Cosmological Probes of Dark Energy*

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Evidence for dark energy:

1) Structure formation: $\Omega_M \simeq 0.3 \pm 0.1$

2) CMB: $\Omega_{TOT} \simeq 1.0 \pm 0.06$

3) Type Ia Supernovae: $0.8 \Omega_M - 0.6 \Omega_\Lambda = -0.2 \pm 0.1$
B Band

as measured

Calan/Tololo SNe Ia

$M_B - 5 \log(h/65)$

days

light-curve timescale
“stretch-factor” corrected

Kim, et al. (1997)
\[ m = M + 5 \log \left( \frac{d_L(z, \Omega_M, \Omega_{\Lambda}, \ldots)}{10 \text{pc}} \right) \]
Parameterize with: \( \Omega_X = \frac{\rho_X}{\rho_c} \)

\[
w = \frac{p_X}{\rho_X}
\]

then \( H^2(z)/H_0^2 = \Omega_M(1 + z)^3 + \Omega_X(1 + z)^{3(1+w)} \)

- \( \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_M + \rho_X + 3p_X) \)
  \( \Rightarrow \) Has (strongly) negative pressure

- Smooth - clusters only at \( \lambda \sim \lambda_{\text{HOR}} \) (if at all)
  \( \Rightarrow \) cannot see it in galaxy surveys

- From observations: \( -1 > w \gtrsim -0.6 \)
  \( \Rightarrow \) important only at \( z \lesssim 1 \), since \( \rho_X/\rho_M \propto (1 + z)^{3w} \).
Fine-Tuning Problems

1) “Why now?”

2) “Why so small?” (The Cosmological Constant, that is)

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \]

but

\[ \rho_\Lambda \lesssim (10^{-3}\text{ eV})^4 \ll (10^{+19}\text{ GeV})^4 \]
A candidate: Quintessence

\[ \ddot{\phi} + 3H \dot{\phi} + V,\phi = 0 \]

\[ m_\phi \sim H_0 \sim 10^{-33} eV \]

\[ w = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \]
Kinematic Tests

\[ r(z) \rightarrow H_0^{-1} \left[ z - \frac{3}{4} z^2 - \frac{3}{4} \Omega_X w z^2 + \cdots \right] \quad \text{for } z \ll 1 \]
Growth of Density Perturbations

\[ w = -1 \]

\[ w = -1/3 \]
Galaxy Cluster Abundance

\[ \frac{dN}{dzd\Omega}(z) = \left[ \frac{dV}{dzd\Omega}(z) \int_{M_{\text{min}}(z)}^{\infty} dM \frac{dn}{dM} \right] \]

Haiman, Mohr & Holder (2000)
CMB and Dark Energy

Location of the first peak: \( l_1 \approx \frac{\eta_{LS}}{\eta_{SH}} \)

\[
\frac{\Delta l_1}{l_1} \approx -0.084 \Delta w - 0.45 \frac{\Delta h}{h} + 0.09 \frac{\Delta \Omega_B h^2}{\Omega_B h^2} - 0.14 \frac{\Delta \Omega_M}{\Omega_M} - 1.25 \frac{\Delta \Omega_0}{\Omega_0}
\]

P. Steinhardt (2000)

![Graph showing the location of the first peak in the CMB spectrum with data points for different models and the corresponding error bars. The x-axis represents the multipole l, and the y-axis represents the peak intensity in \( \mu K \). The graph includes data from COBE (no quadrupole) and other models such as BOOM00, MAXIMA00, QCDM (w=-2/3), and ΛCDM.]
CMB provides a *single* measurement of the distance to LSS, therefore

- Degeneracy in parameter estimation
- Only $w_{\text{eff}}$ is probed
Weak Lensing and Dark Energy

(Huterer 2001)
Power Spectrum of the Convergence

\[ \kappa(\hat{n}, \chi) = \int_0^\chi W(\chi') \delta(\chi') d\chi' \]

\[ \langle \kappa_{lm} \kappa_{l'm'} \rangle = \delta_{l_1 l_2} \delta_{m_1 m_2} P_l^{\kappa} \]

\[ P_l^{\kappa} = \frac{2\pi^2}{l^3} \int_0^{z_s} dz W_1(z) \Delta^2 \left( \frac{l}{r(z)} ; z \right) \]
\[ \Delta^2(k, z) = \delta_H^2 \left( \frac{k}{H_0} \right)^{n+3} T^2(k, z) \frac{D^2(z)}{D^2(0)} T_{NL} \]
Parameters:

\[ \delta_H \quad \Omega_X \quad w \quad \Omega_M h^2 \quad n_s \quad \Omega_B h^2 \quad m_\nu \]

Fisher Matrix formalism:

\[ F_{ij} = - \left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle = \sum_l \frac{1}{(\Delta P_l^\kappa)^2} \frac{\partial P_l^\kappa}{\partial p_i} \frac{\partial P_l^\kappa}{\partial p_j} \]

Results:

![Graph showing the results of different models](image-url)
WL Tomography

0 0.2 0.4 0.6 0.8 1
Ω

-1.0 -0.8 -0.6 -0.4 -0.2 0.0
W

0 0.2 0.4 0.6 0.8 1
Ω_{M}

1 redshift bin
2 bins
10 bins
Can clustering of Dark Energy be seen in WL?

\[ l(l+1)P \]

Ma et al. 1999

\[ \frac{T_q}{T_A} (k, \Omega_m, w_q) \]

\( \Omega_m = 0.4 \)

\( \Omega_m = 0.6 \)

\( w_q = -1/3 \)

\( w_q = -1/2 \)

\( w_q = -2/3 \)

\( k \) (Mpc/h)

\[ 0.0001 \quad 0.001 \quad 0.01 \quad 0.1 \quad 1 \]

\[ 0.9 \quad 1 \quad 1.1 \quad 1.2 \quad 1.3 \quad 1.4 \]

\[ 10^{-7} \quad 10^{-6} \quad 10^{-5} \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \]

\[ f_{SKY} = 1 \]

\[ |l+1| P_l^k / (2\pi) \]

- no clustering
- with clustering

\[ 1 \quad 10 \]
Biases: Non-linear Power Spectrum

\[ \Delta^2(k) \]

Relative difference

\[ \frac{\delta \Omega_X}{\sigma(\Omega_X)} \sim 2.5 \]

\[ \frac{|\delta w|}{\sigma(w)} \sim 4.8 \]
Bispectrum and Skewness

\[ B_{l_1l_2l_3}^K \propto \langle \kappa_{l_1m_1}\kappa_{l_2m_2}\kappa_{l_3m_3} \rangle \]

\[ S_3(\theta) \propto \sum_{l_1l_2l_3} B_{l_1l_2l_3}^K W(l_1\theta)W(l_2\theta)W(l_3\theta) \]
SCIENCE
• Measure $\Omega_M$ and $\Lambda$
• Measure $w$ and $w(z)$

STATISTICAL REQUIREMENTS
• Sufficient (~2000) numbers of SNe Ia
• ...distributed in redshift
• ...out to $z < 1.7$

SYSTEMATICS REQUIREMENTS
Identified & proposed systematics:
• Measurements to eliminate / bound each one to +/-0.02mag

DATA SET REQUIREMENTS
• Discoveries 3.8 mag before max.
• Spectroscopy with S/N=10 at 15 Å bins.
• Near-IR spectroscopy to 1.7 µm.

SATELLITE / INSTRUMENTATION REQUIREMENTS
• ~2-meter mirror
• 1-square degree imager
• 3-channel spectrograph (0.3 µm to 1.7 µm)

Derived requirements:
• High Earth orbit
• ~5 Mb/sec bandwidth
\begin{figure}
\centering
\includegraphics{snla_plot}
\caption{\textbf{SNAP SuperNova Acceleration Probe} 

\textit{\sim 2500 SNe Ia}}
\end{figure}
Supernova Cosmology Project
Perlmutter et al. (1998)

No Big Bang

$\Lambda = 0$

Universe

Target Uncertainty

42 Supernovae

expands forever
recollapses eventually

closed
flat
open
**Dark Energy**

Unknown Component, $\Omega_u$, of Energy Density

Supernova Cosmology Project
Perlmutter *et al.* (1998)

**Equation of State**

$$w = \frac{p_u}{\rho_u}$$

- Flat Universe
- Constant $w$

- Network of cosmic strings
  $$w = \frac{1}{3}$$

- Range of "Quintessence" models

- Cosmological constant
  $$w = -1$$

**SNAP Satellite**

Target Statistical Uncertainty

$$\Omega_M = 1 - \Omega_u$$
Strategies: Optimal Redshifts

(Huterer and Turner 2000)
Beyond Constant $w$: Probing $w(z)$

The difficulty:

$$r(z) = \frac{1}{H_0} \int_0^z \frac{dz'}{\left\{ \Omega_M(1+z')^3 + \Omega_X \exp[3 \int_0^{z'} (1 + w(z'')) d\ln(1 + z'')] \right\}^{1/2}}$$
Probing $w(z)$: $w = w_1 + w'(z - z_1)$

(Cooray and Huterer 1999)
Probing $w(z)$: Reconstruction

(Starobinsky 1998; Huterer and Turner 1999; Chiba and Nakamura 1999)

\[ 1 + w(z) = \frac{1 + z}{3} \frac{3H_0^2\Omega_M(1 + z)^2 + 2(d^2r/dz^2)/(dr/dz)^3}{H_0^2\Omega_M(1 + z)^3 - (dr/dz)^{-2}} \]

\[ V[r(z)] = \frac{1}{8\pi G} \left[ \frac{3}{(dr/dz)^2} + (1 + z) \frac{d^2r/dz^2}{(dr/dz)^3} \right] - \frac{3\Omega_M H_0^2(1 + z)^3}{16\pi G} \]

\[ \frac{d\phi}{dz} = \mp \frac{dr/dz}{1 + z} \left[ -\frac{1}{4\pi G} \frac{(1 + z)d^2r/dz^2}{(dr/dz)^3} - \frac{3\Omega_M H_0^2(1 + z)^3}{8\pi G} \right]^{1/2} \]

**Good:** Non-parametric (e.g., no assumptions about $w(z)$ needed)

**Bad:** The constraint upon $w(z)$ is relatively weak

**Ugly:** Need to fit $r(z)$ with a smooth curve first.
What next?

• Explore the acceleration using future powerful measurements:
  e.g., measure isotropy of expansion using SNAP.

• Work on optimization of proposed methods:
  e.g., weak lensing tomography.

• Combine various measurements in a clever way in order to get to
  \( w(z) \).

• But to understand dark energy, esp. if \( w = -1 \),
  major input will be needed from theory!