First Constraints on the Running of Non-Gaussianity

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We use data from the Wilkinson Microwave Anisotropy probe temperature maps to constrain a scale-dependent generalization of the popular "local" model for primordial non-Gaussianity. In the model where the parameter $f_{\rm NL}$ is allowed to run with scale k, $f_{\rm NL}(k) = f^*_{\rm NL}(k/k_{\rm piv})^{n_{f_{\rm NL}}}$, we constrain the running to be $n_{f_{\rm NL}} = 0.30^{+1.9}_{-1.2}$ at 95% confidence, marginalized over the amplitude $f^*_{\rm NL}$. The constraints depend somewhat on the prior probabilities assigned to the two parameters. In the near future, constraints from a combination of Planck and large-scale structure surveys are expected to improve this limit by about an order of magnitude and usefully constrain classes of inflationary models.

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Introduction.—Non-Gaussianity in the distribution of primordial density fluctuations provides a unique window into the physics of inflation. The magnitude of primordial non-Gaussianity and its dependence on scale provide information about the dynamics of scalar field(s), their interactions, and the speed of sound during inflation. Constraints on non-Gaussianity have traditionally come from the measurements of the three-point correlation function of the cosmic microwave background (CMB) temperature anisotropies. Upper limits from COBE [1] have been improved by two orders of magnitude by the Wilkinson Microwave Anisotropy probe (WMAP) experiment [2]. Moreover, clustering of galaxies and galaxy clusters has also been identified as a powerful probe of non-Gaussianity [3], already leading to interesting constraints that are complementary in their information content to the CMB measurements.

So far, most attention has been devoted to the "local" model of primordial non-Gaussianity, where the primordial Newtonian potential $\phi(x)$ is modified with a quadratic term: $\phi = \phi_G + f_{\rm NL}(\phi_G^2 - \langle\phi_G^2\rangle)$, where ϕ_G is a Gaussian potential [4]. The parameter $f_{\rm NL}$ is currently constrained to be 32 ± 21 by WMAP ([2]; see also Refs. [5,6]) and 28 ± 23 by the large-scale structure [7–9]. Several other non-Gaussian models have been constrained as well (e.g., Refs. [10,11]). However, the "running" with physical scale of these models, which may carry important information about the number of inflationary fields and their interactions [12–23], has not yet been constrained with current data (except for a very rough estimate of the angular-multipole dependence of $f_{\rm NL}$ [11] and implicit constraints on a braneworld-motivated model [24]). Such constraints have only been forecasted for future experiments [25–29]. Constraining the running of non-Gaussianity therefore presents a major new opportunity to probe inflationary physics and is just becoming feasible. In this Letter, we present the first such constraints.

Model.—In this work we consider a physically motivated generalization of the local model, where the parameter $f_{\rm NL}$ is promoted to a function of scale k. In particular, we seek to constrain the two-parameter power-law subclass of the generalized models [26]

$$f_{\rm NL}(k) = f^*_{\rm NL}\left(\frac{k}{k_*}\right)^{n_{f_{\rm NL}}}, \tag{1}$$

where k_* is an arbitrary fixed parameter, leaving $f^*_{\rm NL}$ and $n_{f_{\rm NL}}$ as the parameters of interest in this model. Such scaling is expected in inflation when more than one field dominates or when there is self-interaction, and its signatures in the CMB and LSS have been discussed in the literature [25,26,30]. The parameter $n_{f_{\rm NL}}$ is often, though certainly not always, expected to be $\lesssim O(1)$ in inflationary models, but in our phenomenological model it is allowed to take any value.

Bispectrum and $f^*_{\rm NL}$ estimator.—The primordial bispectrum of the $f_{\rm NL}(k)$ model from Eq. (1) is straightforward to calculate:

$$F(\vec{k}_1, \vec{k}_2, \vec{k}_3) = 2[f_{\rm NL}(k_1)P(k_2)P(k_3) + \text{perm.}], \tag{2}$$

where the full bispectrum is $B(\vec{k}_1, \vec{k}_2, \vec{k}_3) \equiv (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)F(\vec{k}_1, \vec{k}_2, \vec{k}_3)$. Here P is the power spectrum of the primordial curvature perturbations, and δ is the Dirac delta function.

Constraining the running parameter $n_{f_{\rm NL}}$ seems difficult because of the apparent requirement to find an estimator for a parameter in an exponent. To avoid this, we resort to an indirect approach where, for a *fixed* value of $n_{f_{\rm NL}}$, we estimate the parameter $f^*_{\rm NL}$ using modifications of the well-known KSW estimator [31], which is known to be nearly optimal [32,33]. We then iterate over the values of the running $n_{f_{\rm NL}}$ to obtain the full likelihood $\mathcal{L}(f^*_{\rm NL}, n_{f_{\rm NL}})$.

The theoretical expectation for the bispectrum of the temperature anisotropies in the cosmic microwave background can be explicitly evaluated, starting from the definition of the generalized non-Gaussian local model in Eq. (1) to account for the running $n_{f_{\rm NL}}$:

$$B^{\rm theory}_{\ell_1\ell_2\ell_3}(f^*_{\rm NL}, n_{f_{\rm NL}}) = 2f^*_{\rm NL} I_{\ell_1\ell_2\ell_3} \times \int_0^\infty r^2 dr(\alpha_{\ell_1}(n_{f_{\rm NL}}, r) \times \beta_{\ell_2}(r)\beta_{\ell_3}(r) + \text{perm.}), \tag{3}$$

where $I_{\ell_1\ell_2\ell_3}$ is the Gaunt integral and

$$\alpha_\ell(r) \equiv \frac{2}{\pi}\frac{1}{k_{\rm piv}^{n_{f_{\rm NL}}}}\int k^{2+n_{f_{\rm NL}}} t_\ell(k) j_\ell(kr) dk \tag{4}$$

$$\beta_\ell(r) \equiv \frac{2}{\pi}\int k^2 P_\Phi(k) t_\ell(k) j_\ell(kr) dk. \tag{5}$$

Here, t_ℓ is the radiation transfer function, which can be calculated using CAMB [34]. Following KSW [31] we can define new, filtered maps $A(\hat{\mathbf{n}}, r)$ and $B(\hat{\mathbf{n}}, r)$,

$$A(\hat{\mathbf{n}}, r) \equiv \sum_{\ell,m} \alpha_\ell(n_{f_{\rm NL}}, r)\frac{b_\ell}{\tilde{C}_\ell} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}), \tag{6}$$

$$B(\hat{\mathbf{n}}, r) \equiv \sum_{\ell,m} \beta_\ell(r)\frac{b_\ell}{\tilde{C}_\ell} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}). \tag{7}$$

Then, we write down the skewness $S(n_{f_{\rm NL}})$:

$$S(n_{f_{\rm NL}}) \equiv \int r^2 dr \int d^2\hat{\mathbf{n}} A(\hat{\mathbf{n}}, r) B^2(\hat{\mathbf{n}}, r), \tag{8}$$

which requires $n_{f_{\rm NL}}$ as input (through A), and does not require *a priori* knowledge of $f^*_{\rm NL}$.

The observed CMB bispectrum is defined as $B^{\rm obs.}_{\ell_1\ell_2\ell_3} = \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}\rangle$, and $S(n_{f_{\rm NL}})$ therefore reduces to

$$S = \sum_{\ell_1\le\ell_2\le\ell_3} \frac{B^{\rm obs}_{\ell_1\ell_2\ell_3}\tilde{B}^{\rm theory}_{\ell_1\ell_2\ell_3}(f_{\rm NL}=1)}{\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}\tilde{C}_{\ell_3}}, \tag{9}$$

where $\tilde{B}^{\rm theory}_{\ell_1\ell_2\ell_3} = b_{\ell_1} b_{\ell_2} b_{\ell_3} B^{\rm theory}_{\ell_1\ell_2\ell_3}$, and b_ℓ is the beam transfer function.

We now define $F \equiv F(n_{f_{\rm NL}})$, the Fisher matrix for $f^*_{\rm NL}$, equivalent to the cumulative signal-to-noise squared of the theoretical bispectrum for $f^*_{\rm NL} = 1$

$$F(n_{f_{\rm NL}}) = \sum_{\ell_1\le\ell_2\le\ell_3} \frac{(\tilde{B}^{\rm theory}_{\ell_1\ell_2\ell_3}(f^*_{\rm NL}=1))^2}{\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}\tilde{C}_{\ell_3}}. \tag{10}$$

The theoretical expectation for $B_{\ell_1\ell_2\ell_3} \propto f^*_{\rm NL}$, so the cubic KSW estimator for $f^*_{\rm NL}$ is:

$$\hat{f}^*_{\rm NL} = \frac{S}{F}. \tag{11}$$

We used HEALPIX, by way of HealPy, to do the forward and backward spherical harmonic transforms required to obtain the A and B maps.

Cut-sky maps.—Equation (11) works well for a full-sky map, but a sky cut introduces a spurious non-Gaussian signal. To account for the masking of the CMB sky, we make the substitution $S \to S_{\rm cut} = S/f_{\rm sky} + S_{\rm linear}$ [35]. $S_{\rm linear}$ is an addition to skewness from Eq. (8), calibrated to account for partial-sky observations:

$$S_{\rm linear} = -\frac{1}{f_{\rm sky}}\int r^2 dr \int d^2\hat{\mathbf{n}}[A(\hat{\mathbf{n}}, r)\langle B^2_{\rm sim}(\hat{\mathbf{n}}, r)\rangle_{\rm MC} + 2B(\hat{\mathbf{n}}, r)\langle A_{\rm sim}(\hat{\mathbf{n}}, r) B_{\rm sim}(\hat{\mathbf{n}}, r)\rangle_{\rm MC}]. \tag{12}$$

The subscripted filtered maps $A_{\rm sim}$ and $B_{\rm sim}$ are created from Python-produced Monte Carlo realizations of the cut CMB sky; the brackets $\langle\rangle_{\rm MC}$ indicate an average over all 300 Monte-Python maps. The simulated maps were produced using the prescription laid out in Appendix A of the WMAP5 paper [36]; the only difference (aside from using the WMAP7 cosmological model) is that we used a uniform weighting for the maps, rather than the slightly more complicated weighting given there, since it only gives a marginal improvement in estimating $f_{\rm NL}$.

Likelihood evaluation.—To find the likelihood, we first find a χ^2 statistic for $f^*_{\rm NL}$, given a value of $n_{f_{\rm NL}}$. Taking the angular-averaged bispectrum $B_{\ell_1\ell_2\ell_3}$ as our observables, we have:

$$\chi^2(f^*_{\rm NL}, n_{f_{\rm NL}}) = \sum_{\ell_1\ell_2\ell_3} \frac{(B^{\rm obs}_{\ell_1\ell_2\ell_3} - f^*_{\rm NL}\tilde{B}^{\rm theory}_{\ell_1\ell_2\ell_3}(n_{f_{\rm NL}}, f^*_{\rm NL}=1))^2}{\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}\tilde{C}_{\ell_3}} \tag{13}$$

(Again, this works because the theoretical expectation for $B_{\ell_1\ell_2\ell_3} \propto f^*_{\rm NL}$.) Using Eqs. (9) and (10), we can rewrite χ^2 as

$$\chi^2(f^*_{\rm NL}, n_{f_{\rm NL}}) = F\left(f^*_{\rm NL} - \frac{S}{F}\right)^2 + \chi^2_0 - \frac{S^2}{F}, \tag{14}$$

where $\chi^2_0 \equiv \sum_{\ell_1\ell_2\ell_3}(B^{\rm obs}_{\ell_1\ell_2\ell_3})^2/(\tilde{C}_{\ell_1}\tilde{C}_{\ell_2}\tilde{C}_{\ell_3})$ is the goodness-of-fit parameter for the data with respect to the $f^*_{\rm NL} = 0$ case. Note that the numerator of χ^2_0 is an observed quantity, and the denominator is based solely on the theoretical prediction for the power spectrum (as well as a few noise and beam parameters of WMAP). Therefore, χ^2_0 does not depend on $f^*_{\rm NL}$ or $n_{f_{\rm NL}}$ at all. We can use the definition of $\hat{f}^*_{\rm NL}$ in Eq. (11) to rewrite the expression for χ^2 as follows

$$\chi^2(f^*_{\rm NL}, n_{f_{\rm NL}}) = F(f^*_{\rm NL} - \hat{f}^*_{\rm NL})^2 + \chi^2_0 - (\hat{f}^*_{\rm NL})^2 F. \tag{15}$$

For a fixed value of $n_{f_{\rm NL}}$, the χ^2 is, as expected, minimized in $f^*_{\rm NL}$ when $f^*_{\rm NL} = \hat{f}^*_{\rm NL}$, and one obtains $\chi^2_{\rm min}(n_{f_{\rm NL}}) = \chi^2_0 - (\hat{f}^*_{\rm NL})^2 F$.

A more interesting task is to calculate the constraints when $n_{f_{\rm NL}}$ is allowed to vary. With an expression for $\chi^2(f^*_{\rm NL}, n_{f_{\rm NL}})$ in hand, we can write an expression for the likelihood, $\mathcal{L}(f^*_{\rm NL}, n_{f_{\rm NL}}) \propto \exp(-\chi^2/2)$ (dropping the constant term with χ^2_0)

$$\mathcal{L}(n_{f_{\rm NL}}, f^*_{\rm NL}) \propto \exp\left[-\frac{F(f^*_{\rm NL} - \hat{f}^*_{\rm NL})^2}{2}\right]\exp\left[\frac{(\hat{f}^*_{\rm NL})^2 F}{2}\right]. \tag{16}$$

To marginalize over $f^*_{\rm NL}$ is also straightforward

$$\mathcal{L}(n_{f_{\rm NL}}) = \int \mathcal{L}(n_{f_{\rm NL}}, f^*_{\rm NL}) df^*_{\rm NL} \propto \frac{1}{\sqrt{F}} \exp\left[\frac{(\hat{f}^*_{\rm NL})^2 F}{2}\right], \tag{17}$$

where, recall, $F(n_{f_{\rm NL}})$ is defined in Eq. (10).

WMAP7 constraints on $n_{f_{\rm NL}}$.—Figure 1 shows the likelihood $\mathcal{L}$ in the $n_{f_{\rm NL}}$-$f^*_{\rm NL}$ plane, as well as the likelihood for $n_{f_{\rm NL}}$ alone, calculated from the WMAP7 temperature maps. We used a weighted and masked combination of the WMAP V and W band maps with the monopole and dipole subtracted, as recommended by the WMAP team [36]. To extract full information from WMAP maps, we used multipoles out to $\ell_{\rm max} = 800$ for the sums in Eqs. (6), (7), and (10). We did not find a significant improvement between $\ell_{\rm max} = 700$ and $\ell_{\rm max} = 800$; we chose the higher value to be conservative in our analysis.

The quantity χ^2 is independent of our choice for $k_{\rm piv}$, but the likelihood itself is not, since F is inversely proportional to $k_{\rm piv}^{2n_{f_{\rm NL}}}$. The true pivot scale favored by the data is the value of $k_{\rm piv}$ for which the errors in $f^*_{\rm NL}$ are uncorrelated with the errors in $n_{f_{\rm NL}}$. We find this scale by using the likelihood to calculate the covariance matrix $\mathbf{C}$ between $f^*_{\rm NL}$ and $n_{f_{\rm NL}}$

$$\mathbf{C}_{p_i,p_j} = \langle (p_i - \bar{p}_i)(p_j - \bar{p}_j) \rangle. \tag{18}$$

We can easily find the pivot value $k_{\rm piv}$ that diagonalizes the covariance matrix $\mathbf{C}$ (see, e.g., Ref. [27])

$$k_{\rm piv} = k_* \exp\left(-\frac{\mathbf{C}_{f^*_{\rm NL}, n_{f_{\rm NL}}}}{f^*_{\rm NL} \mathbf{C}_{n_{f_{\rm NL}}, n_{f_{\rm NL}}}}\right), \tag{19}$$

where k_* is the (arbitrary) pivot used initially, and $f^*_{\rm NL}$ is the corresponding value used in $\mathbf{C}$. Despite the fact that k_* and $f^*_{\rm NL}$ show up in the expression, $k_{\rm piv}$ does not depend on them: it is a fixed number telling us roughly where the experiment has greatest power (and where normalization and running of $f_{\rm NL}(k)$ are precisely uncorrelated). We find that $k_{\rm piv}^{\rm WMAP7} \approx 0.064h$ Mpc^{-1}. The 68%, 95%, and 99% constraints on $f^*_{\rm NL}$ and $n_{f_{\rm NL}}$ are shown at the left panel of Fig. 1, assuming flat priors on $f^*_{\rm NL}$ and $n_{f_{\rm NL}}$ and $k_* = k_{\rm piv}^{\rm WMAP7} \approx 0.064h$ Mpc^{-1}.

Dependence on the prior.—As with most present-day cosmological measurements, the precise constraints depend on the prior probability on the parameters we are constraining. Even for a simple flat prior on $f^*_{\rm NL}$ and $n_{f_{\rm NL}}$, the actual effective prior depends on the *a priori* chosen pivot in wave number k_*. For example, a flat prior on $(f^*_{\rm NL})^{(1)} \equiv f_{\rm NL}(k_{*,1})$ defined at some pivot $k_{*,1}$ corresponds to a nonflat prior on some $(f^*_{\rm NL})^{(2)} \equiv f^*_{\rm NL}(k_{*,2})$ defined at some other pivot $k_{*,2}$, since $(f^*_{\rm NL})^{(2)} \equiv (f^*_{\rm NL})^{(1)} \times (k_{*,2}/k_{*,1})^{n_{f_{\rm NL}}}$. If we assume some alternate pivot $k_{*,2}$ but hold the flat prior in $f^*_{\rm NL}$, the contours in the $n_{f_{\rm NL}}$-$f^*_{\rm NL}$ plane (left panel of Fig. 1) are stretched vertically by a factor of $(k_{*,2}/0.064h\ {\rm Mpc}^{-1})^{n_{f_{\rm NL}}}$.

We have experimented with different k-pivot values for a flat prior on $f^*_{\rm NL}$ and $n_{f_{\rm NL}}$. We have also investigated other possibilities, such as the prior that assigns equal weight to each decade in $|f^*_{\rm NL}|$ above 0.1 (so uniform in $\log(f^*_{\rm NL})$, but cut off at the arguably lowest-ever observable value of $|f^*_{\rm NL}| = 0.1$ so that the total integrated likelihood is finite). We present the two aforementioned examples, showing constraints on $n_{f_{\rm NL}}$ marginalized over $f^*_{\rm NL}$, in the right panel of Fig. 1. In the end, we decide to quote results for the flat prior and the uncorrelated $k_{\rm piv}$ value from Eq. (19), which most closely follows priors to both non-Gaussian and other cosmological parameters applied in the literature.

Putting it all together, we can get the estimate for $n_{f_{\rm NL}}$ from the WMAP7 data for a flat prior on $f^*_{\rm NL}$ at the pivot $k_{\rm piv}$ from Eq. (19). The 68% (95%) confidence interval is

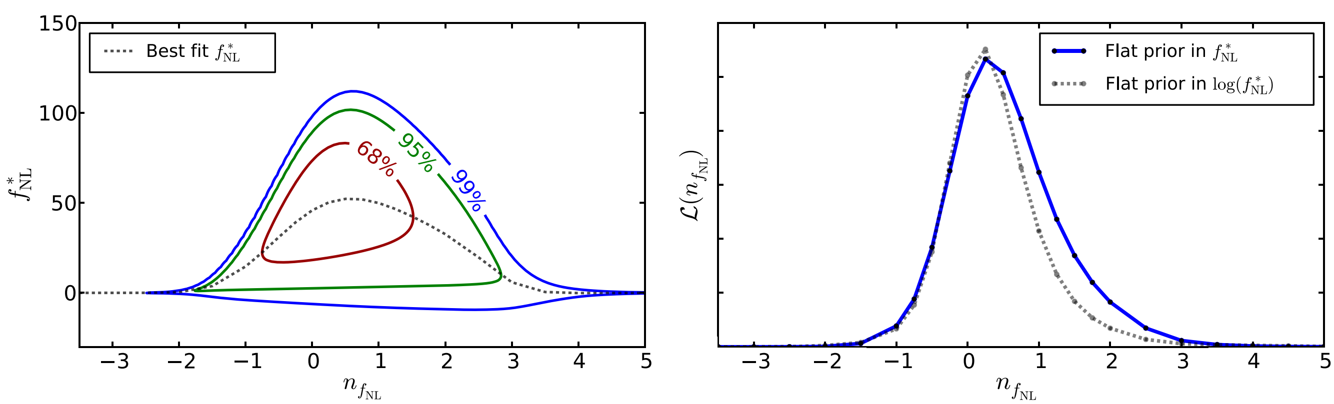

FIG. 1 (color online). Likelihood in the $n_{f_{\rm NL}}$-$f^*_{\rm NL}$ plane (left panel) and marginalized over $f^*_{\rm NL}$ (right panel). The principal constraints, shown in the left panel and with the bold blue curve on the right, correspond to the flat prior on $f^*_{\rm NL}$ at the pivot value where the constraints on $f^*_{\rm NL}$ and $n_{f_{\rm NL}}$ are uncorrelated [see Eq. (19)]. In the right panel we also show the marginalized likelihood for $n_{f_{\rm NL}}$ with a prior on $f^*_{\rm NL}$ that is uniform in $\log(f^*_{\rm NL})$ for $|f^*_{\rm NL}| > 0.1$ and zero otherwise. The dashed curve in the left panel shows the quantity $\hat{f}^*_{\rm NL}$, which is the best-fit value of the parameter $f^*_{\rm NL}$ for a fixed $n_{f_{\rm NL}}$. See text for other details.

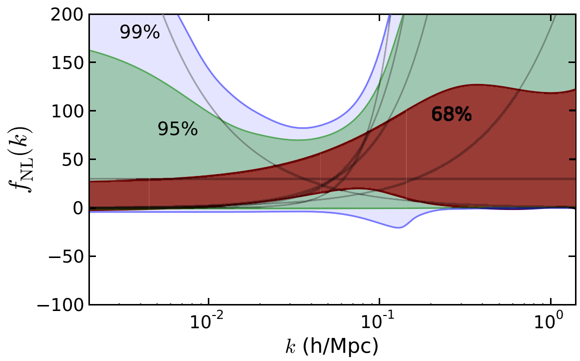

FIG. 2 (color online). Constraints propagated to $f_{\rm NL}(k)$. We also show several models that are reasonable fits to the data (all within the 99% confidence limit of the left panel of Fig. 1) to guide the eye as to how typical models from our ansatz behave.

$$n_{f_{\rm NL}} = 0.30^{+0.78(1.9)}_{-0.61(1.2)}. \quad (20)$$

The current constraints are therefore fully consistent with no running, as Fig. 1 clearly indicates. Figure 2 shows the constraints in the $f_{\rm NL}(k)$ plane together with a few representative models allowed by the data.

Conclusions.—We have presented the first constraints on the scale-dependence of (any form of) non-Gaussianity using the WMAP7 data. The constraints are compatible with zero running, $n_{f_{\rm NL}} = 0$, with very mild ($<$ 1-sigma) preference for a positive value of $n_{f_{\rm NL}}$. We will learn more soon: the Planck data and the data from upcoming large-scale structure surveys should be able to improve constraints on the running of non-Gaussianity by about an order of magnitude [25,28,29], thus shedding important new light on the physics of inflation.

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