# Gravitational lensing by cosmic strings in the era of wide-field surveys 

Dragan Huterer and Tanmay Vachaspati<br>Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106, USA

(Received 7 May 2003; published 12 August 2003)


#### Abstract

Motivated by the recent claim for gravitational lensing by a cosmic string, we reinvestigate the probability of finding such an event with upcoming wide-field surveys. If an observed lensing event is suspected to be due to a string, observations of the vicinity of the event in a circle of diameter $L$ centered on the observed lens should reveal several additional lensing events. For a string located nearby ( $z \lesssim 0.5$ ), we find that observations in a region of size $\approx 1$ arc $\mathrm{min}^{2}$ will see $\sim 100$ objects, of which $\sim 5$ would be lensed by the string, compared to $\sim 0.1$ lensed by conventional sources.


DOI: 10.1103/PhysRevD.68.041301
PACS number(s): 98.80.Cq

## I. INTRODUCTION

Cosmic strings are linear sources of energy momentum, believed to have possibly been produced during a phase transition in the early Universe. Many cosmological signatures of cosmic strings have been investigated over the past few decades [1,2], including large-scale structure formation, gravitational wave spectrum, the effect on cosmic microwave background anisotropies, and gravitational lensing. Observations of the large-scale structure and the cosmic microwave background anisotropy show that strings are not solely responsible for either. This places a constraint on the linear energy density (or, tension) in a string, $\mu \leq 2 \times 10^{22} \mathrm{~g} / \mathrm{cm}$ [1,2], often written in the dimensionless combination $G \mu / c^{2} \leqq 10^{-6}$ or equivalently $\delta \equiv 8 \pi G \mu / c^{2} \leqslant 3 \times 10^{-5}$, where $c$ is the speed of light and $G$ is Newton's constant. Lighter strings do not have the virtue of explaining any major cosmological conundrum, yet are not ruled out, and would have profound implications for particle physics and early Universe cosmology.

Strings lighter than $\delta \simeq 3 \times 10^{-5}$ can also be detected since they act as lineal gravitational lenses. Indeed, since the study of cosmic strings was first initiated, a few gravitational lensing events have been discovered that were suspected to have been sourced by a cosmic string [3,4]. However the suspicion has not been confirmed (e.g. [5]).

Our interest in string lensing was rekindled by the recent claim for a possible lensing by a string [6], and further stoked by the advent of deep, wide-field surveys that will revolutionize the field of observational astrophysics. We build on previous work (e.g. [7-13]) and reexamine estimates of probability of lensing by a cosmic string. In particular, we identify simple yet effective strategies for confirming or refuting the string hypothesis, given one putative lens.

## II. EXPECTED NUMBER OF GALAXIES LENSED BY A STRING

Let us consider lensing by a single open cosmic string. We assume that the string has a correlation length $\xi$ and that its length from any reference point on it, $\vec{r}$, scales as

$$
\begin{equation*}
l=R\left(\frac{R}{\xi}\right)^{a} \tag{1}
\end{equation*}
$$

where $R=\left|\vec{r}-\vec{r}_{1}\right|$ and $l$ is the proper length of the string between $\vec{r}$ and any other point $\vec{r}_{1}$. The parameter $a$ is equal to 1 for a pure random walk of the string (so that $R \propto \sqrt{l}$ ), and 0 for a perfectly straight string.

Cosmic strings do not introduce spacetime curvature, but simply produce a conical spacetime with deficit angle $\delta$ [14,15]. Light propagation from a source in a conical spacetime leads to gravitational lensing with an angular separation between the images given by [14]

$$
\begin{equation*}
\Delta \alpha=\delta \frac{D_{l s}}{D_{o s}} \sin \theta \tag{2}
\end{equation*}
$$

where $D_{o s}$ and $D_{l s}$ are angular diameter distances from observer to source and lens (string) to source, respectively, and $\theta \in[0, \pi]$ is the angle between the string direction and the line of sight. ${ }^{1}$ Note that the lensed object reported in Ref. [6] is located at redshift $z_{s}=0.46$ and its image splitting is reported to be $\Delta \alpha=2^{\prime \prime}$; this, Eq. (2) and the bound $\delta \leq 3$ $\times 10^{-5}$ imply that the putative string is located relatively close to us: $z_{l} \leq 0.25$. (This result is roughly independent of cosmological parameters.) The two similar images of the source in Ref. [6] also indicate that the string should be straight on scales, $\xi \gtrsim \Delta \alpha \sim 2^{\prime \prime}$; if this were not true, the images would be distorted as in Ref. [12]. Note that, here and in the rest of this paper, we quote distances (such as $l, R$ and $\xi$ ) as angles projected on the sky.

To start, we would like to calculate the number of galaxies lensed by a single straight string at redshift $z_{l}$. The probability for lensing for a single galaxy found in the survey at redshift $z_{s}$ due to the infinitely long string located at $z_{l}$ is

$$
\begin{align*}
P\left(z_{l}, z_{s}\right) & \simeq \frac{\langle\Delta \alpha\rangle}{\pi} \Theta\left(\Delta \alpha-\Delta \alpha_{\min }\right) \\
& =\frac{2 \delta}{\pi^{2}} \frac{D_{l s}}{D_{o s}} \Theta\left(\Delta \alpha-\Delta \alpha_{\min }\right) \tag{3}
\end{align*}
$$

[^0]

FIG. 1. Fraction of source galaxies beyond a given redshift $z$, assuming the distribution listed in the text and expected in upcoming wide-field surveys. The vertical arrow denotes the location of the lensed source reported in Ref. [6], while the horizontal bar denotes the range of redshifts allowed for the lens (string) in this case. Note that the vast majority of sources are expected to be at redshifts higher than the reported string redshift $z_{l}$.
where $\langle\Delta \alpha\rangle$ is the angular separation of images averaged over string directions on the sky, $\Delta \alpha_{\text {min }}$ is the angular resolution of the survey, and $\Theta$ is the Heaviside step function. We have assumed that the string appears as a great circle on the sky. More realistically, there will be many ( $\sim 10$ [16-18]) long strings in our horizon, each of which does not cover a great circle on the sky. But we can expect the above estimate to be roughly correct, the greater number of strings compensating for the smaller extent of each of them.

The number of lenses expected is simply

$$
\begin{equation*}
N\left(z_{l}\right)=\int_{z_{l}}^{\infty} P\left(z_{l}, z_{s}\right) \frac{d N_{\mathrm{src}}}{d z}\left(z_{s}\right) d z_{s} \tag{4}
\end{equation*}
$$

where $z_{l}$ is the redshift of the string and $d N_{\text {src }} / d z$ is the number of observable galaxies at redshift $z$ in interval $d z$. By "observable galaxies" we mean those that are part of the source population imaged by the survey. We assume that the redshift dependence of the number density of these galaxies is given by $d N_{\text {src }} / d z \propto z^{2} \exp \left[-\left(z / z_{0}\right)^{2}\right]$ with $z_{0}=1.13$ (which corresponds to median redshift of 1.23) [19]. The surface density of galaxies peaks at $z \sim 1$. If the string is located at $z \lesssim 0.25$, as indicated by the reported string lens candidate [6], most $(\sim 99 \%)$ of galaxies along the line of sight are located behind the string; see Fig. 1.

Upcoming wide-field surveys from space, such as Supernova/Acceleration Probe (SNAP; [20]) and ground, such as Large Synoptic Survey Telescope (LSST [21]), will cover hundreds and tens of thousands of square degrees, perform deep imaging to about 28th and 26th magnitude in $R$ band, and find locations, photometric redshifts and shapes for about $10^{8}$ and $10^{10}$ galaxies, respectively. (Our analysis is also applicable to ongoing or imminent wide-field surveys, such as Canada-France-Hawaii Telescope Legacy Survey [22].) For $\delta \sim 10^{-5}$, this fact and Eq. (4) imply of the order of $10^{2}-10^{5}$ galaxies lensed by the string. This sounds fantastic until we realize that the number of galaxies/quasars
lensed by intervening large-scale structure will be at least two orders of magnitude higher, since the optical depth for lensing has been measured to be around $10^{-3}$ in the JVAS/ CLASS survey [23]. Further complication will be induced by the finite angular resolution of these surveys, as well as spurious candidates, such as binary systems. The signal we are looking for is likely to be dwarfed by the "noise" just described, and it is difficult to expect that future wide-field surveys will either detect cosmic strings through lensing or impose much better constraints on the string scale $\delta$. On a positive side, it is possible that we will get lucky with a serendipitous discovery, such as that reported in Ref. [6], and chances for that are significantly enhanced with future widefield surveys.

## III. CONFIRMING THE LENSING BY A COSMIC STRING HYPOTHESIS

Let us now consider a different problem. Given the observation of one lensing event by a cosmic string, how likely is it that we will find another such event, and what can we do to find it? The fact that the location of one lensing event is known is very helpful, since we know that the string passed through that angular location, and the orientation of the double image tells us the direction of passage projected on the sky.

In the following, we assume that future wide-field surveys, such as SNAP and LSST, will find about 100 galaxies $/ \operatorname{arcmin}^{2}$, which corresponds roughly to 28th magnitude in the $R$ color band. This estimate is conservative; for example, around 200 galaxies/arc $\min ^{2}$ are detectable in the Hubble Deep Field. These galaxies will be fully resolved and their shear will be measured for weak lensing studies.

The shape of the string is described by Eq. (1). There are two limiting cases:
(1) $a=0$ : the string is straight, static and perpendicular to the line of sight. In that case an obvious strategy is to look a distance $L$ along the axis of the observed lens (see Fig. 2, left drawing). From Eq. (2) it follows that the maximum angular splitting for experimentally allowed values of $\delta$ is about $5^{\prime \prime}$, so a sensible strategy is to look $5^{\prime \prime}$ perpendicularly and symmetrically to the aforementioned axis; see Fig. 2. This guarantees that all galaxies lensed by this chunk of the string of length $L$ will be seen. The number of lensed galaxies depends on the string redshift $z_{l}$ as

$$
\begin{align*}
N_{\mathrm{obs}}\left(z_{l}\right)= & L \int_{z_{l}}^{\infty} \frac{d\left(N_{\mathrm{src}} / A\right)}{d z_{s}} \Delta \alpha\left(z_{l}, z_{s}\right) \Theta\left(\Delta \alpha\left(z_{l}, z_{s}\right)\right. \\
& \left.-\Delta \alpha_{\mathrm{min}}\right) d z_{s} \tag{5}
\end{align*}
$$

where $N_{\text {src }} / A$ is the surface density of source galaxies. The expected number of sources for $L=1^{\prime}$ is shown in the top panel of Fig. 3. For a locally straight string and the lens redshift $z \leq 0.5$, it is sufficient to look about $L=1^{\prime}$ along the axis passing between the images. In doing so we are guaranteed to see a few $(2-10)$ additional lensed galaxies. If by any chance the observed lens is suspected to be at higher redshift, we have to survey a longer strip, corresponding to $L$


FIG. 2. Two extreme cases discussed in the text: straight string (left) and string exhibiting a random walk (right). In each case the cosmic string is shown with solid lines, while the area that we need to image in order to find at least one more lens is enclosed with dashed lines. The pair of concentric circles denotes images of the original lensing event.
of perhaps a few arc min (see the bottom panel of Fig. 3). It is also clear that even a limited resolution of the survey of $1^{\prime \prime}$ will not significantly change the strategy (see dashed lines in Fig. 3) since most of the observed lensed sources will be far from the string and therefore sufficiently separated in angle.
(2) $a=1$ : the string exhibits a pure random walk. It is clear that the total length of the string that we need to encompass will be the same as in (1) in order to observe several additional lenses. Since the random walk extends in two directions from the observed lens, we need to make follow-up observations in the circle of radius of $R=\sqrt{L \xi / 2}$ away from the lens (see Fig. 2, right drawing). ${ }^{2}$ This depends on the correlation length $\xi$. If $\xi>L / 2$, the string is straight in the region of interest and the estimate in (1) holds. So we need only consider $\xi<L / 2$. Therefore the largest diameter we need to cover will be for the case of $\xi=L / 2$, giving $2 R=L$ ( $\approx 1^{\prime}$ for the lens candidate of Ref. [6]).

It is now clear that the sufficiently effective strategy for any value of $0 \leqslant a \leqslant 1$ should be the same as in case (2) above: following up candidates in the circle of diameter $L$ $\left(\approx 1^{\prime}\right.$ for $\left.z_{l} \leqq 0.5\right)$ centered on the lens guarantees finding at least a few additional lenses, provided that the original lens was caused by the cosmic string. This statement is quite robust. One can of course imagine that the string conspires and its direction near the lens becomes nearly parallel to the line of sight, but the skeptical astronomer can decrease the odds of this happening by simply following up a larger area around the observed lens. Note also that lensing by conventional sources will not cause confusion, since only about $0.1 \%$ of galaxies are expected to be lensed by large-scale

[^1]

FIG. 3. Top panel: number of galaxies found in the area 5 " $\times 1^{\prime}$ as shown in the left drawing in Fig. 2 (i.e. for $L=1^{\prime}$ ) as a function of the redshift of the string, $z_{l}$. We have assumed the surface density of galaxies of $N_{s} / A=100$ galaxies $/$ arcmin $^{2}$. Solid line assumes a perfect angular resolution of the survey, $\Delta \alpha_{\min }=0$, while the dashed line assumes the resolution of $\Delta \alpha_{\min }=1^{\prime \prime}$. The horizontal line shows the range of redshifts allowed for the string reported in Ref. [6]. Bottom panel: same as above, except we now show the required follow-up length $L$ in order to see 5 additional lensing events ( $N_{\text {obs }}=5$ ), as a function of string redshift $z_{l}$. Note that the two panels contain the same information, and are both shown for clarity.
structure, which implies roughly 1 conventional lens for every 50 cosmic string lenses in the vicinity of the observed event.

A more serious potential problem is that of false positives-chance alignments of galaxies located nearby. It can be seen that this is not a problem for two reasons. First, the galaxies are on average spread apart by $6^{\prime \prime}$, only a small fraction will be closer to each other than $\sim 2^{\prime \prime}$, and those can be separated by photometric redshifts, which are nowadays quite accurate ( $\sigma_{z} \leq 0.05$ [19]). Second, the very few galaxies that accidentally happen to be nearby both in angle and redshift will in general have different shapes while, recall, lensing by a cosmic string does not cause image distortions. Comparing the shapes of candidate lenses is another way of filtering out false positives.

The most serious concern are the binary galaxies, and they are the reason that a careful follow-up of the lens candidates is preferred.

## IV. CONCLUSIONS

Cosmic strings can produce observable lensing signatures even if they are light enough to be irrelevant for structure formation in the Universe. Previous reports of objects being lensed by a cosmic string have not been confirmed after follow-up observations. It remains to be seen what further observations will tell us about the current candidate [6].

Future wide-field surveys, such as SNAP and LSST, will be able to see thousands of lenses caused by a single infinite string with linear density $G \mu / c^{2} \approx 10^{-6}$. However, one cannot guarantee that these surveys will either find evidence for strings or else significantly improve limits on their abundance and energy density simply because the number of lenses caused by cosmic strings will be dwarfed by a much larger number of galaxies gravitationally lensed by largescale structure in the Universe.

Tackling a somewhat different problem, we have argued that confirming or refuting the hypothesis of lensing by the cosmic string, given one reported observation of such an event, is in principle straightforward. Regardless of what
shape the string has, follow-up observations in the circle of diameter $L$ centered on the observed lens will uncover at least a few galaxies with split images. For a string located at $z_{l} \leqq 0.5, L \approx 1^{\prime}$. Since the original observation will presumably report the source redshift $z_{s}$ and splitting $\Delta \alpha$, using Eq. (2) one can bound the value of the string redshift $z_{l}$ and, using Eq. (5), compute the required value of the follow-up diameter $L$ so that at least a few additional multiply imaged galaxies are guaranteed to be seen. Since there will be a total of about 100 galaxies per arc $\mathrm{min}^{2}$, photometric redshifts have a sufficiently good accuracy ( $\sigma_{z} \subseteq 0.05$ ) to select lensing candidates. Furthermore, lensing by large-scale structure will be subdominant in this region and will not cause confusion. The only serious complication is the presence of binary systems which will require a more careful follow-up.

## ACKNOWLEDGMENTS

This work was supported by a Department of Energy grant to the particle astrophysics theory group at CWRU. D.H. thanks Chuck Keeton for useful conversations.
[1] A. Vilenkin and E.P.S. Shellard, Cosmic Strings and Other Topological Defects (Cambridge University Press, Cambridge, 1994).
[2] M.B. Hindmarsh and T.W.B. Kibble, Rep. Prog. Phys. 58, 477 (1995).
[3] L.L. Cowie and E.M. Hu, Astrophys. J. Lett. 313, L33 (1987).
[4] E.L. Turner et al., Nature (London) 321, 142 (1986).
[5] J.N. Hewitt, E.L. Turner, R.A. Perey, and E.M. Hu, Astrophys. J. 356, 57 (1990).
[6] M. Sahzin et al., astro-ph/0302547.
[7] J. Hogan and R. Narayan, Mon. Not. R. Astron. Soc. 211, 575 (1984).
[8] B. Paczynski, Nature (London) 319, 567 (1986).
[9] A. Vilenkin, Nature (London) 326, 772 (1987).
[10] M. Hindmarsh, in The Formation and Evolution of Cosmic Strings, edited by G.W. Gibbons, S.W. Hawking, and T. Vachaspati (Cambridge University Press, Cambridge, 1990).
[11] A.A. de Laix and T. Vachaspati, Phys. Rev. D 54, 4780 (1996).
[12] A.A. de Laix, L.M. Krauss, and T. Vachaspati, Phys. Rev. Lett. 79, 1968 (1997).
[13] F. Bernardeau and J.-P. Uzan, Phys. Rev. D 63, 023005 (2001).
[14] A. Vilenkin, Astrophys. J. Lett. 282, L51 (1984).
[15] J.R. Gott, Astrophys. J. 288, 422 (1985).
[16] A. Albrecht and N. Turok, Phys. Rev. D 40, 973 (1989).
[17] B. Allen and E.P.S. Shellard, Phys. Rev. Lett. 64, 119 (1990).
[18] D.P. Bennett and F.R. Bouchet, Phys. Rev. D 41, 2408 (1990).
[19] A. Refregier et al., astro-ph/0304419.
[20] G. Alderingr et al., SPIE Proceedings Vol. 4835, astro-ph/ 0209550; see http://snap.lbl.gov
[21] J.A. Tyson et al., astro-ph/0209632; see http:// www.dmtelescope.org/dark_home.html
[22] See http://www.cfht.hawaii.edu/Science/CFHLS
[23] I.W.A. Browne et al., Mon. Not. R. Astron. Soc. 341, 13 (2003).


[^0]:    ${ }^{1}$ We ignore the string velocity factors in Eq. (2), since the string is expected to be at most mildly relativistic.

[^1]:    ${ }^{2}$ Taking into account the projection of the random walk on the sky makes the actual required length $L$ larger by $4 / \pi$, but we ignore this small correction.

