A Parametric Texture Model based on Joint Statistics of Complex Wavelet Coefficients

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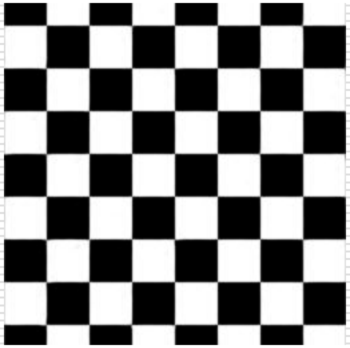
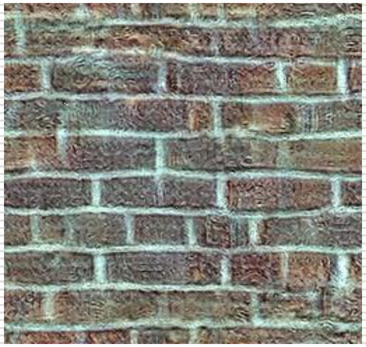
Texture

- What is a Texture?

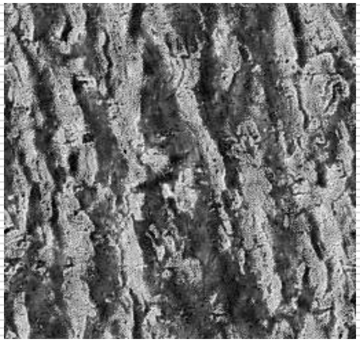
  Texture Images are spatially homogeneous and consist of repeated elements, often subject to randomization in their location, size, color or orientation.



Periodic



Pseudo Periodic



Random

Objective

- To statistically parameterize textures
- Practical applications: Medical Imaging, Video Synthesis, Image Correction, Computer Graphics
- How do we know if such a model exists at all? The answer is given by Julesz conjecture

Julesz conjecture

- Every texture can be modeled as a real 2D Random Field (RF)
- Julesz conjecture hypothesizes that there exist a set of statistical functions such that texture samples drawn from two RF's that are equal in expectation over these statistical functions are visually indistinguishable

$$\mathcal{E}(\phi_k(X)) = \mathcal{E}(\phi_k(Y))$$

Interpreting the Conjecture

- The hypothesis also establishes the importance of human perception as the ultimate criterion for judging texture equivalence
- Now the problem reduces to finding these statistical functions – called the constraint functions.

Some Questions !

- How do we come up with the set of constraint functions?
- How do we test the validity of this set?
  - Exhaustive Search??
  - The Synthesis by Analysis approach!!

Synthesis by Analysis approach

- A library of example textures is used to test these constraints
- Design an algorithm that synthesizes textures satisfying the required statistical constraints
- Compare texture images : Visual Perception

Constraint functions

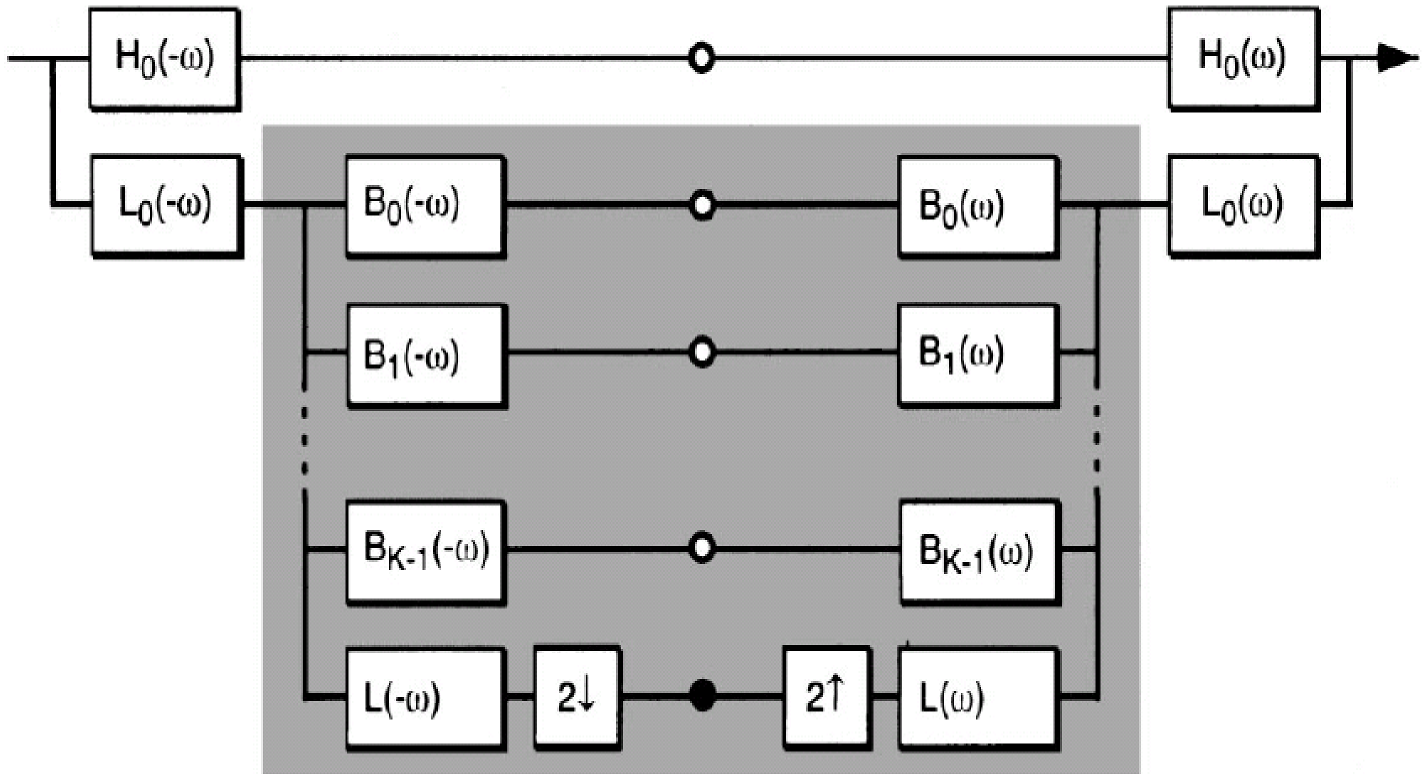
- Choose an initial set of functions and synthesize large number of texture samples
- Select synthesis failures and classify them according to the visual features. Choose group which produces poorest results
- Choose new statistical constraint capturing the visual feature most noticeably missing in the group and incorporate into the synthesis algorithm
- Verify that the new constraint achieves the desired effect of capturing that feature by re-synthesizing the failure group.
- Verify all constraints for redundancy.

Choice of constraint Functions

- We consider four different kinds of constraint functions :
  - Marginal Statistics
  - Coefficient Correlation
  - Magnitude Correlation
  - Phase Statistics

Texture Decomposition

Steerable Pyramid Decomposition



Source: Simoncelli et.al, Steerable Pyramid,1991

Steerable Pyramid Decomposition

$$L(r,\theta) = \begin{cases} 2\cos\left(\frac{\pi}{2}\log_2\left(\frac{4r}{\pi}\right)\right), & \frac{\pi}{4} < r < \frac{\pi}{2} \\ 2, & r \leq \frac{\pi}{4} \\ 0, & r \geq \frac{\pi}{2} \end{cases}$$

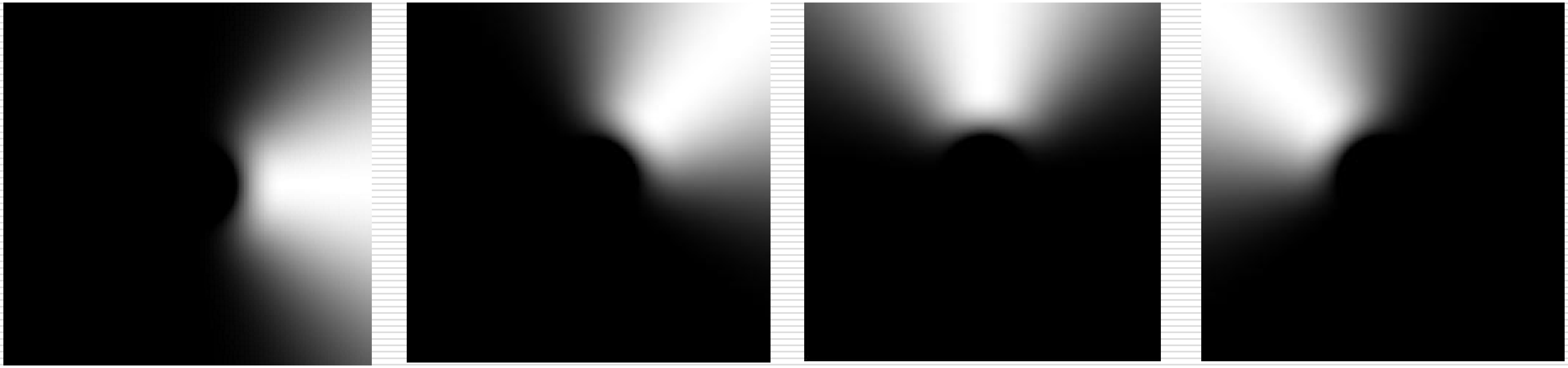
$$H(r) = \begin{cases} \cos\left(\frac{\pi}{2}\log_2\left(\frac{2r}{\pi}\right)\right), & \frac{\pi}{4} < r < \frac{\pi}{2} \\ 1, & r \geq \frac{\pi}{2} \\ 0, & r \leq \frac{\pi}{4} \end{cases}$$

$$G_k(\theta) = \begin{cases} \alpha_K \left[\cos\left(\theta - \frac{\pi k}{K}\right)\right]^{K-1}, & \left|\theta - \frac{\pi k}{K}\right| < \frac{\pi}{2} \\ 0, & \text{otherwise,} \end{cases}$$

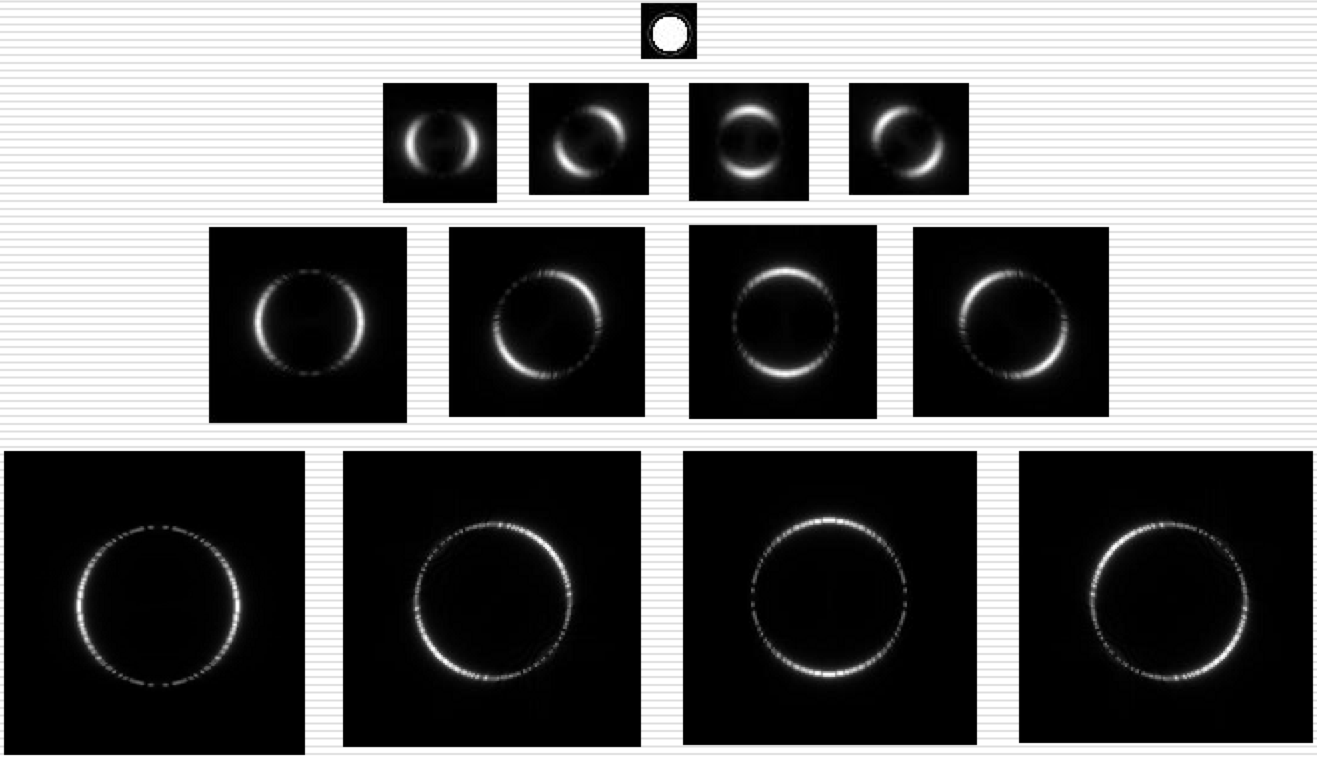
$$B_k(r,\theta) = H(r)G_k(\theta), \quad k \in [0, K-1],$$

Source: Simoncelli et.al, Steerable Pyramid,1991

Steerable Pyramid Decomposition



Steerable Pyramid Decomposition



Source: Simoncelli et.al, Steerable Pyramid,1991

Steerable Pyramid Decomposition

- Advantages :

  → Tight Frame

  → No Aliasing

  → Rotation and Translation Invariance

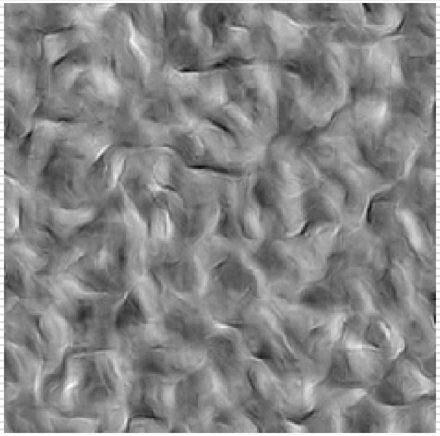
Texture Analysis

Analysis Algorithm

- Extract pixel statistics - Range, Mean, Variance, Skewness and Kurtosis of the Image - ***6 parameters***
- Apply the Steerable Pyramid Decomposition. After the decomposition, we will have a total of N.K+2 images. Let me index them as follows - HP, BP(1,1), BP(1,2)...., BP(1,K), BP(2,1), Bp(2,2)...., BP(2,K), ...,BP(N,1), BP(N,2)...., BP(N,K), LP
- Calculate the variance of the High pass image - ***1 Parameter***
- Obtain the partially reconstructed Low-pass images - Index them by L1, L2, ...,LN
  - Obtain the Skewness and Kurtosis of each of these partially reconstructed lowpass images along with that of the residual low pass image - ***2*(N+1) parameters***
  - Compute Central samples of the auto-correlation of the residual low pass image LP, and the partially reconstructed low pass images L1, L2…LN - ***$(N+1)(M^2+1)/2$ parameters***
- Compute Central samples of the auto-correlation of magnitude of each subband - i.e. from all the Band pass Images - ***$NK(M^2+1)/2$***.
- Compute Cross-Correlation between different subband magnitudes at each scale - ***$NK(K-1)/2$.***
- Compute Cross-Correlation between subbands magnitudes of each scale with the subband magnitudes of the subsequent coarser scale - ***$K^2(N-1)$ parameters***
- Compute the cross-correlation between the real part of all subbands in a particular scale with the real and imaginary parts of the all the phase-doubled subbands at the next coarser scale - ***$2K^2(N-1)$***

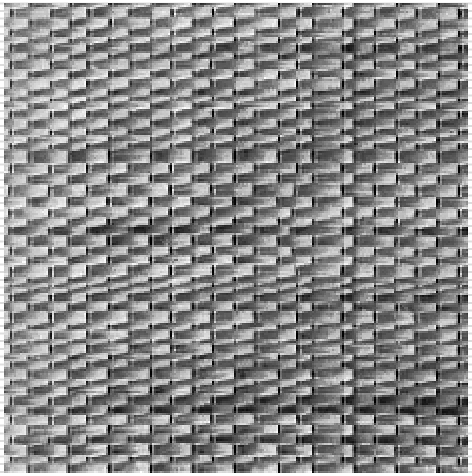
Marginal Statistics

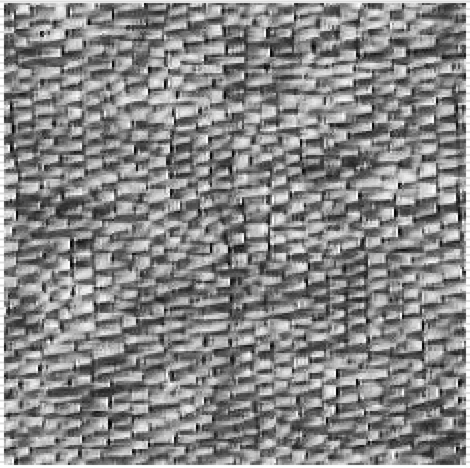
- Express the relative amount of each intensity in the texture

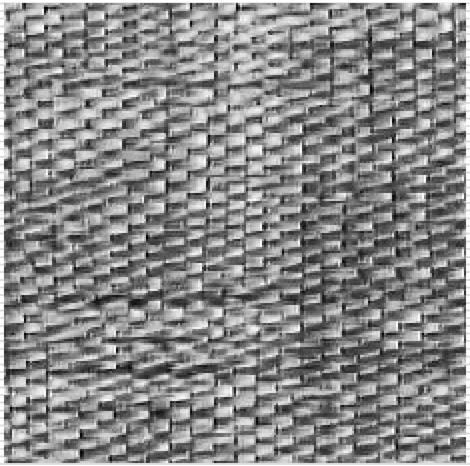


Correlation Coefficient

- Necessary to represent periodic structures and long range correlations

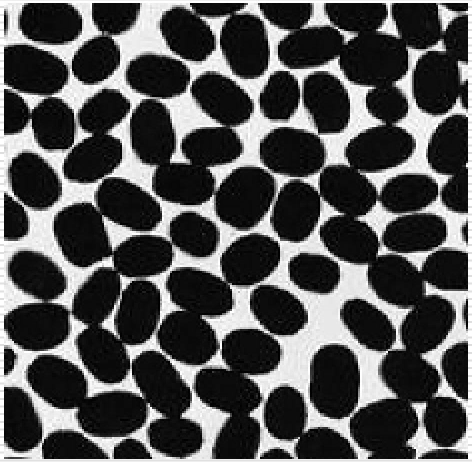






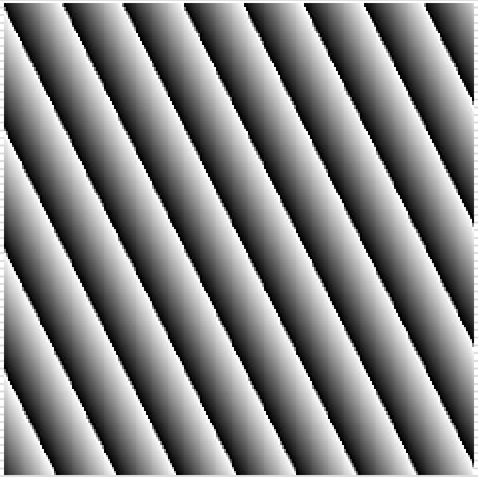
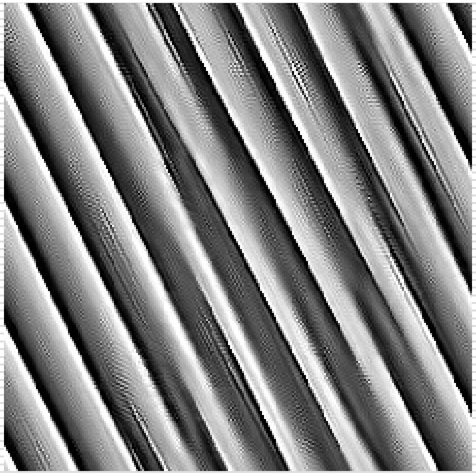
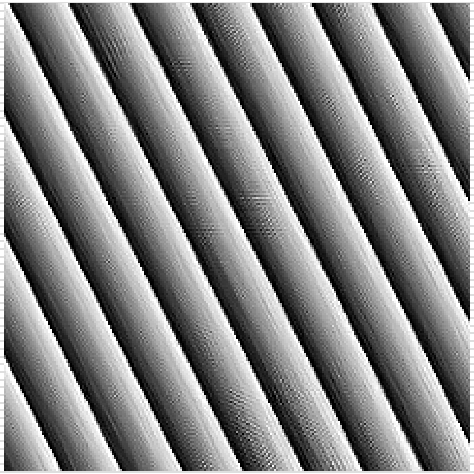
Magnitude Correlation

- Magnitudes capture important structural information about the textures.
- Magnitude correlations between coefficients is present even when the pixels are uncorrelated !



Phase Statistics

- Phase Statistics distinguishes edges from lines.
- They help in representing gradients due to shading and lighting effects.
- Captures relative phase of coefficients of bands at adjacent scales

Texture Synthesis

The Synthesis Algorithm

- We need to synthesize an image which satisfies the constraints extracted during analysis
- We start with a noisy image generated from a Gaussian distribution and impose statistical constraints on this image
- This is essentially projecting the image on to a subspace of textures with the required statistical properties
- However given the large number of constraint functions, it is not always possible to determine the projection operator

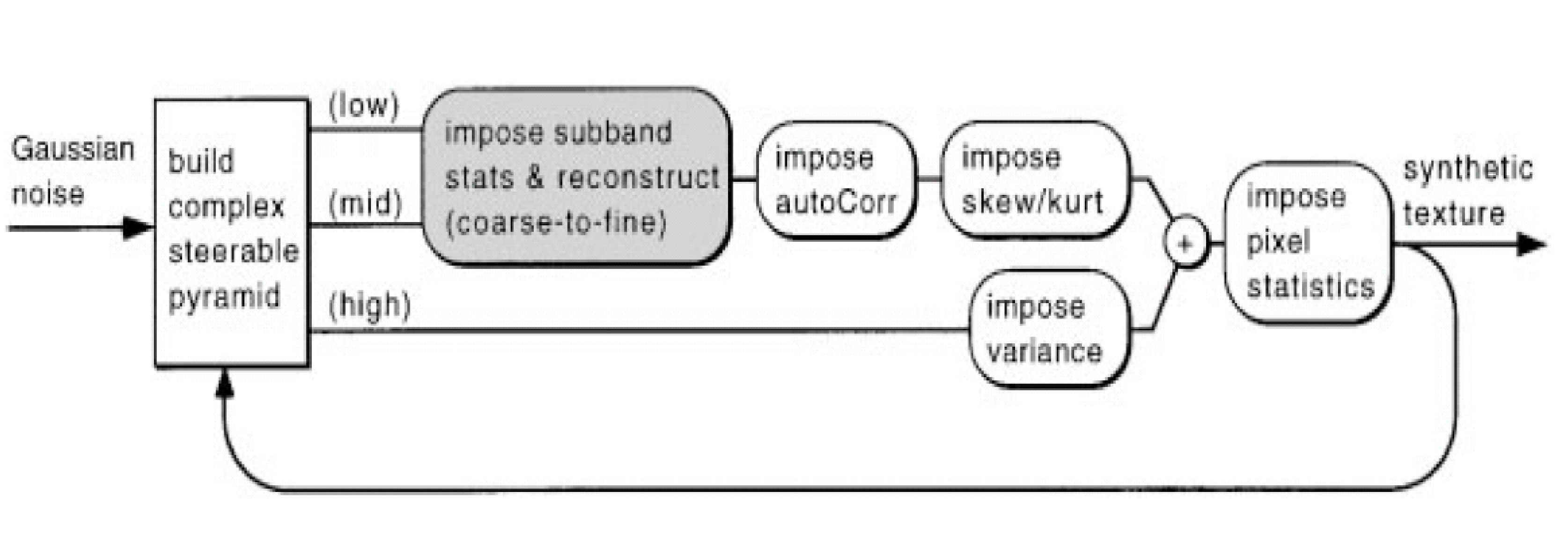
Projection onto Constraint Surfaces

- Instead, the constraints are imposed sequentially rather than simultaneously in an iterative manner
- Issues with convergence – We are not guaranteed that this sequence of operations will converge
- To maximize chances of convergence, every time a particular constraint is imposed, we would like to do so while changing the image as little as possible
- This is done using gradient projection – The image is updated in the direction of the gradient of the particular statistical constraint being imposed

$$\vec{x}' = \vec{x} + \lambda_k \vec{\nabla} \phi_k(\vec{x}),$$

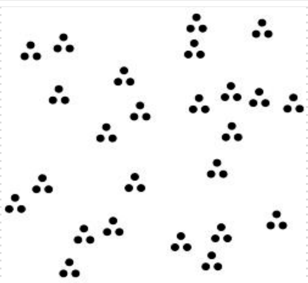
$$\phi_k(\vec{x}') = c_k$$

Top Level Block Diagram for Synthesis

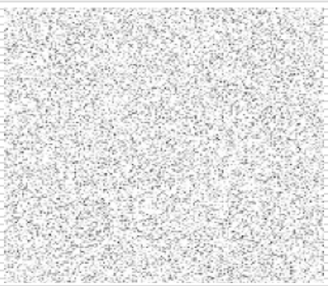


Source: Simoncelli et.al,Texture Synthesis,2003

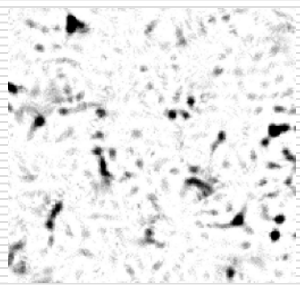
Progress over Iterations



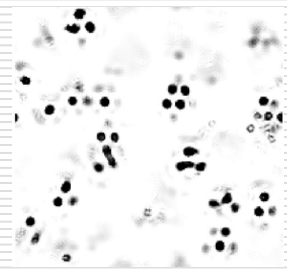
Original Texture



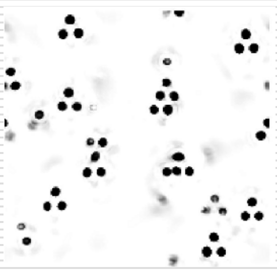
Gaussian Noise



1^{st} iteration

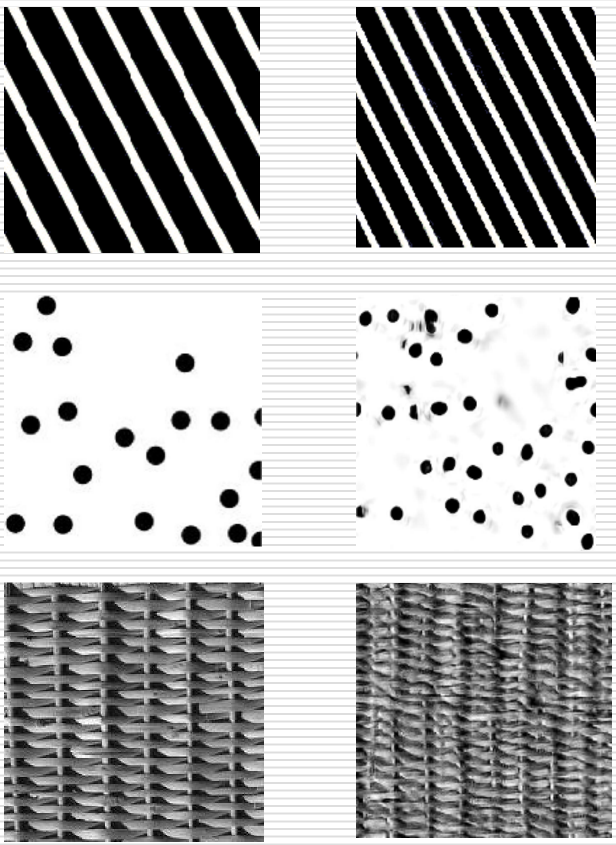


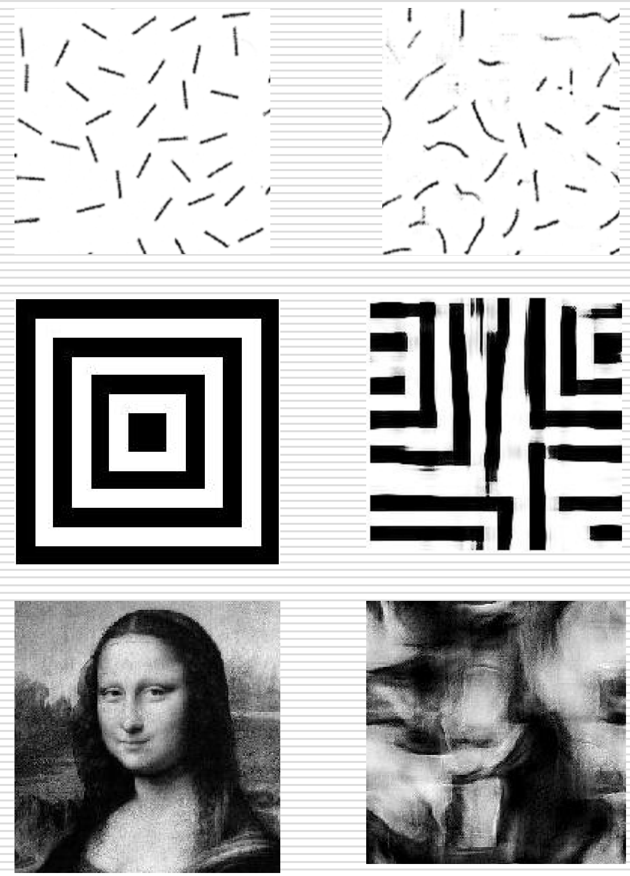
5^{th} iteration



25^{th} iteration

Synthesis Examples





Observations

Choice of Constraint Functions:

- The chosen set of statistical functions are not the complete set of constraint functions
- The constraints used have been determined through observations and reverse-engineering.
- No guarantee that the current constraint set is unique – Another alternative could perform equally as well or perhaps better
- However, the current set is good enough to distinguish a large set of textures and thus is a good descriptor of textures

Convergence Issues

Convergence issues:

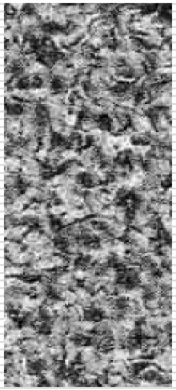
- Convergence has not been proved
- However, the algorithm has almost always converged to an image that is visually indistinguishable from the original texture

Extensions

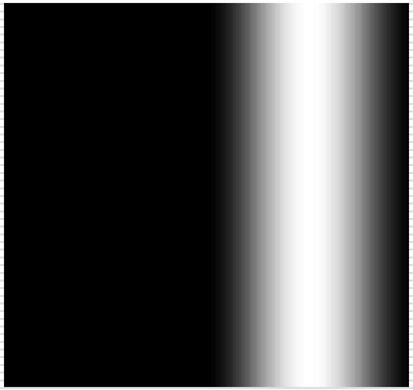
- Constrained Texture synthesis
- Repairing of Defects
- Painting a texture onto a Image
- Mixture of two textures

Extending a Texture

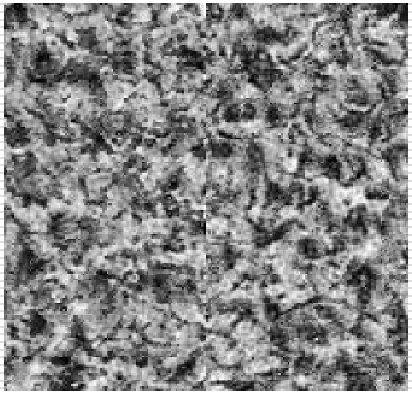
Original Texture



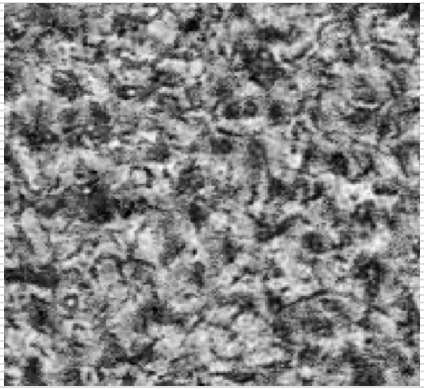
Mask



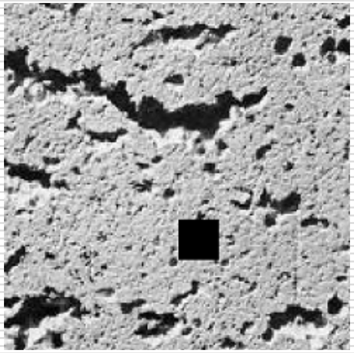
Sharp Mask



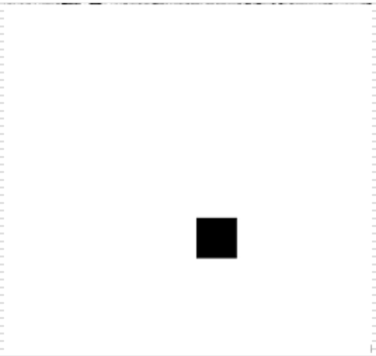
Smooth Mask



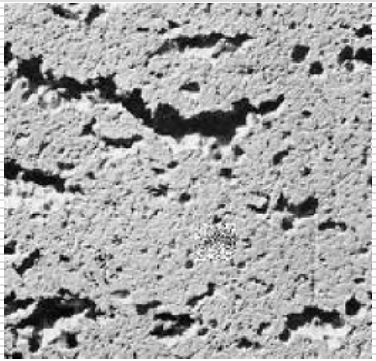
Repairing Defect in a Texture



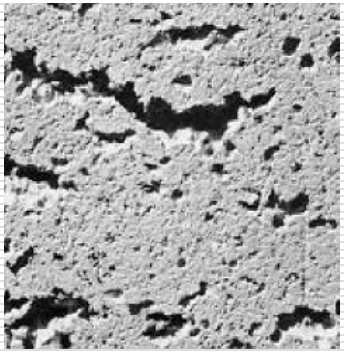
Texture with defect



Mask

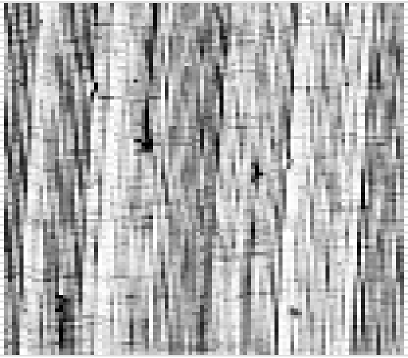


5th iteration

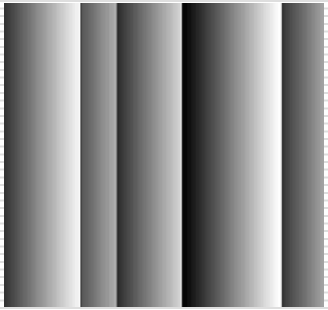


25th iteration

Painting Texture onto an Image



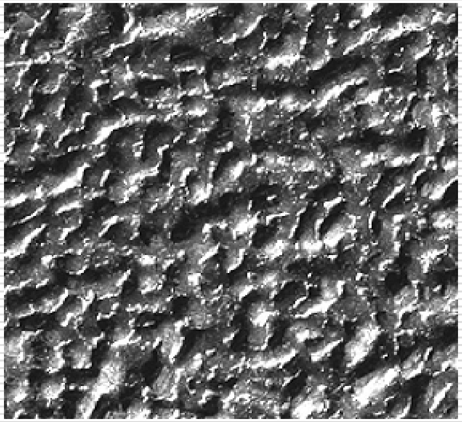
Painting Texture onto an Image



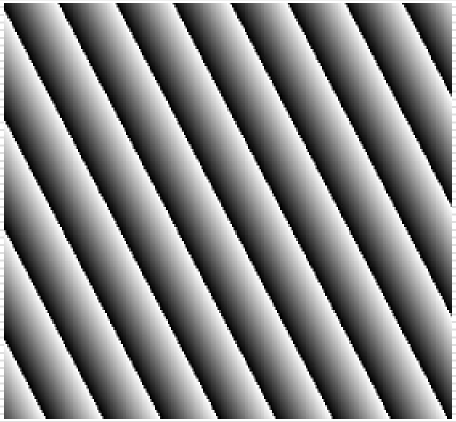
Mixing of two textures

- Done by averaging the parameters of the two textures

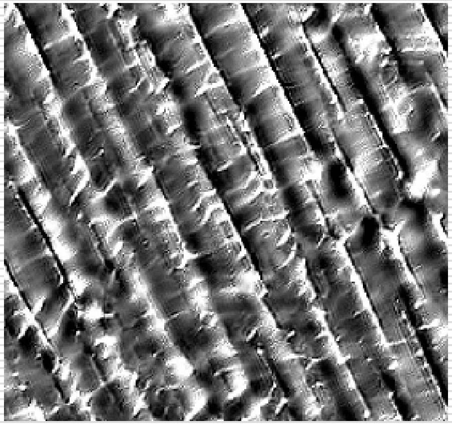
Metal texture



Sawtooth texture



Mixed Texture !



References

- A parametric texture model based on Joint Statistics of Complex Wavelet Coefficients- Simoncelli and Portilla(2003)
- Texture Characterization via Joint Statistics of Complex Wavelet Coefficient Magnitudes- Simoncelli and Portilla(1998)
- A Filter design techinque for Steerable Pyramid Image Transforms
  - Simoncelli et al.(1996)

Thank You !