

## 6.4: Second Fundamental Theorem

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### Exercises

In each problem, find a formula for  $F(x)$  satisfying:

1.  $F'(x) = \cos(x^2)$ ,  $F(1) = 2$

2.  $F'(x) = e^{x^4}$ ,  $F(-3) = 1$

Compute the derivative of each function.

3.  $\int_2^x \sin(2t^2) dt$

5.  $\int_{-t}^{t^2} \sin(x^2 - x) dx$

4.  $2 \cos(x) + \int_{-x}^5 e^{-t^2} dt$

6.  $\int_x^{x+2} \sin(\cos(t)) dt.$

Let  $F(x) = \int_0^x \sqrt{2t^2 + 1} dt$ . Evaluate each of the following.

7.  $F(0)$

8.  $F(2)$

For the same  $F(x)$ , answer the following:

9. Is  $F(x)$  increasing or decreasing?

10. Is  $F(x)$  concave up or concave down?

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### Problems

Let  $f(x) = e^{-x^2}$ .

11. Write a formula for an antiderivative  $F(x)$  of  $f(x)$  satisfying  $F(0) = 0$ .

12. For which values of  $x$  is  $F(x)$  positive? Negative?

13. For which values of  $x$  is  $F(x)$  increasing? Decreasing?

14. For which values of  $x$  is  $F(x)$  concave up? Concave down?

15. Write an expression for the average value of  $F(x)$  on the interval  $[0, 2]$ .

Suppose  $G(x)$  is an antiderivative of  $f(x)$  satisfying  $G(0) = 2$ .

16. If  $F(x) = \int_0^2 f(t) dt$ , what is  $G(2) - F(2)$ ?

17. If  $G(1) = 0$ , what is  $\int_0^1 f(x) dx$ ?

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## Answers

1.  $F(x) = 2 + \int_1^x \cos(t^2) dt$  2.  $F(x) = 1 + \int_{-3}^x e^{t^4} dt$  3.  $\sin(2x^2)$  4.  $-2\sin(x) - e^{-x^2}$   
5.  $2t \sin(t^4 - t^2) + \sin(t^2 + t)$  6.  $\sin(\cos(x + 2)) - \sin(\cos(x))$  7. 0 8. 3.62323 9. Increasing  
10. Concave Up 11.  $F(x) = \int_0^x e^{-t^2} dt$  12. Positive:  $x > 0$ , Negative:  $x < 0$  13. Increasing:  $-\infty < x < \infty$ , Decreasing: nowhere 14. Concave up:  $x > 0$ , concave down:  $x < 0$   
15.  $\frac{1}{2} \int_0^2 \int_0^x e^{-t^2} dt dx$  16. 2 17. -2