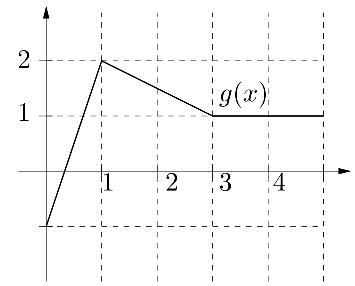
1. [12 points] For all of parts (a)–(d), let $f(x) = 2x - 4$ and let $g(x)$ be given in the graph to the right.



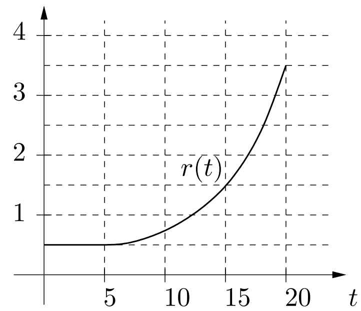
(a) [3 points of 12] Find $\int_1^5 g'(x)\,dx$.

(b) [3 points of 12] Find $\int_0^5 g(x)\,dx$.

(c) [3 points of 12] Find $\int_2^{4.5} g(f(x))\,dx$.

(d) [3 points of 12] Find $\int_0^5 f(x) \cdot g'(x)\,dx$.

3. [12 points] Having completed their team homework, Alex and Chris are making chocolate chip cookies to celebrate. The rate at which they make their cookies, $r(t)$, is given in cookies/minute in the figure to the right (in which t is given in minutes). After $t =$ 20 minutes they have completed their cookie making extravaganza.



(a) [3 of 12 points] Write an expression for the total number of cookies that they make in the 20 minutes they are baking. Why does your expression give the total number of cookies?

(b) [3 of 12 points] Using $\Delta t = 5$, find left- and right-Riemann sum and trapezoid estimates for the total number of cookies that they make.

(c) [3 of 12 points] How large could the error in each of your estimates in (b) be?

(d) [3 of 12 points] How would you have to change the way you found each of your estimates to reduce the possible errors noted in (c) to one quarter of their current values?

3. [10 points] The Great Pyramid of Giza in Egypt was originally (approximately) 480 ft high. Its base was originally (again, approximately) a square with side lengths 760 ft.

(a) [6 points of 10] Sketch a slice that could be used to calculate the volume of the pyramid by integrating. In your sketch, indicate all variables you are using. Find an expression for the volume of the slice in terms of those variables.

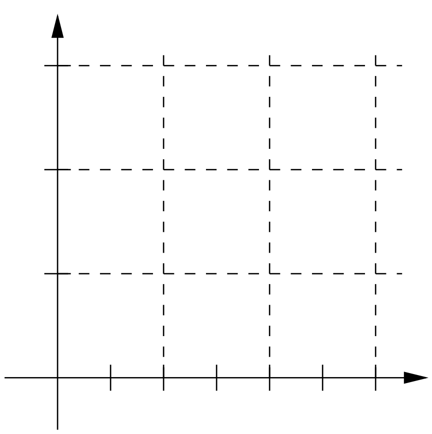
(b) [4 points of 10] Use your slice from (a) to find the volume of the pyramid.

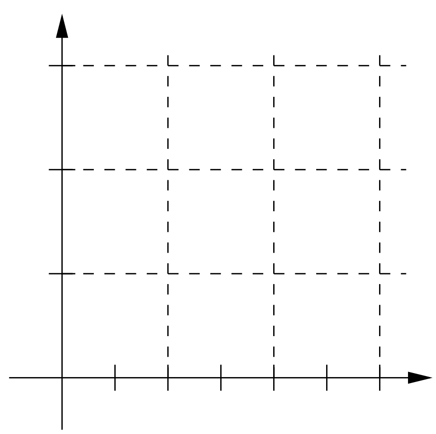
6. [10 points] While eating cookies, Alex notes that the velocity of a student passing by is given, in meters/second, by the data shown below.

t (seconds)	0	1	2	3	4	5	6
$v(t)$ (m/s)	0	0.5	1.5	2	2.5	2.5	3

(a) [5 of 10 points] Using the midpoint rule, find as accurate an estimate as possible for the total distance the student travels in the six seconds shown in the table (use only the given data in your calculation).

(b) [5 of 10 points] Draw two figures on the axes below, one each to illustrate the total distance you are estimating and the estimate you found. Be sure it is clear how your figures illustrate the indicated quantities.





1. [12 points] While at home for Thanksgiving, Alex finds a forgotten can of corn that has been sitting on the shelf for a number of years. The contents have started to settle towards the bottom of the can, and the density of corn inside the can is therefore a function, $\delta(h)$, of the height h (measured in cm) from the bottom of the can. δ is measured in g/cm^3. The can has a radius of 4 cm, and a height of 12 cm.

(a) [3 points of 12] Write an expression that approximates the mass of corn in the cylindrical cross-section from height h to height $h + \Delta h$.

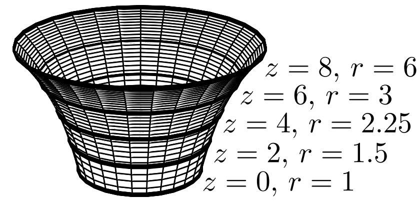
(b) [3 points of 12] Write a definite integral that gives the total mass of corn in the can.

(c) [3 points of 12] If $\delta(h) = 4e^{-0.03h}$, what is the total mass of corn inside the can?

(d) [3 points of 12] Write, but do not evaluate, an expression for the can's center of mass in the h direction. Would you expect the center of mass to be in the top or bottom half of the can? Do not solve for the center of mass, but in one sentence, justify your answer.

4. [16 points] An entrepreneurial University of Michigan Business Squirrel is marketing childrens' buckets with curved sides, as shown in the figure to the right, below. The figure gives the radius of the bucket, r, at different heights, z, from the bottom of the bucket. All lengths are given in inches. Suppose that a child fills one of these buckets with muddy water.

a. [4 points] If the density of the water in the bucket is $\delta(z)$ oz/in^3, write an integral that gives the mass of the water in the bucket.



b. [4 points] If $\delta(z) = (24 - z)$ oz/in^3, estimate the mass using your integral from (a).

c. [8 points] Estimate the center of mass of the bucket.

6. [12 points] Recall that the Great Pyramid of Giza was (originally) approximately 480 ft high, with a square base approximately 760 ft to a side. The pyramid was made of close to 2.4 million limestone blocks, and has several chambers and halls that extended into its center. It is not too far from the truth to suppose that these open areas are located along the vertical centerline of the pyramid, and that we can therefore think of the density of the pyramid varying only along its vertical dimension. Suppose that the result is that the density of the pyramid is approximately $\delta(h) = (0.00011(h-240)^2 + 134.2)$ lb/ft^3, where h is the height measured up from the base of the pyramid.

(a) [6 points of 12] Set up an integral to find the weight W of the pyramid. You need not evaluate the integral to find the actual weight.

(b) [6 points of 12] Give an expression, in terms of integral(s), that tells how far off the ground the center of mass of the pyramid is. Again, you need not evaluate the integral(s). *(Note that you may set up the expression in terms of the weight density without worrying about converting it to a mass density.)*