

## 7.5: Numerical Integration

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### Exercises

1. Estimate  $\int_0^2 \frac{x}{x^2 + x + 1} dx$  using LEFT(4), RIGHT(4), TRAP(4), and MID(4).

For each of the following integrals, specify which of the four integration methods learned in class are guaranteed underestimate the integral.

2.  $\int_{-1}^1 x^2 dx$
3.  $\int_0^2 x^3 dx$
4.  $\int_0^2 \sqrt{x^2 + 1} dx$ ,
5.  $\int_{-2}^0 x^4 dx$ ,
6.  $\int_{-2}^0 -x^4 dx$ ,
7.  $\int_0^5 f(x) dx$ , where  $f'(x) < 0$  and  $f''(x) > 0$ .

For each of the following question, say whether the statement is true or false.

8. At least one method out of LEFT, RIGHT, TRAP, MID will produce an underestimate for any definite integral.
9. If TRAP underestimates a definite integral then MID overestimates the integral.
10. If  $f'(x) = 0$  then TRAP( $n$ ) gives the exact value of the integral  $\int_a^b f(x) dx$  for any choice of  $a$ ,  $b$ , and  $n > 0$ .

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### Problems

Let  $f(x) = \int_2^x te^{-t^3} dt$ .

11. Which of the four integration methods learned in class will give the closest underestimate of  $f(3)$ ?
12. Estimate  $f(4)$  using TRAP(2).
13. Suppose I gave you a special computer that can evaluate  $f(x)$  EXACTLY at any value of  $x$ . Which integration method would you use to obtain the best overestimate of  $\int_3^5 f(x) dx$ ?

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## Answers

1.  $\text{LEFT}(4) = \frac{1}{2} \left( 0 + \frac{2}{14} + \frac{1}{3} + \frac{6}{19} \right) = \frac{215}{399} \approx 0.538847$ ,  $\text{RIGHT}(4) = \frac{1}{2} \left( \frac{2}{14} + \frac{1}{3} + \frac{6}{19} + \frac{2}{7} \right) = \frac{278}{399} \approx 0.69674$ ,  $\text{TRAP}(4) = \frac{1}{2} \left( \frac{215}{399} + \frac{278}{399} \right) = \frac{493}{798} \approx 0.6177944$ ,  $\text{MID}(4) = \frac{1}{2} \left( \frac{4}{21} + \frac{12}{37} + \frac{20}{61} + \frac{28}{93} \right) = \frac{840256}{1469307} \approx 0.5718723$  2. MID 3. LEFT, MID 4. LEFT, TRAP 5. RIGHT, MID 6. LEFT, TRAP 7. RIGHT, MID 8. False 9. False 10. True 11. MID 12. 0.00033546 13. MID