8.2: Applications to Geometry

Problems

Consider the region R which is bounded by the curves $y = x^2$ and $y = x^4$, between $x = 0$ and $x = 1$. Write an integral for each of the following quantities.

1. The area of R.
2. The perimeter of R.
3. The volume of the solid obtained by rotating the region R around the x-axis.
4. The volume of the solid obtained by rotating the region R around the line $x = -1$.

For the next two problems use the table of values of $f(x)$ and $f'(x)$ provided below.

x	0	0.5	1	1.5	2
$f(x)$	1	3	4	5	5.5
$f'(x)$	2	1.5	1.2	1	0.5

5. Estimate the length of the arc defined by the graph of $f(x)$ between $x = 0$ and $x = 2$.
6. Let R be the region bounded by the x-axis, the graph of $f(x)$, and the lines $x = 0$ and $x = 2$. Estimate the volume of the solid obtained by rotating the region R around the x-axis using the trapezoidal rule.
7. Find the volume of the region bounded by $y = e^{-x/3}$, the x-axis, the y-axis, and the line $x = 5$ and whose cross sections perpendicular to the x-axis are equilateral triangles.
8. Consider a solid S whose base is the region bounded by the circle $x^2 + y^2 = 4$ and the y-axis with $0 \leq x \leq 2$ in the xy-plane, and whose cross-sections perpendicular to the x-axis are half ellipses. The major and minor axes of the ellipses satisfy $a = \frac{1}{4}b$. The area of an ellipse is $A = \pi ab$. Write a definite integral of the solid S.

Answers

1. $\displaystyle\int_0^1 x^2 - x^4 \, \mathrm{d}x$ **2.** $\displaystyle\int_0^1 \sqrt{1+4x^2} + \sqrt{1+16x^6} \, \mathrm{d}x$ **3.** $\displaystyle\pi \int_0^1 x^4 - x^8 \, \mathrm{d}x$ **4.** $\displaystyle\pi \int_0^1 (\sqrt[4]{y}+1)^2 - (\sqrt{y}+1)^2 \, \mathrm{d}y$ **5.** 3.228044 **6.** 103.084 **7.** $\dfrac{3\sqrt{3}}{8}\left(1 - e^{-10/3}\right)$ **8.** $\dfrac{\pi}{8}\displaystyle\int_0^2 4 - x^2 \, \mathrm{d}x$