

7.2: Integration by Parts

Exercises

Solve each integral without using a calculator or computer.

1. $\int (x + 1) \sin x \, dx$

4. $\int x^3 e^{-x^2} \, dx$

2. $\int \arcsin x \, dx$, $\left[\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}} \right]$

5. $\int_0^{\pi/2} x \cos x \, dx$

3. $\int x \ln x \, dx$

6. $\int_0^1 x^2 e^x \, dx$

Problems

7. If $f(x)$ is an even function, evaluate $\int_{-1}^1 x^2 f'(x) \, dx$.

8. Compute $\int \sin x \cos x \, dx$ two different ways (substitution and integration by parts) and verify that you get the same answer.

The following problems will help you evaluate the indefinite integral $\int_0^\infty x^n e^{-x} \, dx$, where n is a positive integer. We will talk more about indefinite integrals later in the course.

9. Evaluate $\int x e^{-x} \, dx$.

10. Evaluate $\int x^2 e^{-x} \, dx$.

11. Evaluate $\int x^3 e^{-x} \, dx$.

12. Find a general formula for $\int x^n e^{-x} \, dx$, where $n \geq 0$ is an integer.

13. Let $T > 0$ be a positive constant. Using your previous answer, evaluate $\int_0^T x^n e^{-x} \, dx$.

14. Now, evaluate $\lim_{T \rightarrow \infty} \int_0^T x^n e^{-x} \, dx$.

Answers

- 1.** $-(x+1)\cos x + \sin x + C$ **2.** $x \arcsin x + \sqrt{1-x^2} + C$ **3.** $\frac{x^2}{2} \ln x - \frac{1}{4}x^2 + C$ **4.** $-\frac{x^2}{2}e^{-x^2} - \frac{1}{2}e^{-x^2} + C$ **5.** $\frac{\pi}{2} - 1$ **6.** e^{-2} **7.** 0 **8.** $\frac{1}{2}\sin^2 x + C$ **9.** $-e^{-x}(x+1) + C$ **10.** $-e^{-x}(x^2 + 2x + 2) + C$ **11.** $-e^{-x}(x^3 + 3x^2 + 6x + 6) + C$ **12.** $-e^{-x}(x^n + nx^{n-1} + n(n-1)x^{n-2} + \cdots + n!x + n!) + C$ **13.** $-e^{-T}(T^n + nT^{n-1} + n(n-1)T^{n-2} + \cdots + n!T + n!) + n!$ **14.** $n!$