Reconstructing quintessence

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+ work in progress
Quintessence - a dynamical scalar field

Origin: particle physics (yet unknown)

History: Starting in the late 1980’s, shows up in literature as ‘Rolling Scalar field’, ‘Dynamical Lambda’, ‘Quintessence’.

Features:

- rolls down its (effective) potential
- provides significant energy density $\Omega_Q$ (missing energy?).
- has negative equation of state today

$$w_Q = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} < 0 \quad (-1 \lesssim w_Q \lesssim -0.5)$$

- in addition, quintessence may have other nice properties...
Supernova Ia Search

In flat universe: \( \Omega_M = 0.28 \) [± 0.085 statistical] [± 0.05 systematic]
Prob. of fit to \( \Lambda = 0 \) universe: 1%

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Reconstruction Equations: \( r(z) \rightarrow V(\phi) \)

Assume a Universe where \( \Omega_M + \Omega_Q = 1 \). Then, from the Friedmann equations:

\[
V[r(z)] = \frac{1}{8\pi G} \left[ \frac{3}{(dr/dz)^2} + (1 + z) \frac{d^2 r/dz^2}{(dr/dz)^3} \right] - \frac{3 \Omega_M H_0^2 (1 + z)^3}{16\pi G} \\

\frac{d\phi}{dz} = \frac{dr/dz}{1 + z} \left[ -\frac{1}{4\pi G} \frac{(1 + z) d^2 r/dz^2}{(dr/dz)^3} - \frac{3 \Omega_M H_0^2 (1 + z)^3}{8\pi G} \right]^{1/2}
\]

- Only need to know \( \Omega_M \)
- \( r(z) \) comes in only as \( dr/dz \) and \( d^2 r/dz^2 \)

To demonstrate the feasibility of this approach, we use Monte Carlo simulation.
Monte Carlo demonstration of the potential reconstruction

Pick \( V(\phi) \), \( \Omega_M \), \( H_0 \) and present KE/PE (or eq. of state) of the field.

Compute the evolution of \( \phi \), \( a(t) \) and \( r(z) \) by evolving \( \phi(t) \) and \( a(t) \) back in time

Simulate SNeIa measurements:
\[ r_{\text{sim}}(z_i) = r_{\text{exact}}(z_i) + \delta r_i \]
\( \delta r_i \) taken from a Gaussian distribution

Repeat 1000 times.

Fit the data with a (low-order) polynomial and numerically compute \( V(\phi) \) from the reconstruction equations
Examples of reconstruction

\[ V(\phi) = V_0 \exp(-\beta \phi / m_{Pl}) \quad \beta = 8 \]

\[ V_0 = (2.43 \times 10^{-3} \text{eV})^4 \]

\[ N = 40 \text{ points} \quad z_{\text{max}} = 1.5 \quad \Omega_M = 0.4 \]

\[ 0.01 \quad 0.02 \quad 0.03 \quad 0.04 \quad 0.05 \quad 0.06 \]

\[ V(\phi) / 10^{-10} \text{eV}^4 \]

\[ \sigma = 2\% \quad \sigma = 5\% \]

\[ 0.2 \quad 0.4 \quad 0.6 \]

\[ -0.01 \quad 0.01 \quad 0.03 \quad 0.05 \quad 0.07 \]

\[ \phi / m_{Pl} \]

\[ V(\phi) = V_0[1 + \cos(\phi / f)] \quad V_0 = (4.65 \times 10^{-3} \text{eV})^4 \quad f / m_{Pl} = 0.154 \]

\[ N = 40 \text{ points} \quad z_{\text{max}} = 1.0 \quad \Omega_M = 0.3 \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \]

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\[ \sigma = 2\% \quad \sigma = 5\% \]

\[ -1 \quad 1 \quad 3 \]

\[ 2 \quad 2.5 \quad 3 \quad 3.5 \]

\[ \phi / f \]
Padé Approximants:

- Fit the (simulated) data with 
  \[ r(z) = \frac{z(1 + az)}{1 + bz + cz^2} \]

Summary of potential reconstruction

- Need to know only \( \Omega_M \) and \( \Omega_Q = 1 - \Omega_M \).
- The uncertainty in the reconstruction will decrease as more supernovae are discovered (roughly as \( 1/\sqrt{N} \)).
- Inferring \( d^2r/dz^2 \) from the data is required for reconstruction.
Reconstructing the equation of state

• No need to assume that quintessence is the missing energy!

\[ 1 + w_X(z) = \frac{1 + z}{3} \frac{3H_0^2\Omega_M(1 + z)^2 + 2 (d^2 r / dz^2) / (d r / dz)^3}{H_0^2\Omega_M(1 + z)^3 - (d r / dz)^{-2}} \]

• This gives evidence that beyond \( z \sim 0.8 \) it is difficult to get information about the missing component.
Optimal supernova search strategies

Q: What is the ideal distribution of supernovae in redshift?

Minimize $A \propto [\det(F')]^{-1/2}$

$$m_n = 5 \log[H_0d_L(z_n, \Omega_M, \Omega_\Lambda)] + m_0 + \epsilon_n,$$

$$F_{ij} = -\left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle_x$$

$$= \frac{1}{\Delta m^2} \sum_{n=1}^{N} \frac{\partial m_n(z_n, \Omega_M, \Omega_\Lambda, \ldots)}{\partial p_i} \frac{\partial m_n(z_n, \Omega_M, \Omega_\Lambda, \ldots)}{\partial p_j}$$

$$= \frac{1}{\Delta m^2} \sum_{n=1}^{N} \omega_i(z_n) \omega_j(z_n)$$

(Tegmark et al., astro-ph/9804168)

If we represent the measurements as a sum of delta-functions

$$g(z) = \sum_{i=1}^{BINS} g_i \delta(z - z_i),$$

then

$$F_{ij} = \frac{N}{(\Delta m)^2} \int_0^{\infty} g(z) \omega_i(z) \omega_j(z) \, dz,$$
With two parameters:

\[
\det(F) = \int_0^\infty \int_0^\infty g(z_1) g(z_2) \omega^2(z_1, z_2) \, dz_1 \, dz_2
\]

\[
= \sum_{i,j=1}^{BINS} g_i g_j \omega^2(z_i, z_j)
\]

with

\[
\sum_{i=1}^{BINS} g_i = 1 \quad \text{and} \quad g_i > 0
\]

The result is, for \(\Omega_M - \Omega_{\Lambda}\) case

\[
g(z) = 0.50 \delta(z - 0.44) + 0.50 \delta(z - 1.00),
\]

and for the \(\Omega_M - w_Q\) case

\[
g(z) = 0.50 \delta(z - 0.36) + 0.50 \delta(z - 1.00).
\]
Simulating and fitting the data

![Graph showing simulated data and fit data]

- **Simulated data** (σ=5%)
- **Fit data** (4th order pol.)