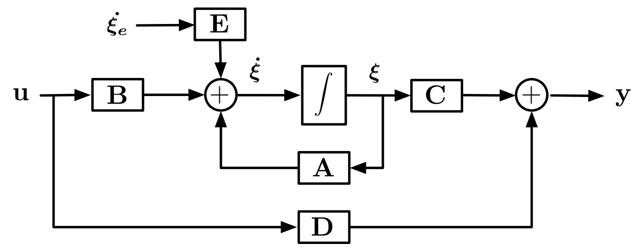

Figure 4: LTI System Block Diagram

present state $\boldsymbol{\xi}(t)$. Bellman expanded on the the equivalence of the open loop trajectory problem and the feedback control problem by introducing the principle of optimality, which is now the cornerstone of dynamic programming [37].

Consider the dynamic process with a scalar control input, $\mathbf{u}(t) = u$:

$$\dot{\boldsymbol{\xi}} = \frac{d\boldsymbol{\xi}}{dt} = f(\boldsymbol{\xi}, u) \tag{9}$$

For this process the performance measure to be minimized is:

$$J = \int_t^T L(\boldsymbol{\xi}, u)d\tau, \tag{10}$$

which corresponds to $f_s(\mathbf{x_p}, \mathbf{x_c})$ in Eqns. (4) – (8). Using the chain rule, $\dot{J} = J'_{\boldsymbol{\xi}} f(\boldsymbol{\xi}, u)$, where $J_{\boldsymbol{\xi}} = \partial J/\partial \boldsymbol{\xi}$. From the performance integral (Eqn. (10)), $\dot{J} = -L(\boldsymbol{\xi}, u)$. Equating both expressions for $\dot{J}$, we obtain the partial differential equation:

$$-H = J'_{\boldsymbol{\xi}} f(\boldsymbol{\xi}, u) + L(\boldsymbol{\xi}, u) = 0 \tag{11}$$

In the optimal control literature H is referred to as the Hamiltonian or H-function. By Bellman's principle of optimality, the optimal control is:

$$u^* = \underset{u \in \Omega}{\operatorname{argmax}}\ H(J_{\boldsymbol{\xi}}, \boldsymbol{\xi}, u) \tag{12}$$

where Ω is the feasible control design space. This can be enforced using the inequality constraint in Eqn. (8). When u^*, the optimal value of the control u, is substituted into Eqn. (11), the control-free Hamilton-Jacobi-Bellman (HJB) equation is obtained. The solution of the HJB equation gives the optimal control as a function of the state.

The HJB equation entails the gradient $J_{\boldsymbol{\xi}}$, and hence requires differentiability of J. The standard HJB equation solution approach is by the method of characteristics. Let:

$$\mathbf{p} = -J_{\boldsymbol{\xi}} \quad \text{(the "costate")} \tag{13}$$

Then Eqn. (11) becomes:

$$H = \mathbf{p}' f(\boldsymbol{\xi}, u) - L(\boldsymbol{\xi}, u) = 0 \tag{14}$$

The Pontryagin Minimum Principle (PMP), derived by methods of the calculus of variations, starts with the Hamiltonian function in Eqn. (14), without identifying the costate $\mathbf{p}$ with -$J_{\boldsymbol{\xi}}$, eliminating the requirement that J is differentiable. The relationship between the Dynamic Programming approach in Eqn. (11) and the Minimum Principle approach in Eqn. (14) is proven in [22].

Continuing with the Pontryagin formulation, let $u(\tau)^*$ be the optimal control in the sense that the value of J using u^* is smaller than it is for any other control signal $u(\tau)$. According to the Minimum Principle, the necessary conditions for the control signal $\{u^*(\tau) : \tau \in [t,T]\}$ to be optimal are:

1. The H-function in Eqn. (14) has an absolute minimum at $u(\tau) = u^*(\tau)$:
$$H(\boldsymbol{\xi}^*, \mathbf{p}^*, u^*) \leq H(\boldsymbol{\xi}^*, \mathbf{p}^*, u) \tag{15}$$
2. The value of the Hamiltonian function H is zero:
$$H(\boldsymbol{\xi}^*, \mathbf{p}^*, u^*) = 0 \tag{16}$$

These observations lead to the dynamic and costate canonical representation [7,22,36]:

$$\dot{\xi} = \frac{\partial H}{\partial \mathbf{p}}(\boldsymbol{\xi}, \mathbf{p}, u) = f(\boldsymbol{\xi}, u) \tag{17}$$

$$\dot{\mathbf{p}} = -\frac{\partial H}{\partial \boldsymbol{\xi}}(\boldsymbol{\xi}, \mathbf{p}, u) = -\frac{\partial L}{\partial \boldsymbol{\xi}} - \mathbf{p}' f(\boldsymbol{\xi}, u) \tag{18}$$

If Eqn. (9) is Linear Time Invariant (LTI) and the integrand of Eqn. (10) possesses a form that is quadratic in state and control with an infinite horizon time interval, i.e.,

$$J = \int_t^\infty (\boldsymbol{\xi}'\mathbf{Q}\boldsymbol{\xi} + u'Ru)d\tau,$$

then Eqns. (17) and (18) are fashioned by:

$$\dot{\xi} = \frac{\partial H}{\partial \mathbf{p}}(\boldsymbol{\xi}, \mathbf{p}, u) = \mathbf{A}\boldsymbol{\xi}^*(t) + \mathbf{B}u^*(t) \tag{19}$$

$$\dot{\mathbf{p}} = -\frac{\partial H}{\partial \boldsymbol{\xi}}(\boldsymbol{\xi}, \mathbf{p}, u) = -\mathbf{Q}\boldsymbol{\xi}^*(t) - \mathbf{A}'\mathbf{p}(t) \tag{20}$$

$$0 = \frac{\partial H}{\partial u}(\mathbf{p}, u) = R_u u^* + \mathbf{B}\mathbf{p}^* \tag{21}$$

where $\mathbf{Q}$ is an $n \times n$ positive definite matrix known as the *state weighting matrix*, and R_u is a scalar known as the *control weighting factor* [22]. $\mathbf{A}$ and $\mathbf{B}$ are the LTI system matrices. Solving (21) for u^* and substituting into (19) yields a set of 2n linear homogenous differential equations:

$$\begin{bmatrix} \dot{\boldsymbol{\xi}}^* \\ \dot{\mathbf{p}}^* \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{R}^{-1}\mathbf{B}' \\ -\mathbf{Q} & -\mathbf{A}' \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}^* \\ \mathbf{p}^* \end{bmatrix} \tag{22}$$

This system of differential equations can be solved for the optimal state trajectory. Using the boundary conditions, $\mathbf{p}^*$ can be eliminated and the optimal control u^* is:

$$u^* = -R_u^{-1}\mathbf{B}'\mathbf{M}\boldsymbol{\xi} \tag{23}$$

where $\mathbf{M}$ is the solution to the matrix Riccati equation:

$$\dot{\mathbf{M}} = -\mathbf{M}\mathbf{A} - \mathbf{A}'\mathbf{M} - \mathbf{Q} + \mathbf{M}\mathbf{B}R_u^{-1}\mathbf{B}'\mathbf{M} \tag{24}$$

Note the dependence of $\mathbf{M}$ on t has been removed for convenience. $\mathbf{M}$ is an $n \times n$ symmetric matrix; therefore $n(n+1)/2$ first-order differential equations must be solved to find $\mathbf{M}$.

3.2 Control Subproblem

In the optimal control subproblem, we consider a special form of the *state regulator* problem described in Section 3.1. In the extension of this problem presented in this section, a finite horizon quadratic cost functional is minimized subject to linear differential constraints, with the distinguishing characteristic that both the cost functional and the differential constraints are functions of time, t, *and* the linking variables, $\mathbf{z}$. In considering this special problem, we show that the linking variables impose an additional condition to the first order conditions of the Minimum Principle of Pontryagin (PMP) by taking the following steps:

1. Define precisely the PMP for this special state regulator problem.
2. Demonstrate how to obtain the optimal state trajectory, $\boldsymbol{\xi}(t)^*$, for this special state regulator problem using the PMP.

P2, in the PMP framework, is:

$$\min_{u(t),\mathbf{z}_2} \quad \frac{1}{2}\int_{t_0}^{t'} (\boldsymbol{\xi}(t,\mathbf{z}_2)'\mathbf{Q}\mathbf{x}(t,\mathbf{z}_2) + u(t)'\mathbf{R}u(t))dt + \mathbf{v}\mathbf{c}_2' + \|\mathbf{w} \circ \mathbf{c}_2\|_2^2$$
$$\text{subject to:} \quad \dot{\boldsymbol{\xi}}(t,\mathbf{z}_2) = \mathbf{A}(\mathbf{z}_2)\boldsymbol{\xi}(t,\mathbf{z}_2) + \mathbf{B}u(t) \tag{25}$$

The first term of the objective function is a quadratic cost integral with finite horizon, and the minimization is performed with respect to both $u(t)$ and $\mathbf{z}_2$. The additional terms comprise the ALC penalty function that helps manage interactions between plant and control design subproblems, introduced in Eqns. (7) and (8). Note that $\mathbf{c}_2' = \mathbf{c}_2'(\mathbf{z}_1, \mathbf{z}_2) = \mathbf{z}_1 - \mathbf{z}_2$. Thus, Eqn. (25) is comprised of two important parts: A quadratic cost integral that measures the cost on the system dynamics and control, and a penalty function that aims to enforce system consistency. The two parts of the objective function are weakly coupled as expressed in the dynamic state vector $\boldsymbol{\xi}(t,\mathbf{z}_2)$. This coupling arises from the fact that the dynamic state is the solution of:

$$\boldsymbol{\xi}(t,\mathbf{z}_2) = \boldsymbol{\xi}_0 e^{\mathbf{A}(\mathbf{z}_2)} + \int_{t_0}^{t} e^{\mathbf{A}(\mathbf{z}_2)(t-\tau)}\mathbf{B}u(\tau)d\tau,$$

i.e., dynamic plant response depends not only on time, but also on plant characteristics $\mathbf{z}_2$ that vary with plant design $\mathbf{x_p}$ (when $\mathbf{z}_1$ and $\mathbf{z}_2$ are consistent). To solve optimal control problems using the PMP, knowledge of the boundary conditions or transversality conditions is required. We assume that t_f is fixed ($t_f - T = 0$), the problem ends at $t = T$, and the final state is specified: $\boldsymbol{\xi}(t_f) = x(T) = x_f$. Next we form the H-function for the co-design subproblem using Eqn. (14):

$$\begin{aligned} H = & \frac{1}{2}\left[(\boldsymbol{\xi}(t,\mathbf{z}_2)'\mathbf{Q}\boldsymbol{\xi}(t,\mathbf{z}_2) + u(t)'\mathbf{R}u(t))\right] \\ & + \mathbf{v}\mathbf{c}_2' + \|\mathbf{w} \circ (\mathbf{c}_2)\|_2^2 \\ & + \mathbf{p}(t)\left[\mathbf{A}(\mathbf{z}_2)\boldsymbol{\xi}(t,\mathbf{z}_2) + \mathbf{B}u(t)\right] \end{aligned} \tag{26}$$

For Eqn. (26), conditions (15) and (16) become:

$$H(\boldsymbol{\xi}^*, \mathbf{p}^*, u^*, \mathbf{z}_2^*) \leq H(\boldsymbol{\xi}^*, \mathbf{p}^*, u, \mathbf{z}_2) \tag{27}$$

$$H(\boldsymbol{\xi}^*, \mathbf{p}^*, u^*, \mathbf{z}_2^*) = 0 \tag{28}$$

To meet the above conditions, we compute the first order necessary conditions for optimality:

$$\dot{\boldsymbol{\xi}}(t,\mathbf{z}_2)= \frac{\partial H}{\partial \mathbf{p}} = \mathbf{A}(\mathbf{z}_2)\boldsymbol{\xi}(t,\mathbf{z}_2)+\mathbf{B}u(t) \quad (29)$$

$$\dot{p}(t,\mathbf{z}_2)= \frac{\partial H}{\partial \boldsymbol{\xi}} = -\boldsymbol{\xi}(t,\mathbf{z}_2)\mathbf{Q}-\mathbf{A}(\mathbf{z}_2)\mathbf{p}(t) \quad (30)$$

$$0= \frac{\partial H}{\partial u} = u(t)\mathbf{R}+\mathbf{p}(t)\mathbf{B} \quad (31)$$

$$0= \frac{\partial H}{\partial \mathbf{z}_2} = \Psi(\boldsymbol{\xi},\mathbf{p},\mathbf{z}_2) \quad (32)$$

Equations (29) – (32) are the first order necessary conditions for **P2**. They are a system of differential algebraic equations (DAEs) which must be solved simultaneously. Equations (29) and (30) are ordinary differential equations (ODEs), and Eqns. (31) and (32) are the algebraic equations. In some problems, it may be possible to eliminate either Eqn. (31) or Eqn. (32), or both. In that case, Eqns. (29) – (32) collapse into a system of ODEs that can be solved simultaneously for the optimal trajectory. Note that at this point the linking variable $\mathbf{z}_2$ can be either solved for explicitly or computed using a numerical solution. Due to the nonlinear relationship between $\mathbf{x}_p$ and $\mathbf{A}$ in the circuit problem presented in the next section, the algabraic equations could not be eliminated.

Now we prove the existence and uniqueness of the global optimum for Eqn. (26) in the scalar case while noting that it can be extended to the vector case with ease. We start with the Weierstrass Theorem to help show existence of a global minimum.

Lemma 1 (Weierstrass Theorem). *Let $\mathbb{S}$ be a compact subset of a finite-dimensional real vector space, $\mathbb{V}$, and let f be the mapping $f:\mathbb{S}\rightarrow\mathbb{R}$. If f is a continuous function and $x\in\mathbf{dom}(f)$, then the mapping f attains a global maximum and a global minimum.* □

For the control subproblem, it is also important to establish the conditions under which a unique solution will be available. The following result will prove the existence and uniqueness of the optimum for our specific problem.

Theorem 1 (P2 Unique Minimum). *Let $\mathbb{S}$ be a compact subset of $\mathbb{R}$ and let:*

$$H=\frac{1}{2}(\xi^2(z_2)q+u^2R)+vz_2+\|wz_2\|_2^2+p(a(z_2)\xi(z_2)+bu)$$

be the scalar H-function under consideration where $\xi,u\in\mathbb{S}$ and $w,v,z_2,R\in\mathbb{S}_+$. If $q>0$, then the function H always attains a unique minimum in $\mathbb{S}$.

Proof. Since $\xi,u\in\mathbb{S}$ and $w,v,z_2,q,R\in\mathbb{S}_+$ with $q>0$ assumed and since it is easy to show that H is a continuous function, we conclude that there exists:

$$H_{min} = \{\forall\xi(z_2)\in\mathbb{S}: f(\xi_{min}(z_2))\leq f(\xi(z_2))\}$$
$$H_{max} = \{\forall\xi(z_2)\in\mathbb{S}: f(\xi_{max}(z_2))\geq f(\xi(z_2))\}$$

That is, we conclude that a global minimum and a global maximum exist for H. This establishes the *existence* of the optimum.

To show *uniqueness* let us consider the Hessian for our H-function. If the Hessian is positive semidefinite $\forall\xi\in\mathbb{S}$, then H is convex, which implies no duality gap and by implication means a unique solution exists [17]. After taking the second partial of H with respect to ξ:

$$\frac{\partial^2 H}{\partial \xi^2}={z_2}^2q=0$$

Since $z_2\in\mathbb{S}$ and $q>0$ and the Hessian is positive semidefinite, a unique global minimum exists for H. □

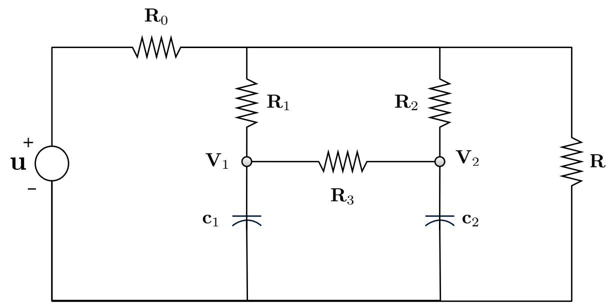

Figure 5: Circuit design example.

4 Circuit Design Example

A circuit design problem, adapted from [19], is presented here to clarify the ALC co-design formulation. The circuit to be design is illustrated in Fig. 5. The objective is to regulate the voltages V_1 and V_2 and to choose an optimal value for the capacitance c_1 to minimize:

$$J=\int_t^\infty(\boldsymbol{\xi}(t,c_1)'\mathbf{Q}\boldsymbol{\xi}(t,c_1)+u'Ru)d\tau$$

$$\text{subject to:} \quad \dot{\boldsymbol{\xi}}(t,c_1)=\mathbf{A}(c_1)\boldsymbol{\xi}(t,c_1)+\mathbf{B}u$$

Note that c_1 is a function of the plant design variables $\boldsymbol{\rho}$, defined below. In the formulations that follow, the copy of c_1 in the plant design problem is z_1, and the copy in the control design problem is z_2. The differential equations that govern the dynamic response of the circuit can be put into linear state space form, as shown in Fig. (4), defined by the matrices:

$$\dot{\boldsymbol{\xi}}(t,c_1)=\mathbf{A}(c_1)\boldsymbol{\xi}(t,c_1)+\mathbf{B}u(t)$$

$$\mathbf{A}(c_1) = \begin{bmatrix} \frac{-a_{11}}{c_1} & \frac{a_{12}}{c_1} \\ \frac{a_{21}}{c_2} & -\frac{a_{22}}{c2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b1 \\ b2 \end{bmatrix},$$

where:

$$\begin{aligned}
a_{11} &= -\; [R_3^{-1} + D_1(R_2^{-1} + R_0^{-1} + R^{-1})] \\
a_{12} &= [R_3^{-1} + R_2^{-1} D1] \\
a_{21} &= [R_3^{-1} + R_1^{-1} D2] \\
a_{22} &= -\; [R_3^{-1} + D_2(R_1^{-1} + R_0^{-1} + R^{-1})] \\
D_1 &= [1 + R_1(R_0^{-1} + R_2^{-1} + R^{-1})]^{-1} \\
D_2 &= [1 + R_2(R_0^{-1} + R_1^{-1} + R^{-1})]^{-1} \\
b_1 &= R_0^{-1} D_1 \\
b_2 &= R_0^{-1} D_2
\end{aligned}$$

For the control subproblem the desired state conditions are constant. The output matrix $\mathbf{C}$ is the identity matrix. In a spherical capacitor, the capacitance c_1 depends on the outer radius ρ_b and inner radius ρ_a:

$$c_1 = \frac{4\pi\varepsilon\rho_a\rho_b}{\rho_b - \rho_a}$$

where $\varepsilon = 8.85 \times 10^{-12} \frac{F}{m}$ is the permittivity constant in free space. Packaging requirements are given by:

$$\rho_a \leq \rho_b \leq \rho_{bmax}, \qquad \rho_a + \rho_g \leq \rho_b$$

where ρ_{bmax} is a packaging constraint and ρ_g is the minimum allowable gap between ρ_a and ρ_b. Note that if $\rho_a + \rho_g \leq \rho_b$ is satisfied, then $\rho_a \leq \rho_b$ is also satisfied, eliminating the need for the latter constraint. The simultaneous optimization formulation is:

$$\begin{aligned}
\min_{u,\boldsymbol{\rho}=[\rho_a,\rho_b]} \quad & \int_t^\infty \left(\boldsymbol{\xi}(t,\boldsymbol{\rho})'\mathbf{Q}\boldsymbol{\xi}(t,\boldsymbol{\rho}) + u' R_u u\right) dt \\
\text{subject to:} \quad & \dot{\boldsymbol{\xi}}(t,\boldsymbol{\rho}) = \mathbf{A}(\boldsymbol{\rho})\boldsymbol{\xi}(t,\boldsymbol{\rho}) + \mathbf{B}u \\
& \text{rank}[\mathbf{B}, \mathbf{A}(\boldsymbol{\rho})\mathbf{B} \ldots \mathbf{A}(\boldsymbol{\rho})^{n-1}\mathbf{B}] = \dim(\boldsymbol{\xi}(t,\boldsymbol{\rho})) \\
& \text{eig}(\mathbf{A}(\boldsymbol{\rho}) - \mathbf{BK}) \leq 0 \\
& \rho_b \leq \rho_{bmax}, \qquad \rho_a + \rho_g \leq \rho_b,
\end{aligned}$$

where the rank and eigenvalue constraints ensure stability and controllability. The nested optimization formulation is given by:

$$\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & J^*(\boldsymbol{\rho}) \\
\text{subject to:} \quad & \rho_b \leq \rho_{b\max} \\
& \rho_a + \rho_g \leq \rho_b \\
\text{where:} \quad & J^*(\boldsymbol{\rho}) = \min_u \int_t^\infty (\boldsymbol{\xi}(t,\boldsymbol{\rho})'\mathbf{Q}\boldsymbol{\xi}(t,\boldsymbol{\rho}) + u' R_u u)\, dt \\
\text{subject to:} \quad & \dot{\boldsymbol{\xi}}(t,\boldsymbol{\rho}) = \mathbf{A}(\boldsymbol{\rho})\boldsymbol{\xi}(t,\boldsymbol{\rho}) + \mathbf{B}u \\
& \text{rank}[\mathbf{B}, \mathbf{A}(\boldsymbol{\rho})\mathbf{B} \ldots \mathbf{A}(\boldsymbol{\rho})^{n-1}\mathbf{B}] = \dim(\boldsymbol{\xi}(t,\boldsymbol{\rho})) \\
& \text{eig}(\mathbf{A}(\boldsymbol{\rho}) - \mathbf{BK}) \leq 0
\end{aligned}$$

The ALC formulation is given by:

P1:

$$\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & \phi(z_1(\boldsymbol{\rho}), z_2, v_1, w_1) \\
\text{subject to:} \quad & \rho_b \leq \rho_{bmax} \\
& \rho_a + \rho_g \leq \rho_b
\end{aligned}$$

P2:

$$\begin{aligned}
\min_{u,z_2} \quad & \int_t^\infty \boldsymbol{\xi}(t,z_2)'\mathbf{Q}\boldsymbol{\xi}(t,z_2) + u' R_u u \, dt + \phi(z_1, z_2, v_2, w_2) \\
\text{subject to:} \quad & \dot{\boldsymbol{\xi}}(t,z_2) = \mathbf{A}(z_2)\boldsymbol{\xi}(t,z_2) + \mathbf{B}u \\
& \text{rank}[\mathbf{B}, \mathbf{A}(z_2)\mathbf{B} \ldots \mathbf{A}(z_2)^{n-1}\mathbf{B}] = \dim(\boldsymbol{\xi}(t,z_2)) \\
& \text{eig}(\mathbf{A}(z_2) - \mathbf{BK}) \leq 0,
\end{aligned}$$

where z_2 is the target value for c_1, which is the linking variable in this example problem, set by **P2**, and z_1 is the value of c_1 achieved by **P1**. The H-function for this problem becomes:

$$\begin{aligned}
H = {} & p_1 (b_1 u - a_{11}\xi_1 + a_{12}\xi_2) + p_2 (b_2 u + a_{21}\xi_1 + a_{22}\xi_2) \\
& + \frac{R_u u^2}{2} + \frac{q_1}{2}\xi_1^2 + \frac{q_2}{2}\xi_2^2 + v_2(z_1 - z_2) + w_2^2 (z_1 - z_2)^2. \quad (33)
\end{aligned}$$

The first two terms of Eqn. (33) are "costate" terms (see Eqn. (14)). The remaining terms constitute L, the instantaneous cost, in Eqn. (14). The first order optimality conditions for this problem are:

$$H_{\xi_1} = a_{21} p_2 - a_{11} p_1 + q_1 \xi_1 \qquad (34)$$

$$H_{\xi_2} = a_{12} p_1 - a_{22} p_2 + q_2 \xi_2 \qquad (35)$$

$$H_{p_1} = b_1 u - a_{11}\xi_1 + a_{12}\xi_2 \qquad (36)$$

$$H_{p_2} = b_2 u + a_{21}\xi_1 - a_{22}\xi_2 \qquad (37)$$

$$H_{z_2} = w_2^2 (2z_2 - 2z_1) - v_2. \qquad (38)$$

Eqns. (34) – (38) are a system of DAEs that need to be solved simultaneously. This system was solved numerically in MATLAB™with the following circuit parameters:

$$\begin{aligned}
& R = 20M\Omega, \quad R_0 = 10M\Omega, \quad R_1 = 50M\Omega \\
& R_2 = 30M\Omega, \quad R_3 = 4M\Omega, \quad c_2 = 1\mu F \\
& q_1 = 10, \quad q_2 = 1, \quad \rho_g = 0.20\,\text{mm}, \quad \rho_{bmax} = 0.5\,\text{m}
\end{aligned}$$

After eleven ALC outer loop iterations, with a convergence tolerance 1×10^{-4} on the Euclidian norm of the consistency constraint $z_1 - z_2 = 0$, ALC converged, where $z_1^* = 0.1303$ μF and $z_2^* = 0.1304$ μF. The corresponding time histories of V_1 and V_2 are plotted in Fig. 6, where the initial conditions are $V_1(0) = 15$ and $V_2(0) = -2.5$. Both states converge quickly to the setpoint.

5 Conclusion

The design of physical systems and their controllers is a set of coupled tasks. The sequential design process utilized in current industry practice does not account fully for plant and controller coupling; it can result in suboptimal designs, and may

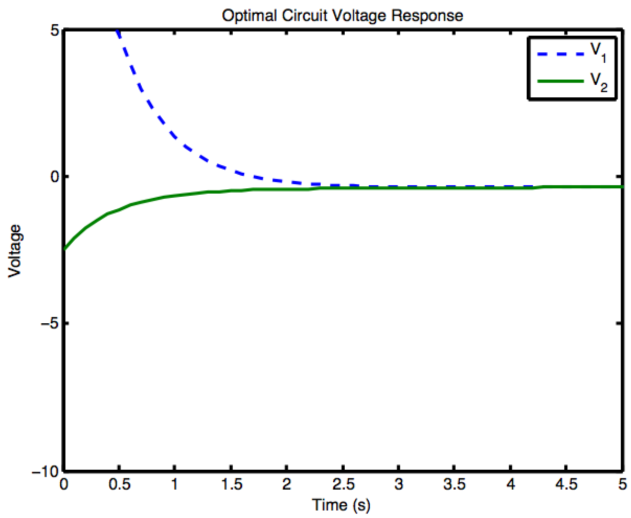

Figure 6: Optimal response of V_1 and V_2 using $c_1 = 0.1304\mu F$.

even fail to produce feasible system designs for particularly challenging system design problems. System design methods that account for plant and controller coupling have been developed, including the nested approach that utilizes existing optimal control techniques, and has been proven to produce system optimal solutions. The nested approach, however, has limitations. In this article we introduced a new approach to co-design; the control design problem was integrated with plant design using augmented Lagrangian coordination, a decomposition-based design optimization approach. A control subproblem was defined that incorporates linking variables and the associated penalty functions to help manage interactions between control design decisions and the rest of the system design.

The first order conditions for the control design subproblem **P2** were derived using PMP. This direct solution to the ALC control subproblem not only obtains optimal control inputs and state trajectories, but optimal linking variable values, which are time-invariant. The solution presented here is limited to co-design problems with uni-directional coupling, and no shared design variables. Generalizing this approach to problems with feedback coupling and shared design variables is a topic for future work. The ALC co-design method presented here was demonstrated using a circuit design problem.

This approach to system design is valuable because it allows domain experts to continue advancing specialized analysis and design techniques, but within a framework that provides formal management of system interactions. In current practice these interactions typically are managed only informally. Adopting a more holistic system design approach can enable designers to improve system performance because of the mechanisms in place to account for interactions, while providing freedom to focus deeply on domain expertise.

REFERENCES

[1] Allison, J., 2008. "Optimal partitioning and coordination decisions in decomposition-based design optimization". Ph.D. dissertation, University of Michigan.

[2] Drăgan, M., 2002. "On decomposing large sparse systems of nonlinear equations". In the proceedings of the 4th International Workshop on Symbolic and Numeric Algorithms for Scientific Computing, Timişoara, Romania.

[3] Kusiak, A., and Larson, N., 1995. "Decomposition and representation methods in mechanical design". *Journal of Mechanical Design, Transactions of the ASME,* **117B**, pp. 17–24.

[4] Michelena, N., and Papalambros, P., 1997. "A hypergraph framework for optimal model-based decomposition of design problems". *Computational Optimization and Applications,* **8**(2), pp. 173–196.

[5] Sobieszczanski-Sobieski, J., and Haftka, R., 1997. "Multidisciplinary aerospace design optimization: Survey of recent developments". *Structural Optimization,* **14**(1), pp. 1–23.

[6] Bendsoe, M., Olhoff, N., and Taylor, J., 1987. "On the design of structure and controls for optimal performance of actively controlled flexible structures". *Mech. Struct.,* **15**, pp. 265–295.

[7] Athans, M., and Falb, P., 1966. *Optimal control : an introduction to the theory and its applications*. McGraw-Hill, New York.

[8] Fathy, H., 2003. "Combined plant and control optimization: Theory, strategies and applications". Ph.D. dissertation, University of Michigan.

[9] Cramer, E. J., Dennis, J. E., Frank, P. D., Lewis, R. M., and Shubin, G. R., 1994. "Problem formulation for multidisciplinary optimization". *SIAM Journal of Optimization,* **4**, pp. 754–776.

[10] Allison, J., Kokkolaras, M., and Papalambros, P., 2007. "On selection of single-level formulations for complex system design optimization". *Journal of Mechanical Design, Transactions of the ASME,* **129**(9).

[11] Allison, J., Kokkolaras, M., and Papalambros, P., 2009. "Optimal partitioning and coordination decisions in decomposition-based design optimization". *Journal of Mechanical Design, Transactions of the ASME,* **131**(8), pp. 081008–1–081008–8.

[12] Tosserams, S., Etman, L., and Rooda, J., 2007. "An augmented Lagrangian decomposition method for quasiseparable problems in MDO". *Structural and Multidisciplinary Optimization,* **34**(3).

[13] Tosserams, S., Etman, L., and Rooda, J., 2008. "Augmented Lagrangian coordination for distributed optimal de-

sign in MDO". *International Journal for Numerical Methods in Engineering,* **73**(13), pp. 1885–1910.

[14] Kim, H. M., Michelena, N. F., Papalambros, P. Y., and Jiang, T., 2003. "Target cascading in optimal system design". *Journal of Mechanical Design, Transactions of the ASME,* **125**(3), pp. 474–480.

[15] Stephanopoulos, G., and Westerber, W., 1975. "The use of hestenes' method of multipliers to resolve dual gaps in engineering system optimization". *J. Optim. Theory App.,* **15**, pp. 285–309.

[16] Ruszczynski, R., 1995. "On convergence of augmented lagrangian decomposition method for sparse convex optimization". *Mathematics of Operations Research,* **20**(3), pp. 634–656.

[17] Bertsekas, D., 1999. *Nonlinear Programming*, second ed. Athena Scientific, Belmont, MA.

[18] Allison, J., and Papalambros, P., 2010. "Consistency constraint allocation in augmented Lagrangian coordination". *Forthcoming in the Journal of Mechanical Design, Transactions of the ASME.*

[19] Friedland, B., 1986. *Control System Design: An Introduction to State Space Methods*. Dover.

[20] Pontryagin, L., Boltyanskii, V., Gamkrelidze, R., and Mishchenko, E., 1962. *The mathematical theory of optimal processes (International series of monographs in pure and applied mathematics)*. Interscience Publishers.

[21] Bryson, A. E., and Ho, Y. C., 1969. *Applied Optimal Control*. Blaisdell, New York.

[22] Kirk, D., 1970. *Optimal Control Theory: An Introduction*. Dover Publications.

[23] Kalman, R., 1960. "Contributions to the theory of optimal control". *Boletin de la Sociedad Mathematica Mexicana,* **5**, pp. 102–119.

[24] Kalman, R., 1960. "A new approach to linear filtering and prediction problems". *Journal of Basic Engineering, ASME,* **82**, pp. 35–45.

[25] Reyer, J., and Papalambros, P., 2000. "An investigation into modeling and solution strategies for optimal design and control". In the Proceedings of DETC'00, ASME 2000 Design Engineering Technical Conferences and Computers and Information in Engineering Conference, DETC2000/DAC-14253.

[26] Hale, A., Lisowski, R., and Dahl, W., 1985. "Optimal simultaneous structural and control design of maneuvering flexible spacecraft". *Journal of Guidance, Control and Dynamics,* **8**(1), pp. 86–93.

[27] Isermann, R., 1996. "Modeling and design methodology for mechatronic systems". *IEEE/ASME Trans. Mechatron.,* **1**(1), pp. 16–28.

[28] Nazari, S., Brittain, K., and Haessig, D., 2005. "Rapid prototyping and test of a c4isr ku-band antenna pointing and stabilization system for communications on-the-move". *IEEE Military Communications Conference 2005,* **3**(1), pp. 1528–1534.

[29] Fathy, H., Reyer, J., Papalambros, P., and Ulsoy, G., 2001. "On the coupling between the plant and controller optimization problems". *Proceedings of the American Control Conference,* **1**(1), pp. 1864–1869.

[30] Sunar, M., and Rao, S., 1993. "Simultaneous passive and active control design of structures using reduced-size models". *Journal of Aicraft,* **29**.

[31] Belvin, W. K., and Park, K. C., 1990. "Structural tailoring and feedback control synthesis: An interdisciplinary approach". *Journal of Guidance,* **13**.

[32] Peters, D., Papalambros, P., and A.G., U., 2009. "On measures of coupling between the artifact and controller optimal design problems". In the Proceedings of the ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, DETC2009-86868.

[33] Mutapcic, A., and Boyd, S., 2009. "Processor speed control with thermal constraints". *IEEE Transactions on Circuits and Systems I*, pp. 56–59.

[34] Bertsekas, D. P., and Tsitsiklis, J. N., 1997. *Parallel and Distributed Computation: Numerical Methods*. Athena Scientific, Belmont, MA.

[35] Papalambros, P., and Wilde, D., 2000. *Principles of Optimal Design: Modeling and Computation*, second ed. Cambridge University Press, Cambridge, UK.

[36] Friedland, B., 1996. *Advanced Control System Design*. Prentice Hall.

[37] Bellman, R., 1957. *Dynamic Programming*. Princeton University Press, Princeton, NJ.