Analytical Target Cascading in Aircraft Design

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Analytical Target Cascading (ATC) is a product development tool that computes component design specifications such that the final system design is consistent and meets design targets. ATC is useful for complex product design that must be approached by decomposition, and facilitates concurrent design activities. While ATC has been applied successfully to automotive design, this article introduces the application of ATC to aircraft design, and discusses how it can be congruent with current design practice. ATC is used to solve an aircraft design problem where several flight regimes are considered separately. ATC can be used to balance low-fidelity system analysis and component-level multidisciplinary design optimization (MDO) activities. Finally, ATC may be used to coordinate overall aircraft design, with MDO employed to solve tightly coupled disciplinary problems that exist within ATC elements.

I. Introduction

Analytical Target Cascading (ATC) is a multi-level optimization methodology for coordinating the design and development of complex systems.^{1,2} ATC was developed to assist in early product development stages, and is intended for design problems with a hierarchical structure. Figure 1 illustrates a simplified hierarchical decomposition of aircraft design, where *i* specifies the level and *j* specifies the element within the hierarchy (additional terminology is discussed in the following section). Child element analyses in the hierarchy generate responses that are required as inputs to respective parent elements. Detail and fidelity typically increase progressively down the hierarchy. ATC has natural application to the coordination between systemlevel analysis already in use for aircraft conceptual design,³ and multidisciplinary design optimization (MDO) methods used for aircraft preliminary design.

Top level system targets are cascaded through all elements in the hierarchy such that the top level targets are met as closely as possible, and that the entire system is consistent. Once element design specifications have been obtained, the individual design tasks may be completed concurrently and independently. Design groups can have confidence that the overall objectives will be met and the system will be consistent, since system interactions were considered during the target cascading process. This approach improves efficiency, reduces the need for iterations at later design stages, prolongs design freedom, and facilitates the use of legacy design tools.

An ATC approach to aircraft design is presented in this article, and a detailed example for aircraft design over multiple regimes is put forward. In this example (and other complex system design problems), difficulties are encountered when attempting to solve with an all-in-one (AiO) approach. It was found that ATC overcame these difficulties and produced results superior to the AiO solution.

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Figure 1. Simplified hierarchical decomposition of an aircraft design

II. ATC Formulation

In the ATC process, an optimization problem is formulated for each element in the system hierarchy (Figure 1), which coordinates the interactions of any child elements, while ensuring optimality and feasibility of its own design. The solution to this optimization problem produces targets for child element problems. The child element seek to match these targets as closely as possible. This result is communicated back up to the parent element, which then adjusts the child element targets appropriately. This is repeated until responses match the targets, resulting in a consistent and optimal system design. Note that element optimization problems are solved to completion before any targets or responses are communicated to other elements. This is in contrast to many MDO methods that utilize nested optimization where lower level problems are solved to completion at each top level optimization.

This article uses the ATC notation as presented by Allison *et al.*⁴ An ATC problem is formulated by identifying shared quantities that couple elements (such as shared variables or analysis interactions), making copies in appropriate elements, and assigning penalties for inconsistencies among these shared quantities. In the general ATC architecture, elements provide targets for other elements, and these elements seek to meet these targets, subject to local design constraints. Targets are used to coordinate consistency between elements, i.e., matching of shared quantities. A coordination strategy is used to execute each element design problem iteratively, holding inputs from other elements fixed, until compatibility constraints are satisfied within a desired tolerance. In the event that a consistent solution is unattainable, then insights gained from the process can be used to modify targets or adjust the feasible space in order to achieve a feasible solution. The formulation for the optimization problem P_{ij} associated with element j at level i is given in Equation (1).

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The responses \mathbf{R}_{ij}^i are outputs of the analysis \mathbf{r}_{ij} of element j at level i required by the parent element as analysis inputs. \mathbf{R}_{ik}^{i} are also outputs of the analysis \mathbf{r}_{ij} , but are required by child element k. \mathbf{R}_{ij}^{i-1} are targets set by the parent element p at level i-1 for \mathbf{R}_{ij}^i . $\mathbf{R}_{(i-1)p}^i$ are targets set by element j at level i for $\mathbf{R}_{(i-1)p}^{i-1}$, which are responses generated by the parent element p at level i-1 that are inputs to the analysis of element j. The shared variables^a \mathbf{y}_{ij}^i are the design variables required at element j that are shared with other elements, as determined by element j at level i. The vector \mathbf{y}_{ip}^{i-1} is comprised of shared variable targets set by the parent element p at level i - 1 for child elements at level i. The binary valued selection matrix \mathbf{S}_j is multiplied by the aggregate vector \mathbf{y}_{ip}^{i-1} to choose the targets that correspond to \mathbf{y}_{ij}^i . \mathbf{S}_j is also used to form the vector of corresponding shared variable penalty weights from \mathbf{w}_{ip}^y . The \circ operator denotes term by term vector multiplication, such that each term in a weighting vector is multiplied by the term in the deviation vector with the same index. This allows every shared value to have assigned to it a weight expressing the relative importance of consistency for that shared value. \mathbf{S}_p and \mathbf{S}_k are the matrices that select what outputs of analysis \mathbf{r}_{ij} are to be passed to the parent element and child element k, respectively. The fourth and sixth term of the objective enforce penalties for deviation between targets set by element j for all elements k that belong to the set of all children \mathcal{C}_{ij} of element j and the corresponding responses and linking variables determined at level i + 1. The fifth term of the objective enforces consistency between targets set by child elements and the corresponding responses from element j.

Several values computed by other elements are inputs to problem P_{ij} and are held fixed during the optimization of that element. These parameters include both targets and responses sent to an element from a parent $(\mathbf{R}_{ij}^{i-1}, \mathbf{R}_{(i-1)p}^{i-1}, \mathbf{S}_j \mathbf{y}_{ip}^{i-1})$, and targets and responses sent from a child to an element $(\mathbf{R}_{ik}^{i+1}, \mathbf{R}_{(i+1)k}^{i+1})$, $\mathbf{y}_{(i+1)k}^{i+1}$). ATC convergence properties have been proven for a certain class of coordination strategies.⁵

III. Aircraft Design Example

This example considers the design of an aircraft for multiple flight regimes. A Boeing 747-400 design was optimized over three mission segments—take-off, cruise, and second segment climb (SSC)—in order to assure satisfactory performance under typical B747 operating conditions. This approach can be viewed as designing a separate aircraft that is ideal for each regime, and then coordinating these designs toward a single aircraft design that performs well in all regimes. This is similar to a product family design approach, except that complete commonality is enforced. Future work will involve the design of separate aircraft for distinct missions using ATC. The aircraft analysis model used for this design problem employs purely algebraic relations, yet captures important relations and interactions.

The design problem is to find the design variable values \mathbf{x} (Table 1) that minimize the gross take-off weight (GTOW), subject to the performance and design constraints $\mathbf{g}(\mathbf{x})$ listed in Table 1. Several of the constraints imposed are due to Federal Aviation Regulations (FAR). Two important parameters are also listed in Table 1. The formal design problem is presented in Equation (2). All design variables are constrained to be nonnegative and less than upper bounds \mathbf{x}_u .

$$\begin{array}{ll} \min_{\mathbf{0} \leq \mathbf{x} \leq \mathbf{x}_u} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{g}(\mathbf{x}) \leq \mathbf{0} \end{array} \tag{2}$$

A. Multiple Regime Analysis

This section conceptually describes the calculation of objective and constraint functions over all three flight regimes. The analysis is based on Roskam's method,⁶ commonly used for aircraft preliminary design.⁷ Additional relations were obtained from MacMillin.⁸ The partitioned analysis is depicted in Figure 2. Each regime analysis calculates the objective and constraint functions as specified in the figure. All analyses depend on the design variables, while the take-off and SSC analyses also require *GTOW* as an input. In addition, the take-off lift coefficient is required as an input to the SSC analysis. Observe that the first 14 design variables are shared variables. Only x_{15} is a local design variable (associated with the cruise regime). The partitioning was performed in a way that enhances the separability of each regime analysis. Each individual analysis can be executed independently if supplied with the inputs specified in Figure 2.

^aAlso termed linking variables in other ATC publications.

variable	description	$\operatorname{constraint}$	description
x_1	Wing root chord	g_1	Range $\geq 13,500$ Km
x_2	Wing tip chord	g_{2-4}	Passenger, cargo, luggage constraints
x_3	Wing span	g_5	Fuel Capacity ≤ 0.55 GTOW
x_4	Wing quarter chord sweep angle	g_6	Fuselage Volume $\geq 2,500 \text{ m}^3$
x_5	Horiz. tail root chord	$g_{7,8}$	Center of mass range
x_6	Horiz. tail tip chord	g_9	Structural load factor $n = \text{lift/weight} \le 2.5$
x_7	Horiz. tail span	g_{10-13}	Balanced field length range
x_8	Horiz. tail quarter chord sweep angle	g_{14-18}	Take-off control and lift-off reqt., pitching
x_9	Vert. tail root chord		moment, landing gear force, vert. accel.
x_{10}	Vert. tail tip chord	$g_{19,20}$	Climb gradient and take off thrust reqt.
x_{11}	Vert. tail span	$g_{21,22}$	Cruise control force requirement
x_{12}	Vert. tail quarter chord sweep angle	g_{23}	Cruise thrust ≤ 0.9 Take-off thrust
x_{13}	Fuselage mid-section length	g_{24}	Available cruise fuel mass
x_{14}	Fuselage max-width	$g_{25,26}$	Minimum tail areas
x_{15}	Nacelle—fuselage centreline distance	g_{27-28}	Engine location range
parameter	description	$g_{27,29}$	Minimum fuselage mid-section length
p_1	Cruise Speed = 0.85 M	g_{30-55}	Implicit geometric and design constraints
p_2	Max take-off thrust = $281.57 \cdot 10^3$ N/engine		

Table 1. Aircraft design variables, parameters, and constraints



Figure 2. Diagram of partitioned analysis structure

The design variables are used to construct geometric information, such as lifting surface areas, fuselage volume, and the aerodynamic center. Geometry and fixed design parameters are then used to calculate regime specific aerodynamic forces and the aircraft weight. Finally, these fundamental analysis results are used to determine regime specific performance metrics. The weight, geometry, and performance data required for the design problem is completely determined at the end of this process.

B. Design Formulation and Implementation

The design problem in Equation (2), using parameters specific to a B747 design, was solved with sequential quadratic programming,⁹ using the analysis approach described above. This approach is also referred to as the all-in-one (AiO) method, or the multidisciplinary feasible (MDF) method,¹⁰ since at every optimization iteration a complete system analysis is performed such that all disciplines (regimes) are feasible (consistent). This initial design solution provides a benchmark to evaluate the ATC solution results.

Although the design problem involves only 15 variables and an algebraic analysis, difficulties were encountered using the AiO approach. A feasible solution to Equation (2) was unattainable. Only after the fuel capacity constraint was relaxed to $0.56 \times GTOW$ could a feasible solution be found. In addition, non-zero positive lower variable bounds had to be introduced to decrease the size of the design space, and to help the optimizer converge to a solution. The ATC approach presented in this section facilitated a more effective search of the design space, yielding a feasible solution without constraint relaxation. Most importantly, AiO produced a GTOW of $3.5 \cdot 10^6$ N, while ATC yielded a superior GTOW of $2.9 \cdot 10^6$ N. The rest of this section develops the ATC formulation.

The design problem as presented in Equation (2) has a scalar objective. In reality, aircraft design involves many competing objectives, some of which were handled as constraints in the design problem described above. An alternative, scalarized multiobjective formulation is given in Equation (3). The scalarized objective seeks to minimize the equally weighted sum of GTOW and three constraint functions that were converted to objective functions. The new objectives are cruise range $(f_1(\mathbf{x}) = -R_{cr})$, take-off distance $(f_2(\mathbf{x}) = -S_{TO})$, and second-segment climb gradient $(f_3(\mathbf{x}) = -\gamma_2)$.

$$\min_{\mathbf{0} \le \mathbf{x} \le \mathbf{x}_u} \quad f(\mathbf{x}) + \sum_{i=1}^3 f_i(\mathbf{x})$$
subject to $\mathbf{g}(\mathbf{x}) \le \mathbf{0}$ (3)



Figure 3. ATC Formulation 1

This formulation is amenable to solution by multilevel optimization methods, such as ATC, since both the objective and constraints are separable. Several options exist for constructing an ATC formulation for this multi-regime aircraft design problem. The first option, shown in Figure 3, illustrates a three-level approach that employs an intermediate element. This approach was motivated by the directional functional dependencies and associated natural hierarchy illustrated in Figure 2. Note that this is not a traditional object-based decomposition, and that no 'system' exists. Rather, this is an analysis-based ATC formulation. The ATC indices are shown in parentheses for clarity, and the second subscript that normally indicates the element within a level is dropped since each level in this case has only one element. Observe that terms with a 0 superscript are fixed external targets for 'local' objectives. Since all sets of shared variables contain the same set of design variables, the subscripts are dropped, and the superscript indicates the level at which they are computed. Because the shared variable targets set for a child element are the same as the shared variables at the parent, the $\mathbf{y}_{(i+1)j}^{i}$ term in the decision variable set is redundant and dropped. The problem is recast as a target cascading problem, rather than a pure design optimization problem. The targets used were:

 $GTOW^{(0)}_{(3)} = 2.9 \cdot 10^6 \text{ N} \quad R^{(0)}_{cr(3)} = 13,500 \text{ km} \quad S^{(0)}_{TO(2)} = 1800 \text{ meters} \quad \gamma^{(0)}_{2(1)} = 15\%$

The latter three targets were obtained from original constraint values. Several optimizations were performed to determine the lowest GTOW target that yielded a consistent solution. Note that utilizing a weighting update method enables the designer to find the optimal objective value(s) with a single ATC execution.

It was found that this ATC problem allowed for the omission of explicit coordination of the communication of GTOW and $C_{L_{TO}}$ between elements, i.e., these two terms were passed directly as input parameters to the required elements instead of making independent copies in the appropriate elements and forcing agreement with penalty terms. Accordingly, GTOW and $C_{L_{TO}}$ do not appear as decision variables in the intermediate and lower levels since no copies are made. Observe that this is a special case. Omitting explicit coordination of analysis responses from the ATC formulation will not work in general. The resulting reduced design freedom may render some element optimization problems infeasible, particularly if bi-directional analysis interactions exist.

Penalty term weights, not shown explicitly in the formulation, were found to be of critical importance to the ATC solution. Rather than employing a weighting update strategy,^{11,12} weights were chosen *a priori* and left fixed throughout the ATC solution process. A trial and error method was used to determine appropriate penalty weights. Although weighting update methods add some complexity, they eliminate the trial and error phase of weight selection, and target values do not need to be identified beforehand or determined repeated executions (such as was done for the *GTOW* target). The augmented Lagrangian weighting update method, although not used here, has been shown to improve efficiency by up to three orders of magnitude.¹² These factors dramatically extend the practical applicability of ATC, and point to direct implementation of ATC in design organizations. A weighting update method was not used here because this work is a first step for ATC in aircraft design. Weighting update methods will be incorporated into future work.

The ATC coordination process is as follows. The input values for the SSC problem $(\mathbf{y}^{(2)}, GTOW^{(3)}, \text{ and } C_{L_{TO}}^{(2)})$ are fixed parameters with respect to this problem, and an initial guess must be made for these values to proceed with the first execution of the level 1 problem. After the SSC problem is solved to completion, the resulting $\mathbf{y}^{(1)}$ values are provided to the level 2 problem as fixed input parameters. Initial guesses for $\mathbf{y}^{(3)}$ and $GTOW^{(3)}$ are required for the first execution of the level 2 problem. Once the level 2 problem is complete, fixed values for $\mathbf{y}^{(2)}$ are provided to the level 3 problem. After this is solved, a complete ATC outer loop has been completed. Updated values for $GTOW^{(3)}$, $C_{L_{TO}}^{(2)}$, $\mathbf{y}^{(2)}$, and $\mathbf{y}^{(3)}$ are recorded for use during the following ATC outer loop iteration. This process is repeated until the system is found to be consistent, or until a stable solution is found if a consistent system is unrealizable. In the latter case, external targets may be infeasible, and should be updated using information obtained from the initial ATC solution.

C. Results

After convergence, the design variables were consistent within 0.01%, and the objective function targets were matched within 0.01%. The resulting optimal design vector was:

$$\mathbf{x}^{*} = [14.2, \ 0.850, \ 65.1, \ 49.9, \ 8.50, \ 3.55, \ 22.50, \ 43.3, \ 9.91, \ 4.00, \ 14.7, \ 40.0, \ 33.8, \ 6.67, \ 11.7]^{\mathrm{T}}$$
(4)

The ATC solution produced a GTOW of $2.9 \cdot 10^6$ N, matching the associated target. At the ATC solution \mathbf{x}^* , constraints on the fuselage volume, maximum fuel mass and range were all active. Constraints on the horizontal and vertical tail areas were also active due to the tradeoff between aircraft weight and controllability. The tip chord lower bound was active since reducing tip chord sizes results in reduced weight and increased fuel capacity. The lower bounds were required because low Reynolds number effects that normally proscribe zero tip chords were not accounted for. The take-off regime constraints on lift-off requirements were active, and the SSC regime had no active constraints. Figure 4 depicts the optimal design produced by ATC compared to that of the original B747. The most notable difference is the reduced lifting

surface area, associated with the optimal design's weight reduction. All lifting surface sweep angles have increased, reducing drag and helping to compensate for the reduced lift force (due to less lifting surface area), resulting in an acceptable $\frac{L}{D}$ ratio.



Figure 4. Comparison of original and ATC optimised B747 designs

The fixed penalty weight approach required substantial computation time. Several thousand function evaluations were needed for this problem. Computation time and system consistency had a strong dependence on the choice of penalty weights.¹³ An efficient weighting update method, such as described in Michalek¹¹ or Tosserams,¹² would reduce computational expense substantially.

D. Remarks

In some cases ATC requires more computation time than AiO. However, ATC offers more flexibility for design space exploration, and may find superior solutions relative to AiO. The ATC solution computed a design with a 17% lower *GTOW* than AiO, while remaining feasible with respect to all original design constraints. An AiO approach may not be successful for complex system designs. Difficulties arose during the AiO solution of the simple design problem used here, while ATC found a better solution without any constraint relaxation. It is expected that the capabilities of ATC will be illustrated more aptly when more realistic aircraft design problems are addressed. ATC is amenable to distributed design processes. It is a natural fit to existing design organizations due to its ability to efficiently coordinate subsystem design problems toward an optimal system design. Finally, by utilizing weighting update methods and parallelism, ATC can in some cases be faster than AiO. In fact, the number of outer-loop iterations between system design and subsystem design can be as low as three or four. This property validates the organizational congruency argument, since inclusion of human design groups in the ATC process is feasible. Such an implementation can lead to rapid identification of optimal system designs by design organizations.

The three-level approach presented was motivated by the analysis structure and illustrates an ATC formulation with an intermediate element. Alternative ATC formulations may be considered. Simplified representations of two alternative strategies are illustrated in Figure 5. Both are bi-level approaches, and exhibit computational advantage in that only two levels must be coordinated and that problems at the lower level may be solved in parallel. Also, observe that the intermediate element in Figure 3 must match shared variables from above and below, resulting in increased problem stiffness. The second bi-level approach introduces an auxilliary element at the top level in order to coordinate the shared variables. No analysis is performed at the top level—the optimization objective is to minimize the penalty terms with respect to

the values that must be coordinated. The system and local objectives are all optimized within the elements at the lower level. An advantage of this approach is that shared variables exist only between elements on the same level. Exploration of these alternatives will be the subject of future work, and it is expected that improved solution efficiency will be realized.



Figure 5. Simplified schematics of two alternative ATC formulations

IV. ATC in Aircraft Design

The preceding example illustrates how ATC can be applied to high-level aircraft design where multiple flight regimes are considered separately. This section proposes additional techniques for using ATC in aircraft design that are complementary to existing design methods.

A. System–Component Coordination

Current aircraft design practices utilize system analysis tools during the conceptual design stage. The results from this step are then used as a starting point for the more involved preliminary design stage. Sophisticated design tools, such as MDO, are typically used at this stage. A formal method that effectively coordinates existing system-level and high-fidelity design activities could help designers realize superior results and guide design efforts more efficiently. ATC was developed under this paradigm, and is naturally suited for this type of coordination.⁴ For example, ATC provides a framework to formalize communication between system and subsystem designs, and allows the communication to be made in a form natural to the design problems, i.e., subsystem designers seek to match performance targets and be consistent with shared variables. The math-based ATC process is very efficient and produces designs that are optimal for the entire system. Design groups can come to agreement in just a handful of iterations.

B. Nested ATC-MDO

Transitioning to a system or object-based design environment has the advantages of improved responsiveness, reduced design cycle times and costs, and better support for concurrent engineering practices. Some industries, such as aerospace, however, still require the disciplinary expertise provided by a function-based design organization. A solution that combines the advantages of both approaches is a matrix organization.¹⁴ An optimization framework has been proposed that maps to matrix-structured design organizations:⁴ nested ATC-MDO. ATC is used to coordinate the system design problem, viewing the product as a hierarchical composition of systems, subsystems, and components. Within each of these subsystems may exist multiple coupled disciplines. In the nested ATC-MDO framework an MDO method is used to solve these multidisciplinary problems within ATC elements. This approach provides both system coordination and functional depth, and is an example of how ATC and MDO can be used in a complementary manner. This requires minimal modifications to the existing design environment, since legacy MDO methods and other design tools can remain intact.

For example, ATC could coordinate the subproblems illustrated in Figure 1, and the aeroelastic design associated with the wing subproblems may be handled with existing MDO techniques. The formulation details vary depending on what MDO approach is used. Allison *et al.*⁴ have demonstrated collaborative optimization (CO) nested within ATC. The individual disciplinary feasible (IDF) approach has also successfully been used within ATC.

V. Conclusion

Several concepts for use of ATC in aircraft design were presented, and a detailed example of design for multiple flight regimes was worked out in detail. It was shown that ATC addresses difficulties encountered during the AiO solution approach, and produces superior design results. ATC addresses several concerns for modern aircraft design. It provides a means to shift more completely toward a system-oriented design environment with a minimum of modification to current practices. Coordination between system-level and high-fidelity subsystem design activities by ATC is an efficient means to identify optimal system designs. It is hoped that through concerted efforts to demonstrate and clarify ATC's applicability, ATC will be more widely adopted and become an effective design tool in the aerospace industry in addition to the automotive industry.

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