

7.1: Substitution

Exercises

Solve each integral without using a calculator or computer.

1. $\int \frac{x}{x^2 + 1} dx$

6. $\int (x^3 + 3)^2 dx$

2. $\int \frac{1}{(2x + 1)^3} dx$

7. $\int \frac{\cos(\ln(x))}{x} dx$

3. $\int e^{\sin(x)} \cos(x) dx$

8. $\int \cos(6x) \sin^3(6x) dx$

4. $\int x^3 (x^4 + 2)^2 ((x^4 + 2)^3 + 1)^4 dx$

9. $\int \frac{e^{3x}}{2 + e^{3x}} dx$

5. $\int z^4 (z^5 - 1)^3 dz$

10. $\int_{\pi/4}^{\pi/2} \cos(x)(3 \sin^2(x) + 1) dx$

Problems

11. Find a formula that does not involve integrals for an antiderivative of $f(x) = (x + 2)^{20}$ satisfying $F(1) = 2$.

12. If $f(x)$ is an odd function and $\int_2^7 f(x) dx = 2$, evaluate $\int_{-2}^1 f(1 - 3x) dx$.

13. If $g(x)$ is an even function and $\int_0^2 g(x) dx = 3$, evaluate $\int_{-1}^1 g(2x) dx$.

Answers

1. $\frac{1}{2} \ln |x^2 + 1| + C$ 2. $-\frac{1}{4(2x+1)^2} + C$ 3. $e^{\sin(x)} + C$ 4. $\frac{1}{60} \left((x^4 + 2)^3 + 1 \right)^5 + C$ 5. $\frac{1}{20} (z^5 - 1)^4 + C$ 6. $\frac{1}{7}x^7 + \frac{3}{2}x^4 + 9x + C$ 7. $\sin(\ln(x)) + C$ 8. $\frac{1}{24} \sin(6x) + C$ 9. $\ln |2 + e^{3x}| + C$ 10. $3 - \frac{3\sqrt{2}}{4}$
11. $2 + \frac{1}{21}(x+2)^{21} - \frac{3^{21}}{21}$ 12. $\frac{2}{3}$ 13. 3