8.4: Mass and Density

Problems

1. An ice cream cone has a height of 15 centimeters and the diameter at the top is 5 centimeters. The cone is filled with soft-serve ice cream so it completely fills the cone, but does not go above the top of the cone. The ice cream has a constant density of 2 grams per cubic centimeter.

(a) Write an expression for the approximate mass of ice cream contained in a circular slice that is located h_i centimeters from the bottom tip of the cone and has a thickness of Δh centimeters. Your answer should be in terms of h_i and Δh. Include units.

(b) Find the mass of the ice cream in the cone, including units. [Integrate by hand]

(c) Find $\overline{h}$, the height above the tip of the cone of the center of mass of the ice cream. Include units. [Integrate by hand]

2. A metal plate with density $\delta = 5\,\text{kg/m}^2$ has a shape bounded by the curve $y = \sqrt{x}$, the line $x = 9$, and the x and y axes. Here, x and y are measured in meters.

(a) Find the mass of the plate. Include units. [Integrate by hand]

(b) Find the x and y coordinates of the center of mass. Include units. [Integrate by hand]

3. The number of trees in a particular forest is $\delta(r) = 100 - \frac{1}{2}r^2$ trees/mi^2 where r is the number of miles from the center of the forest. Write an integral for the number of trees within 10 miles of the center of the forest.

Answers

1. (a) $\frac{\pi}{18} h_i^2 \Delta h$ grams (b) $\frac{125\pi}{2}$ grams (c) $\frac{45}{4}$ cm **2.** (a) $90\,\text{kg}$ (b) $\overline{x} = 5.4\,\text{m}, \overline{y} = 1.125\,\text{m}$

3. $\displaystyle\int_0^{10} 2\pi r \left(100 - \frac{1}{2} r^2\right) \mathrm{d}r$