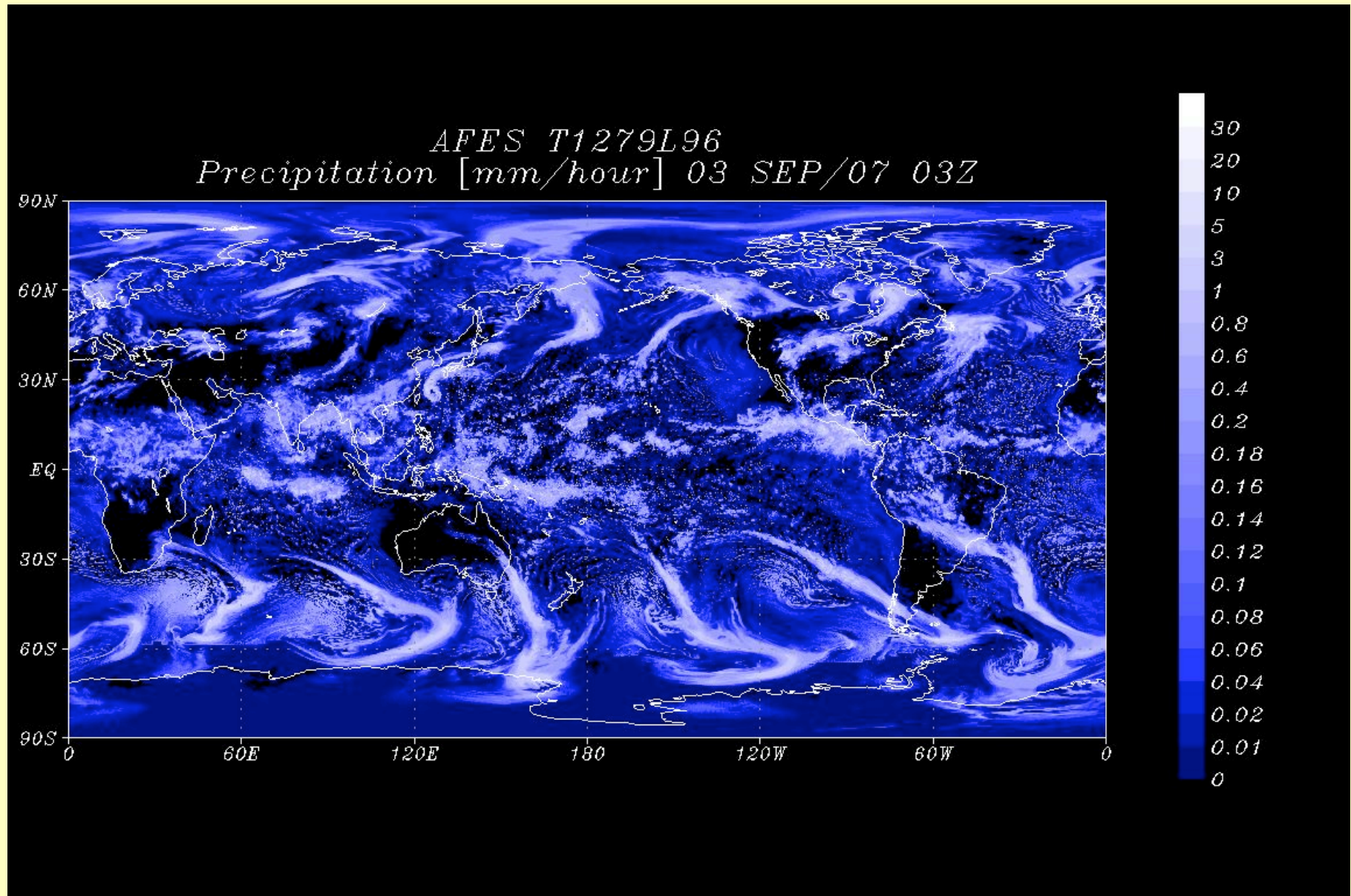


# ***Adaptive Grids for Atmospheric General Circulation Models***

*Christiane Jablonowski  
University of Michigan*

*NCAR ASP Colloquium,  
June/5/2008*

# Multi-Scale Processes



## ***Some observations***

- ✧ Typical length scale of
  - ✧ Rossby waves in midlatitudes:  $O(10,000-1,000 \text{ km})$
  - ✧ Meso-scale systems, e.g. mountain waves:  
 $O(100 \text{ km})$
  - ✧ Convective cloud clusters / thunderstorms:  $O(10 \text{ km})$
  - ✧ Single clouds, convection:  $O(1 \text{ km})$
  - ✧ Hail ?
  - ✧ Rain droplet ?
  - ✧ Molecular diffusion ?
- ✧ Typical resolution of today's global weather and climate models: horizontal  $\Delta x$ , vertical ?
- ✧ 'Resolved' features are typically of order  $6 \Delta x$

## ***More observations***

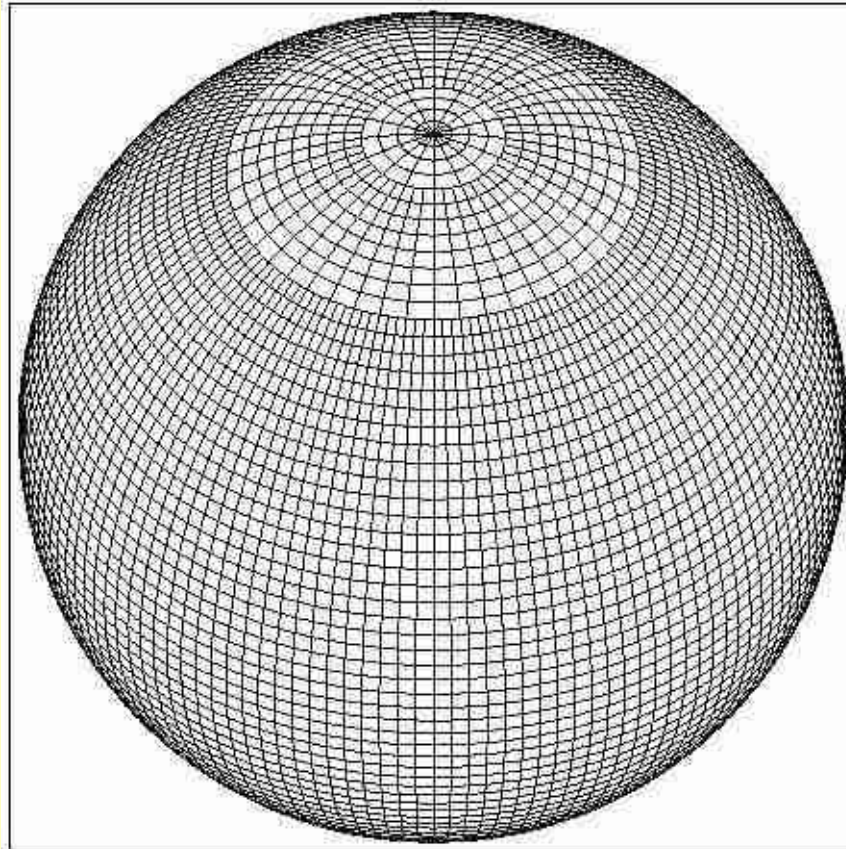
- ✧ Atmospheric flow transitions from the hydrostatic to the nonhydrostatic regime at scales around  $\Delta x \approx 10$  km
- ✧ Our 'dream' resolutions for global weather and climate models are cloud-resolving and lie around  $\Delta x \approx 1$  km
- ✧ Doubling the horizontal resolution increases the computational costs by a factor of 8
- ✧ Doubling the vertical resolutions adds another factor of 2
- ✧ We need to increase our computational power by a factor of  $\approx 10,000$
- ✧ and we need to be more creative:
  - ✧ invent new algorithms
  - ✧ apply Adaptive Mesh Refinement (AMR) to bridge the gap soon



# ***Adaptive Grids for 3D Atmospheric Models***

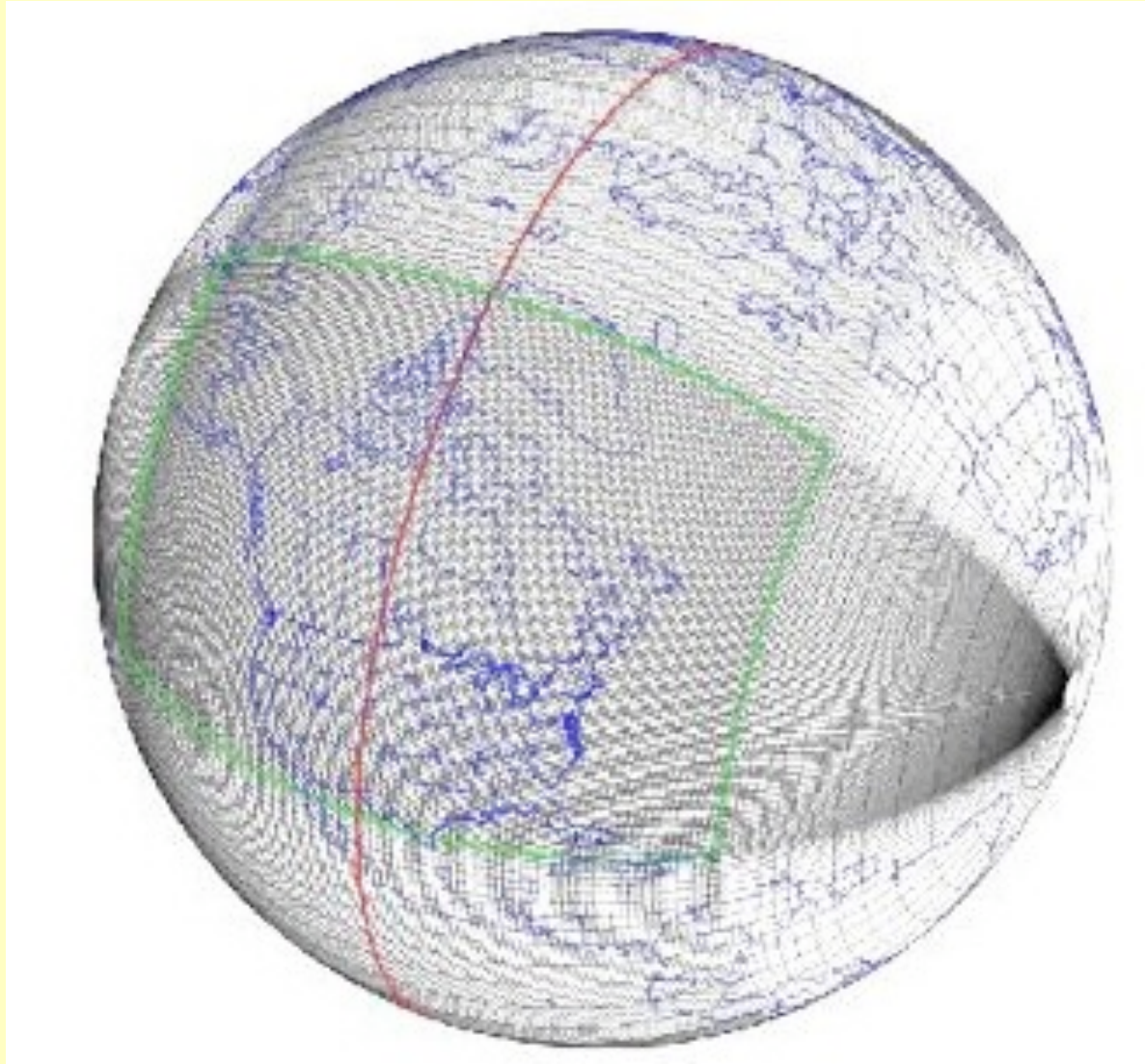
- ✧ Statically adaptive grids
  - ✧ Reduced grid
  - ✧ Stretched grids
  - ✧ Transformed grids (e.g. Schmidt coordinate transformation)
  - ✧ Unstructured grids
  - ✧ Nested grids
- ✧ Dynamically adaptive grids
  - ✧ Irregular data structures: triangulated grids  
(Bacon et al., MWR 1998, Gopalakrishnan et al., MWR 2002)
  - ✧ Regular data structures: block-structured lat-lon grid  
(Skamarock et al., JCP 1989, Hubbard and Nikiforadis, MWR 2003, Jablonowski et al., MWR 2006)
  - ✧ Cubed sphere with spectral element formulation:  
A. St-Cyr, S. Thomas and J. Dennis (St-Cyr et al., MWR (2008))

## ***Static Adaptations: Reduced Grids***



- Number of grid cells in longitudinal direction is reduced towards high latitudes
- Keeps the resolution more uniform, allows longer time steps

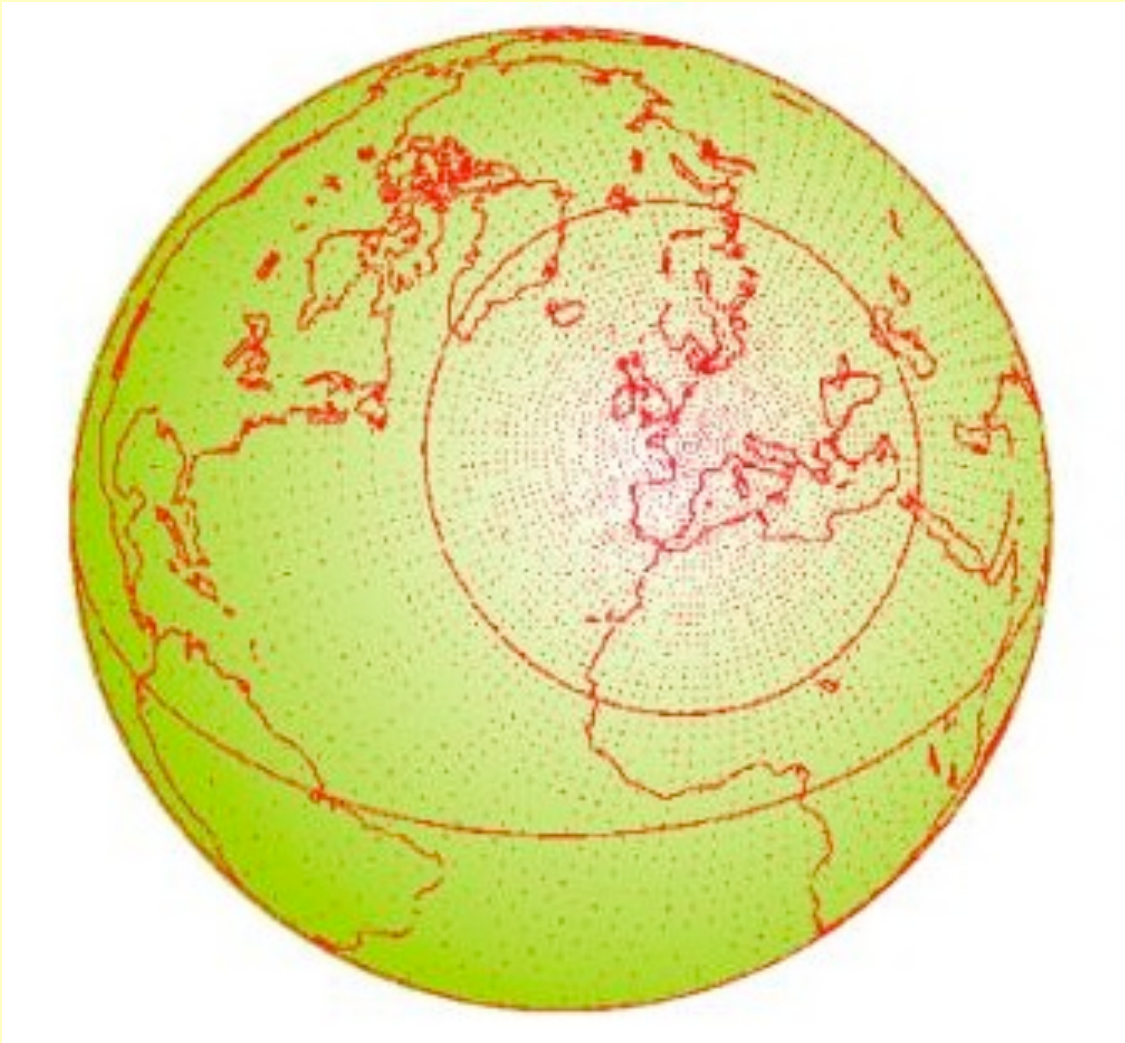
## ***Static Adaptations: Stretched Grids***



GEM  
Canadian Model

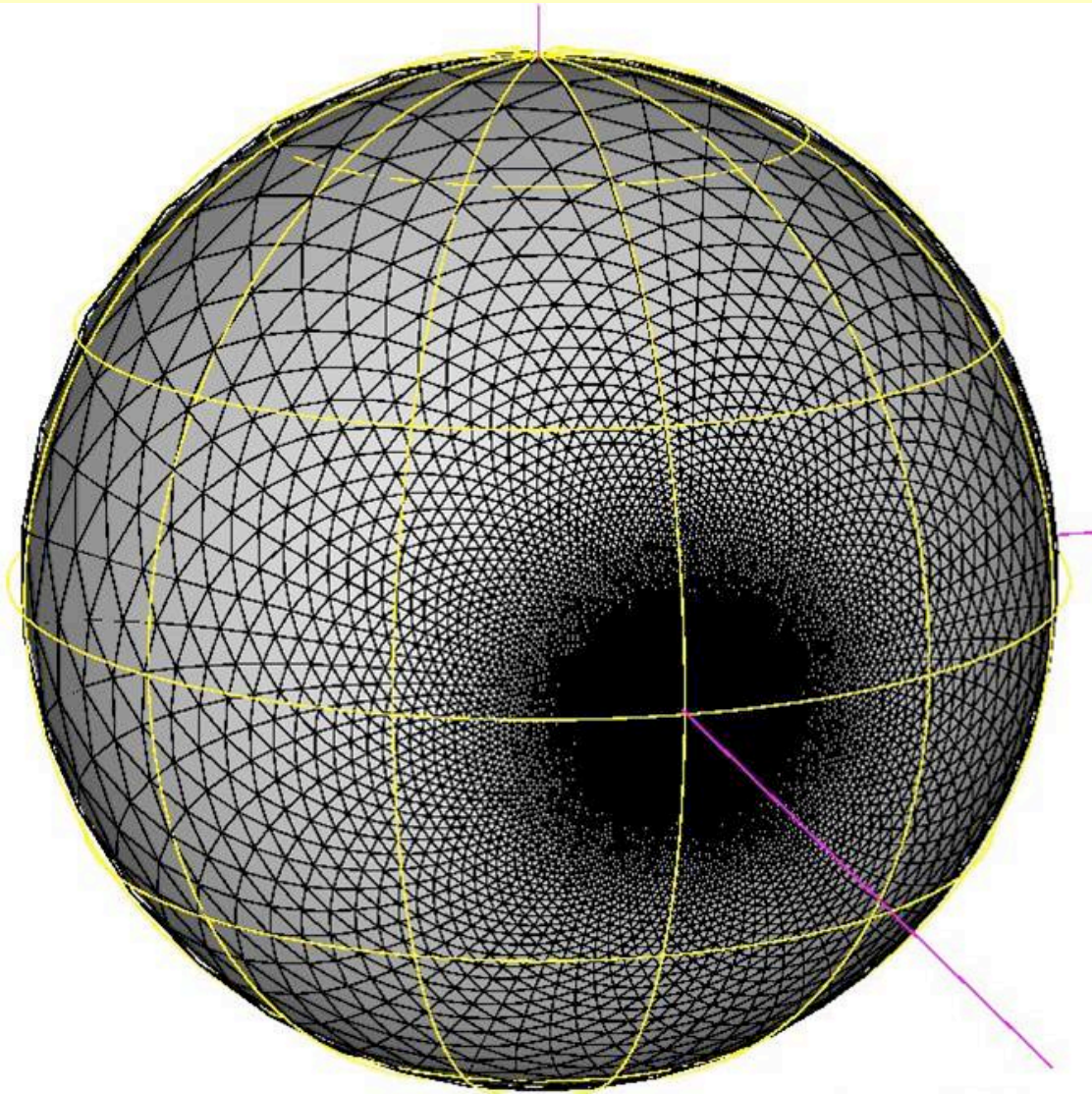


***Static Adaptations:  
Rotated and transformed (Schmidt) lat-lon grid***



Model Arpege  
Meteo France

# ***Static Adaptations: Stretched Icosahedral Grid (Schmidt transformation)***



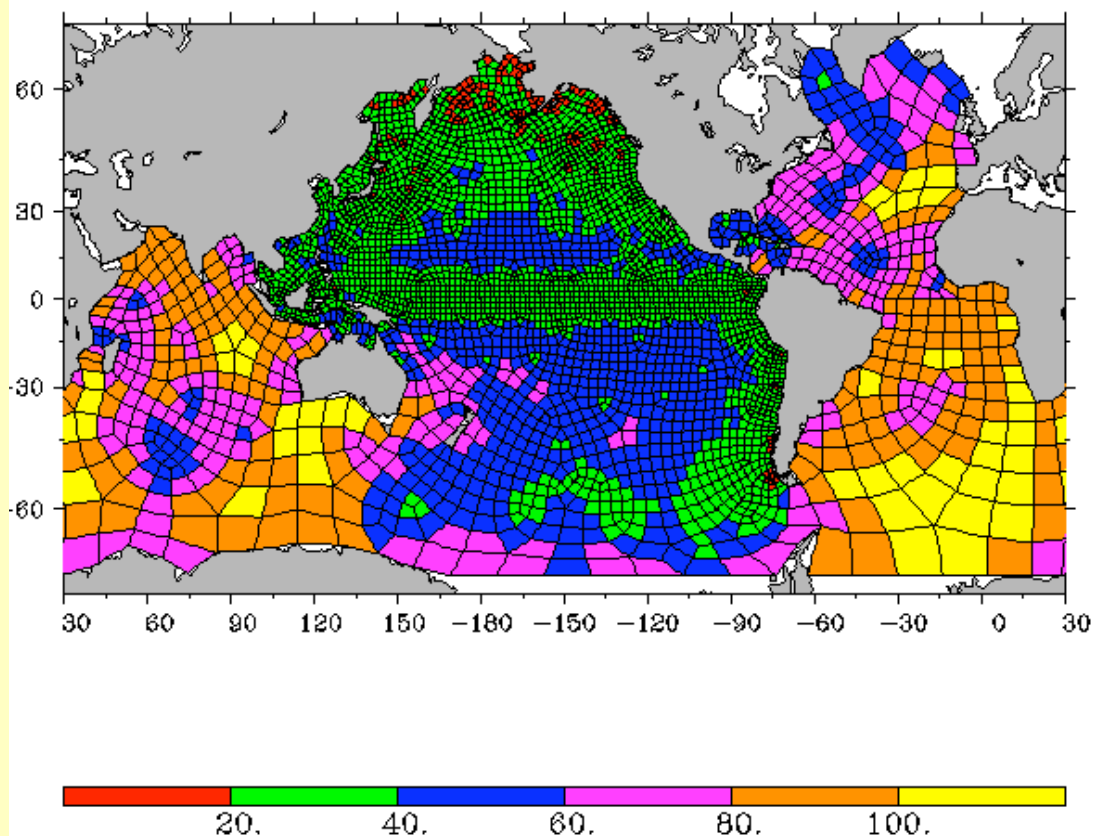
lon: 15W

Courtesy of  
H. Tomita  
(Frontier Research System  
for Global Change, Japan)



# ***Static Adaptations: Unstructured Grids***

Model SEOM: Spectral Element Ocean Model, here 3552 elements with 64 collocation points



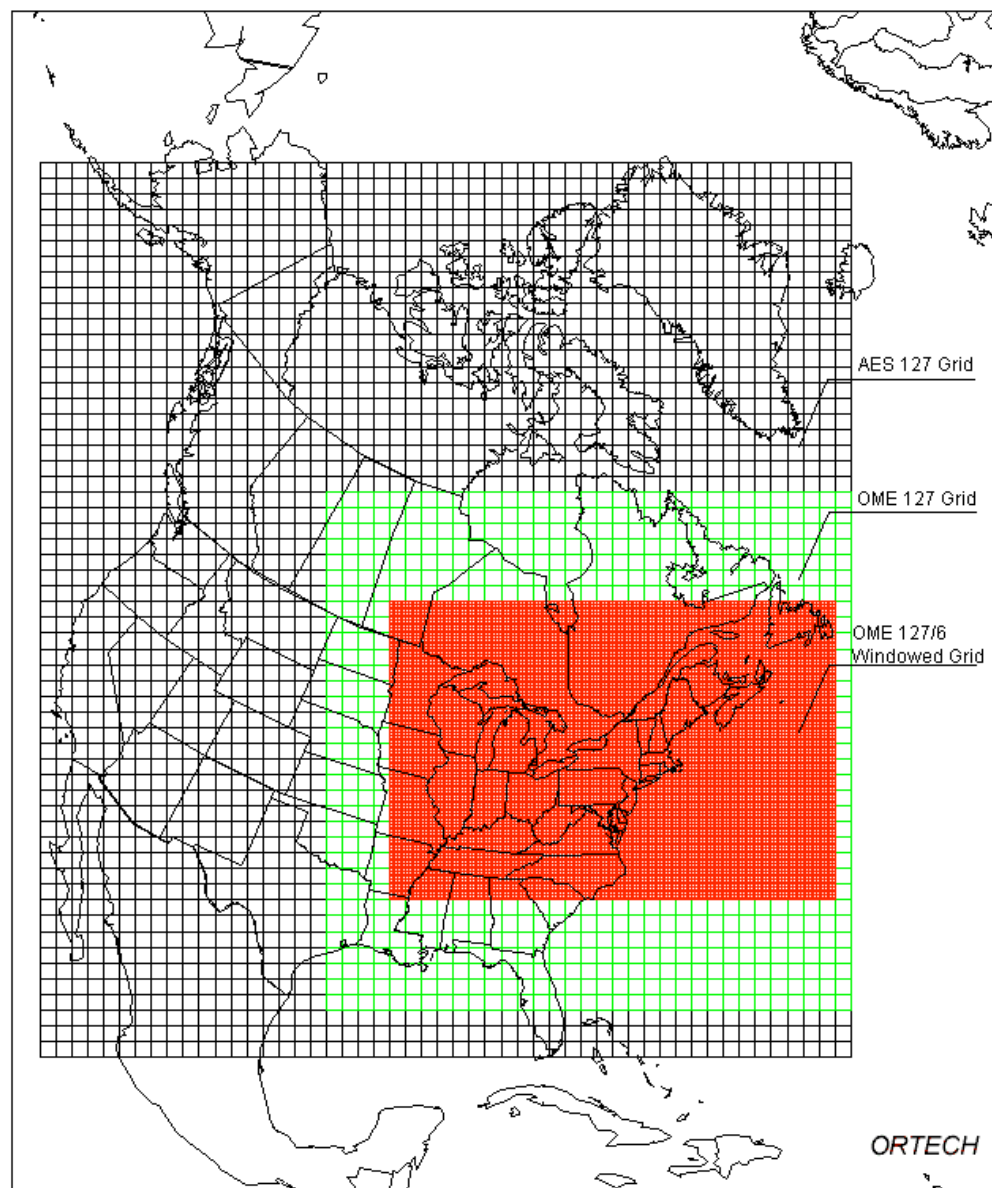
Average grid spacing (km) within each element

Spectral elements allow flexible configurations:

h and p refinements possible (compare to D.B. Haidvogel's presentation)

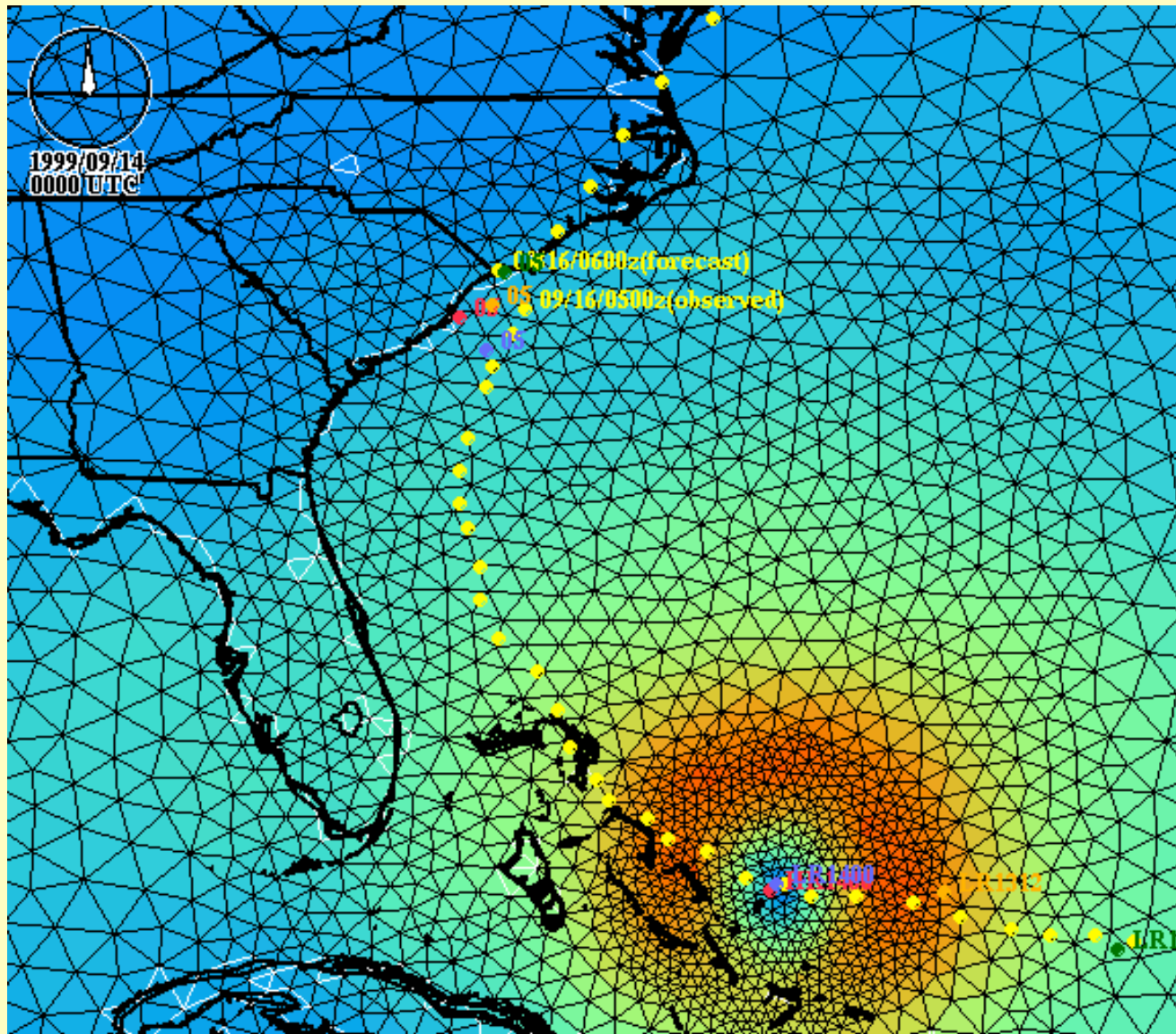
Source:  
Rutgers University

# ***Static adaptations: (Multiple) Nested Grids***



Canadian Model

# ***Dynamic Adaptations: Irregular Triangular Grid***



Hurricane Floyd  
(1999)

OMEGA model

Courtesy of  
A. Sarma (SAIC,  
NC, USA)

Colors indicate the wind speed

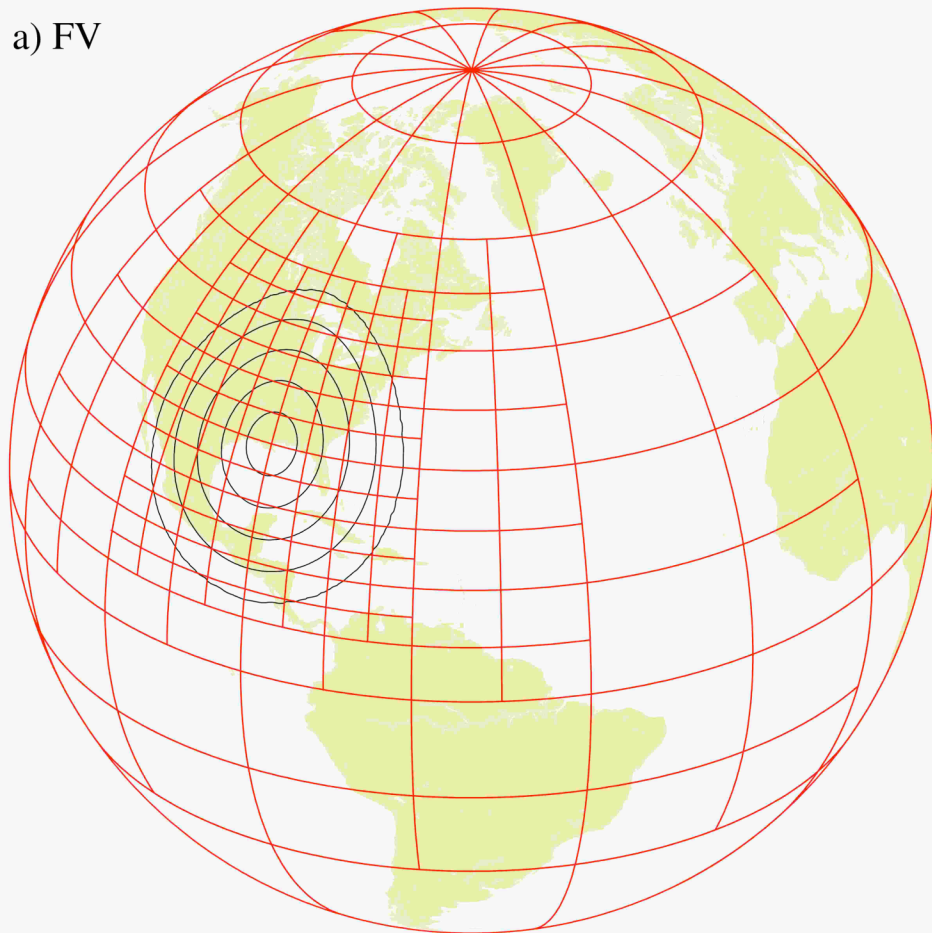
## ***Two Adaptive Shallow Water Models***

- ✧ AMR Comparison is based on a joint paper with Amik St-Cyr and collaborators from NCAR, MWR (2008)
- ✧ The AMR Spectral Element Model (SEM) was mainly developed by A. St-Cyr, J. Dennis & S. Thomas (NCAR), 2D shallow water (SW) version of the 3D dycore HOMME
- ✧ The AMR FV model is documented in Jablonowski (2004), Jablonowski et al. (2004, 2006)
- ✧ FV support by S.-J. Lin (GFDL)
- ✧ Contributors to the AMR FV model are R. Oehmke (NCAR), Q. Stout, J. Penner, B. van Leer, K. Powell (UM)

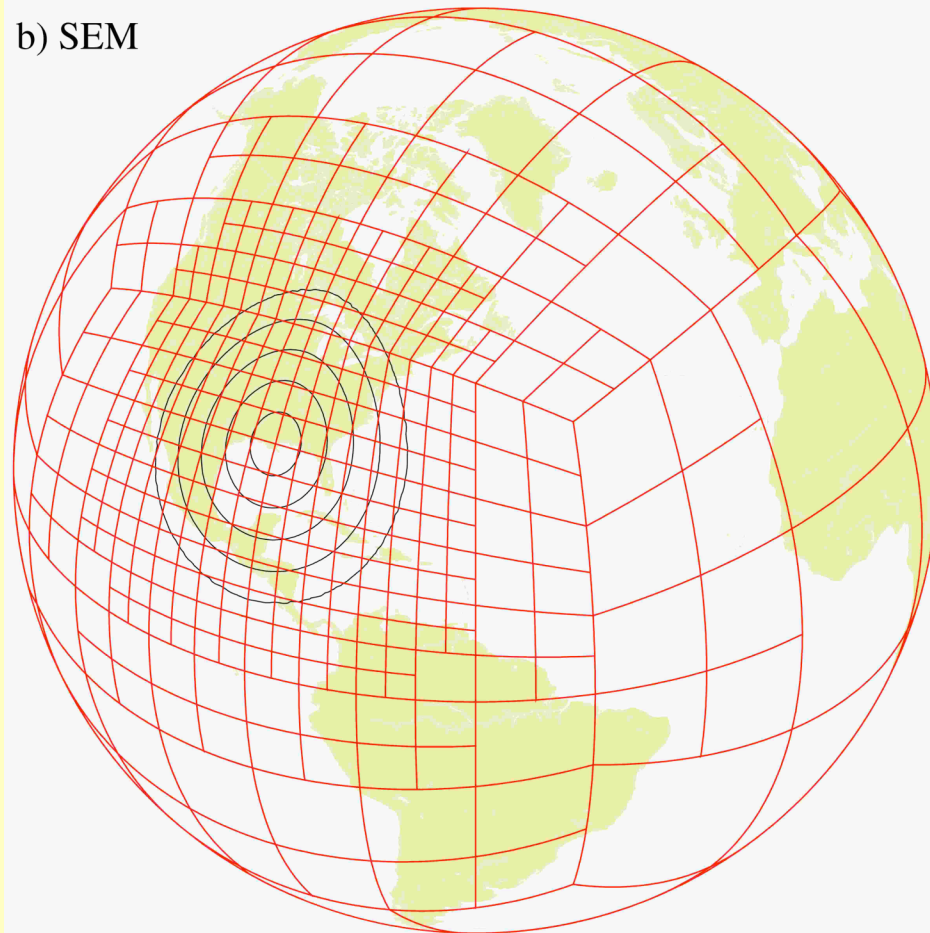


# ***Adaptive Mesh Refinement (AMR): Latitude-Longitude Grid versus Cubed Sphere***

a) FV



b) SEM



Latitude-Longitude grid:  
Model FV

Cubed-sphere grid:  
Model SEM



# ***Shallow Water Equations***

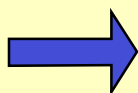
Momentum equation in vector-invariant form

$$\frac{\partial \vec{v}_h}{\partial t} + (\zeta + f) \vec{k} \times \vec{v}_h + \vec{\nabla} (K - \nu D + g(h + h_s)) = 0$$

Continuity equation

$$\frac{\partial h}{\partial t} + \vec{\nabla} \cdot (h \vec{v}) = 0$$

only in FV



$v_h$  horizontal velocity vector

$\zeta$  relative vorticity

$f$  Coriolis parameter

$K = 0.5 \cdot (u^2 + v^2)$  kinetic energy

$D$  horizontal divergence,  $\nu$  damping coefficient

$h$  depth of the fluid,  $h_s$  height of the orography

$g$  gravitational acceleration

## ***Finite Volume (FV) Shallow Water Model***

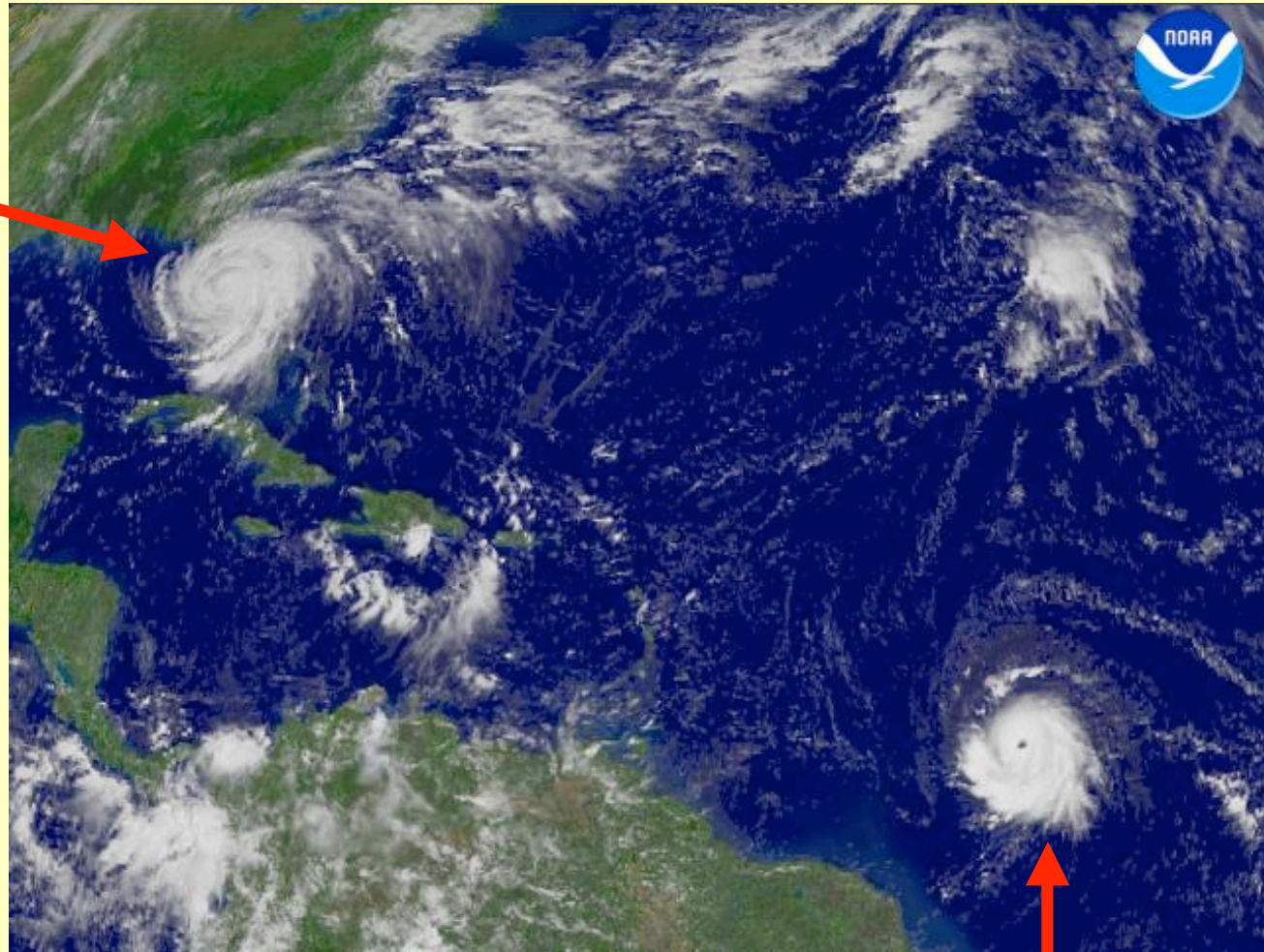
- Developed by Lin and Rood (1996), Lin and Rood (1997)
  - 3D version available (Lin 2004), built upon the SW model:
    - hydrostatic dynamical core used for climate and weather predictions
  - ➔ ● Currently part of NCAR's, NASA's and GFDL's General Circulation Models
- **Numerics:** Finite volume approach
  - conservative and monotonic transport scheme
  - van Leer second order scheme for time-averaged numerical fluxes
  - PPM third order scheme (Piecewise Parabolic Method) for prognostic variables
  - Staggered grid (Arakawa D-grid), C-grid for mid-time levels
  - Orthogonal Latitude-Longitude computational grid

# ***Spectral Element (SEM) Shallow Water Model***

- Documented in Thomas and Loft (2002), St-Cyr and Thomas (2005), St-Cyr et al. (2007)
  - 3D version available (no 3D AMR version though)
  - 3D hydrostatic dynamical core is part of NCAR's CAM model (model HOMME)
- **Numerics:** Spectral Elements
  - Non-conservative (no longer as we learned from Mark and Aimé) and non-monotonic (still true)
  - Allows high-order numerical method
  - Spectral convergence for smooth flows
  - GLL and GL collocation points
  - Non-orthogonal cubed-sphere computational grid

# ***Features of Interest in a Multi-Scale Regime***

Hurricane  
Frances

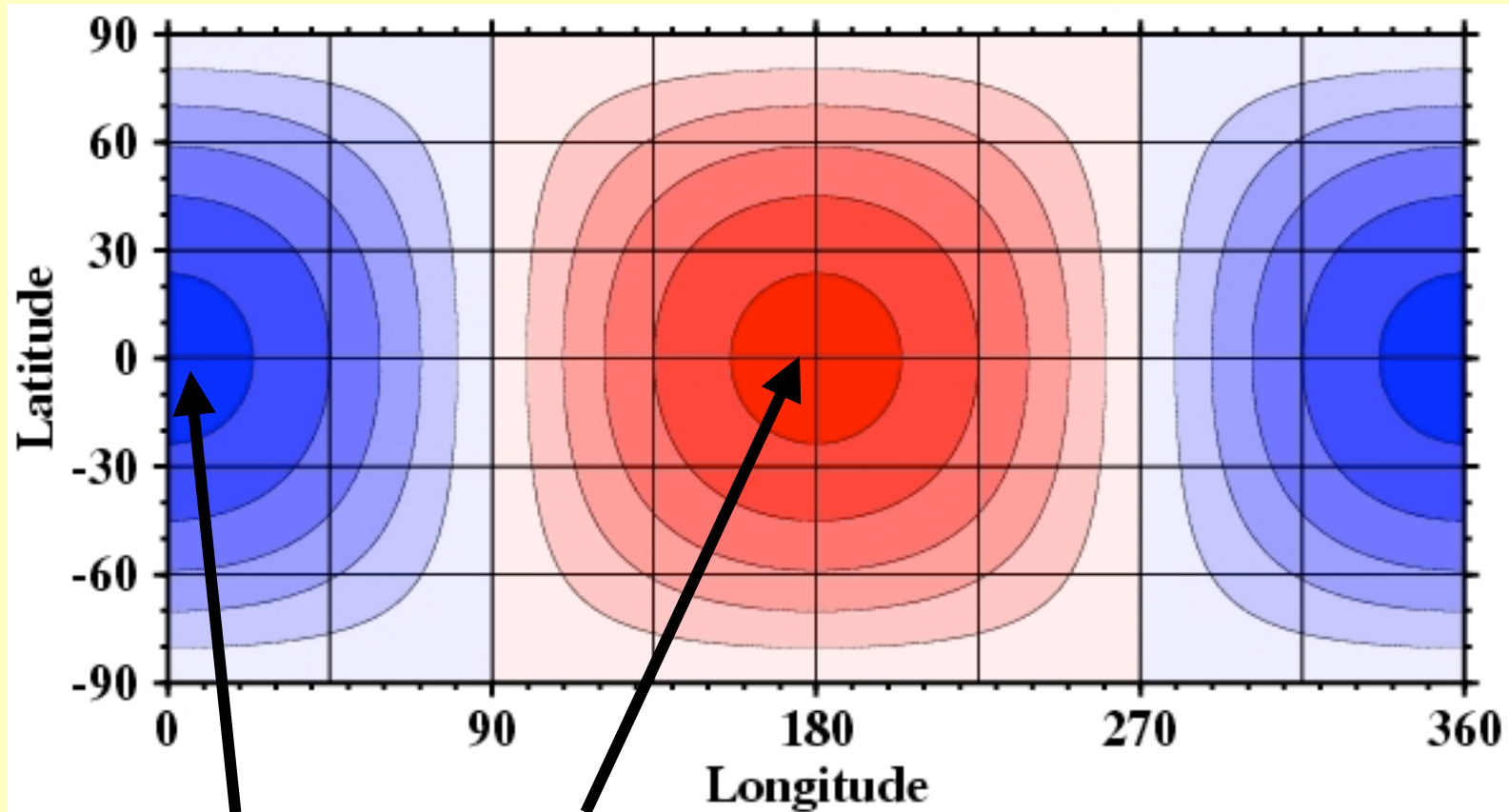


September/5/2004

Hurricane Ivan

# ***Idealized assessment of cyclones in SW models***

Combines solid body rotation and idealized cyclogenesis test case



'Hurricane  
Frances'

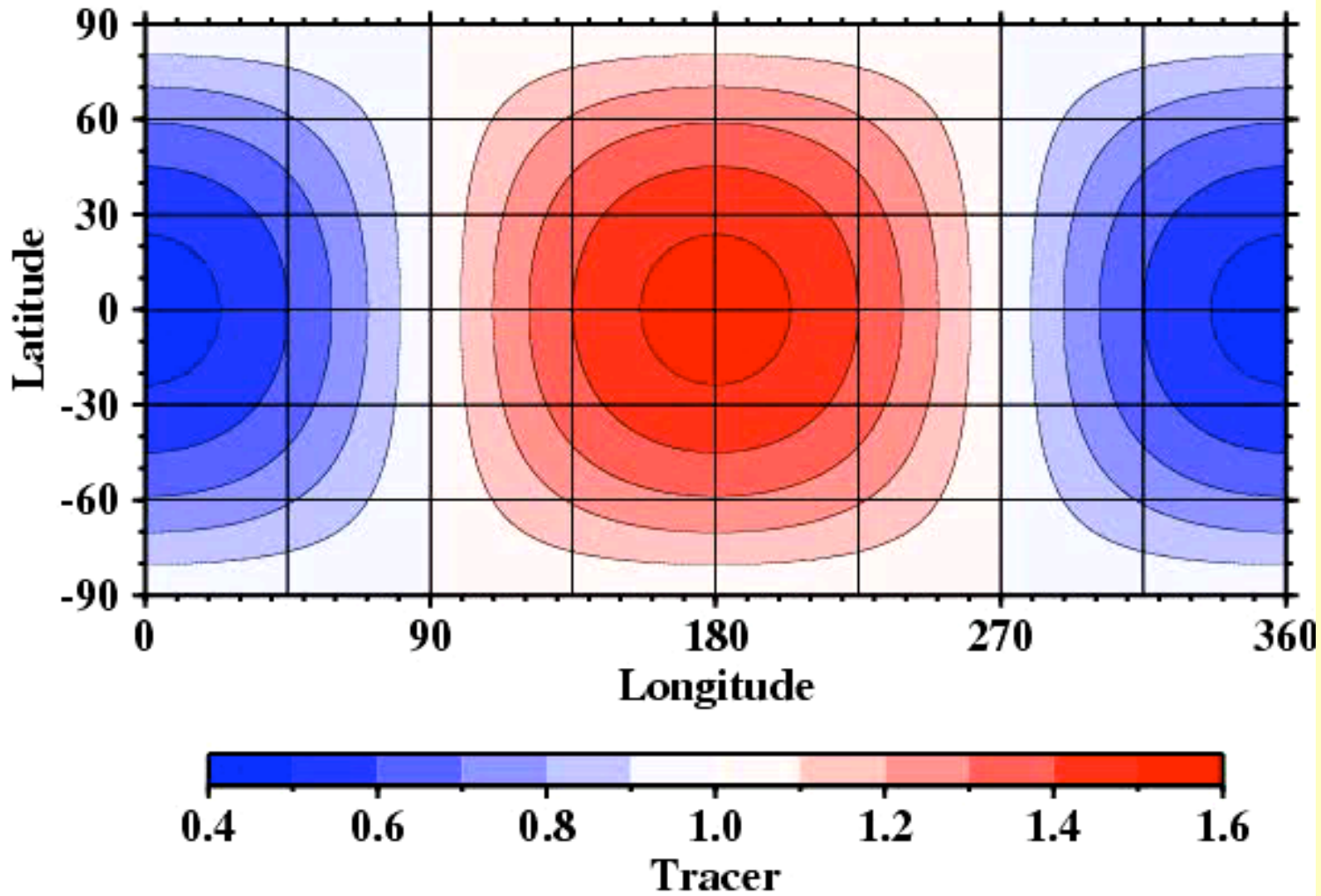
'Hurricane  
Ivan'

Nair and Jablonowski,  
MWR, 2008



# ***Idealized assessment of cyclones in SW models***

**Moving vortex**



FV SW  
model,  
5°x5° initial  
resolution

2 dynamic  
refinement  
levels  
1.25°x1.25°

dynamic ref.  
criterion:  
gradient

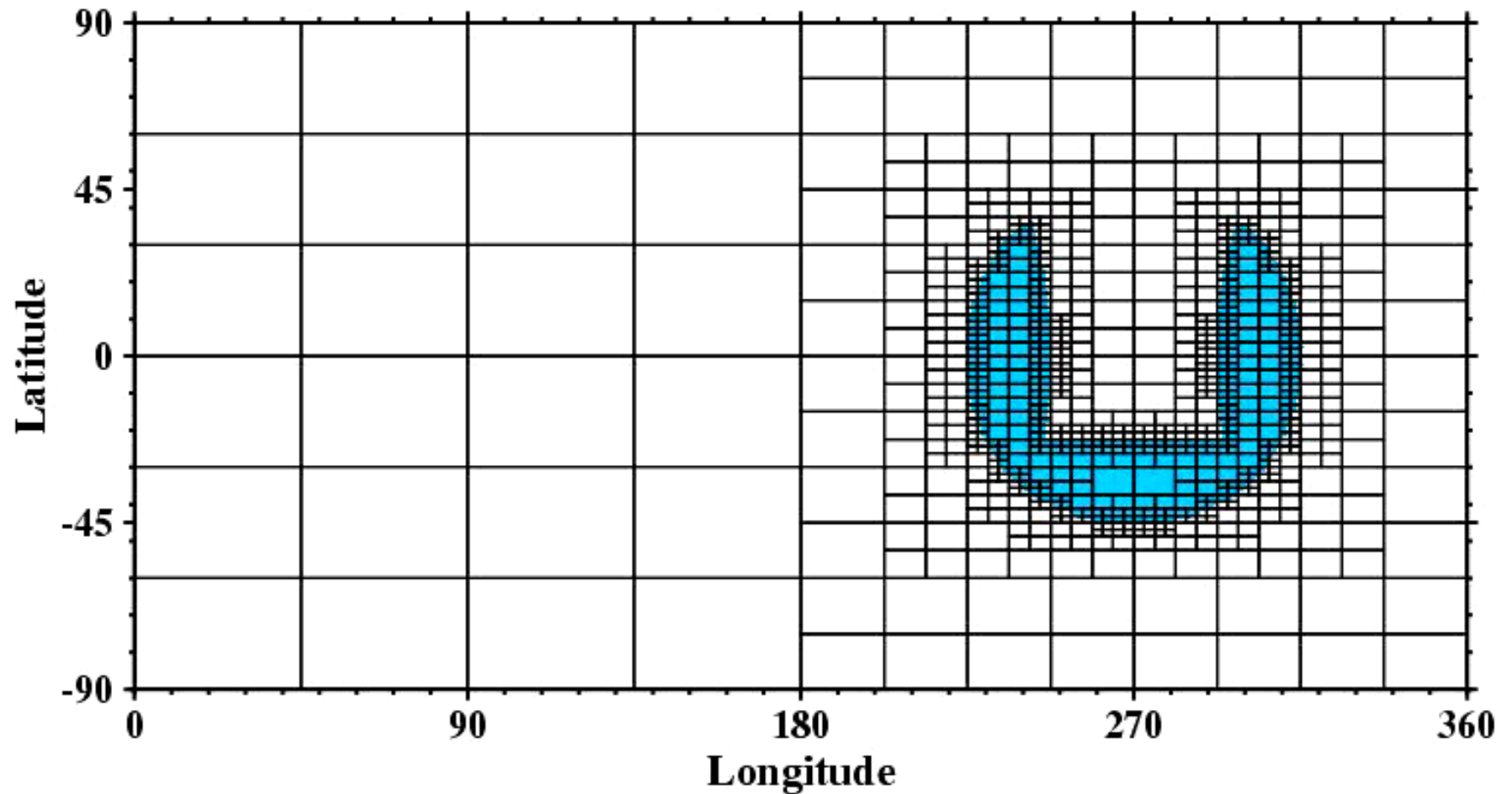
12-day  
simulation

# ***Overview of the AMR comparison***

- ✧ 2D shallow water tests: Williamson et al. (1992)
  - + extensions of the shallow water test suite
    - ✧ Dynamic refinements for pure advection
      - ✧ Slotted cylinder
      - ✧ Cosine bell advection test
    - ✧ Dynamic refinements and refinement criteria:
      - Flow over a mountain
- ✧ Barotropic instability test (Galewsky et al., Tellus 2004)
- ✧ 3D Baroclinic waves (Jablonowski and Williamson, QJRMS (2006) & NCAR Technical Report (2006))

# ***AMR Transport of a Slotted Cylinder***

**Transport of a slotted cylinder,  $\alpha=30^\circ$**

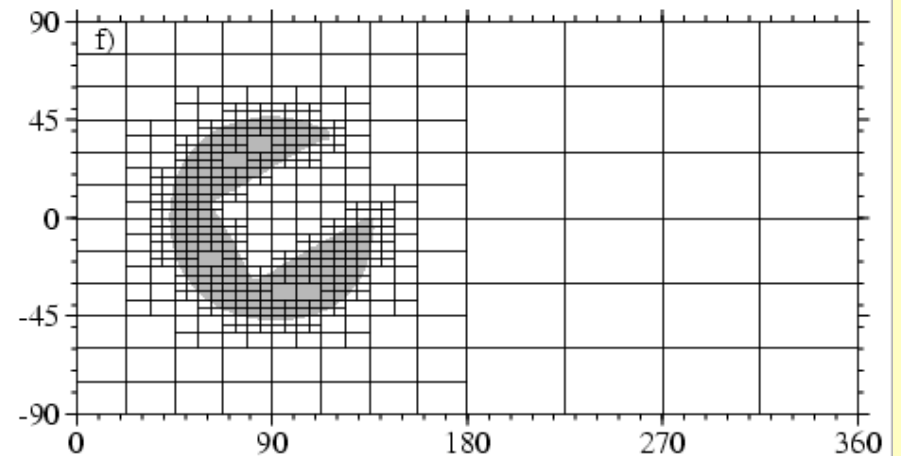
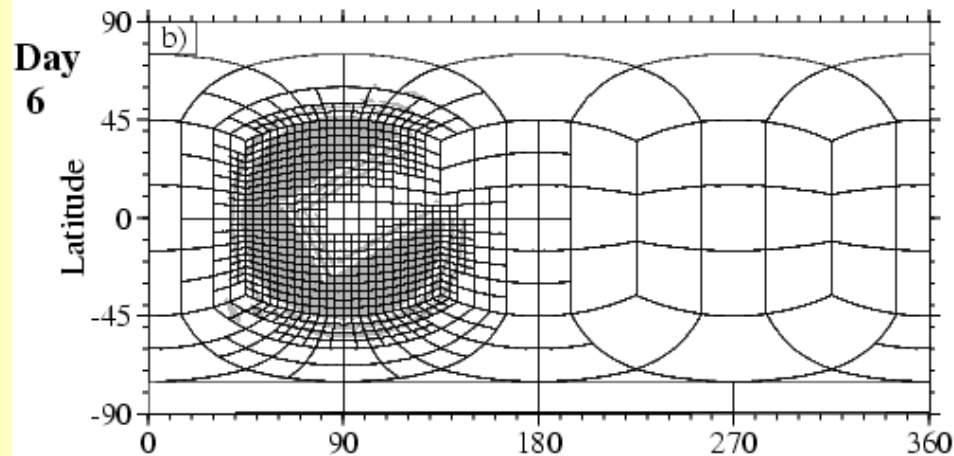
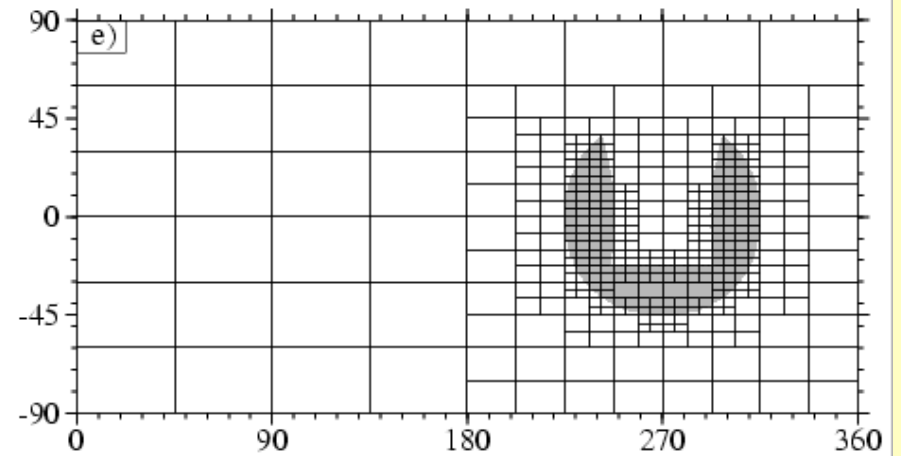
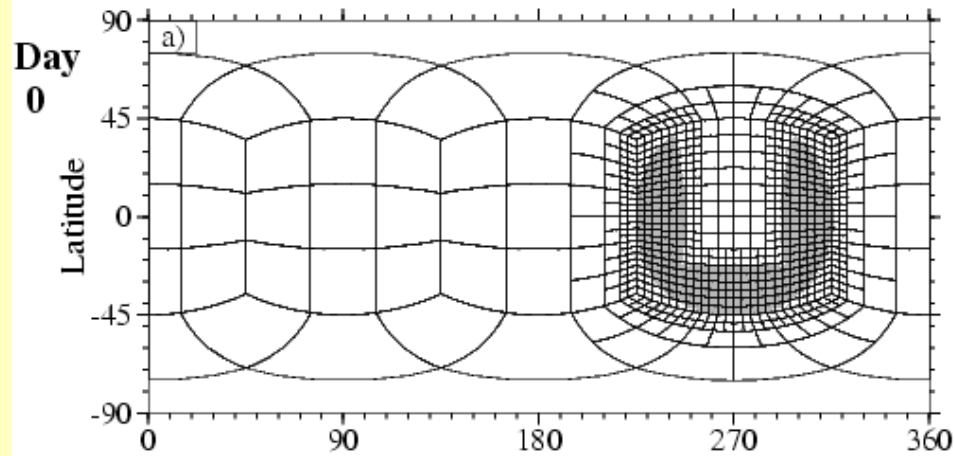


Model FV

# ***Transport of a Slotted Cylinder***

SEM

FV

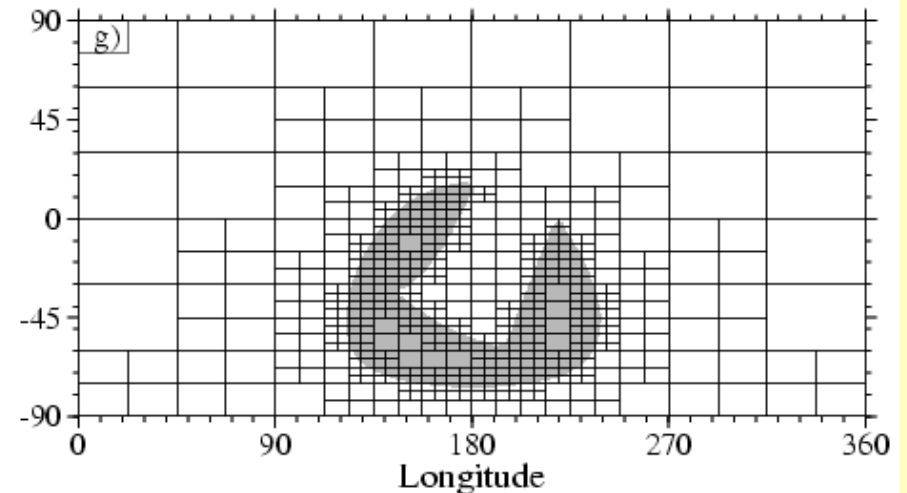
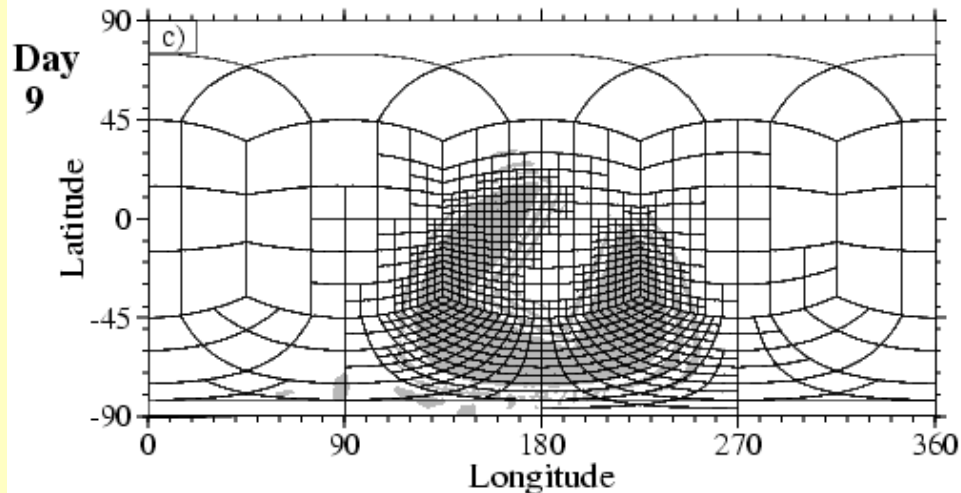


5 x 5 deg base grid, 3 refinement levels

# Transport of a Slotted Cylinder

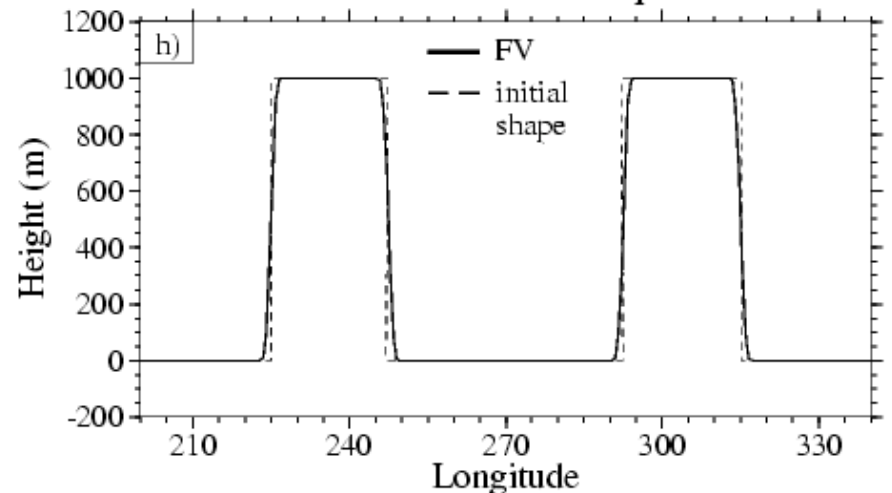
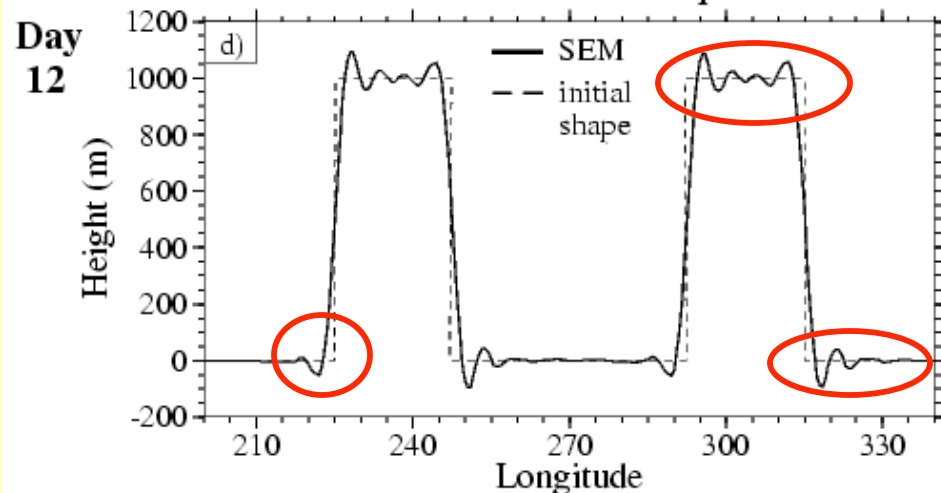
SEM

FV



Cross section at the equator

Cross section at the equator

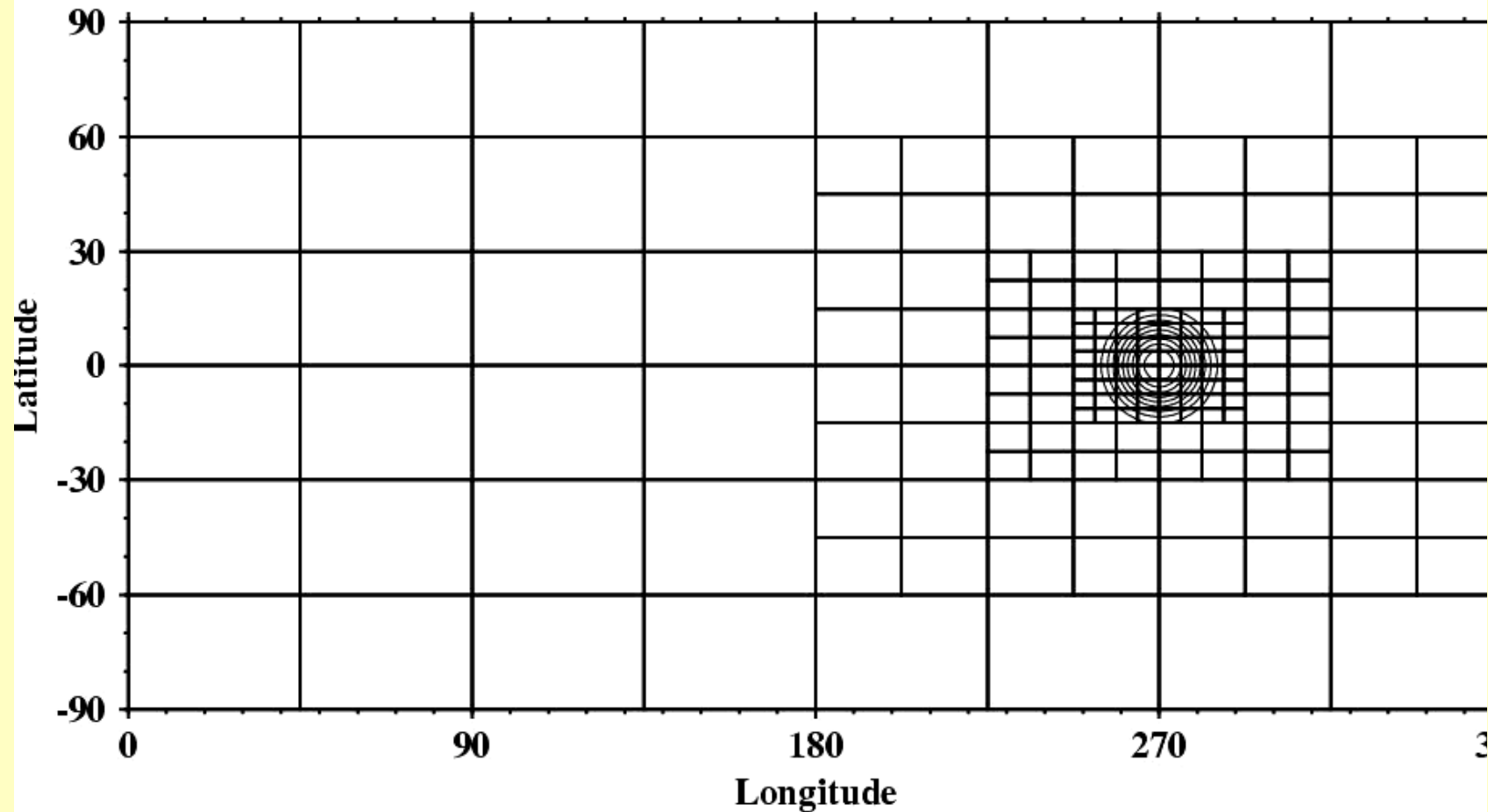


- Slotted cylinder is reliably detected and tracked
- Over- and undershoots in SEM, FV monotonic



# ***Advection of a Cosine Bell with $\alpha = 90^\circ$***

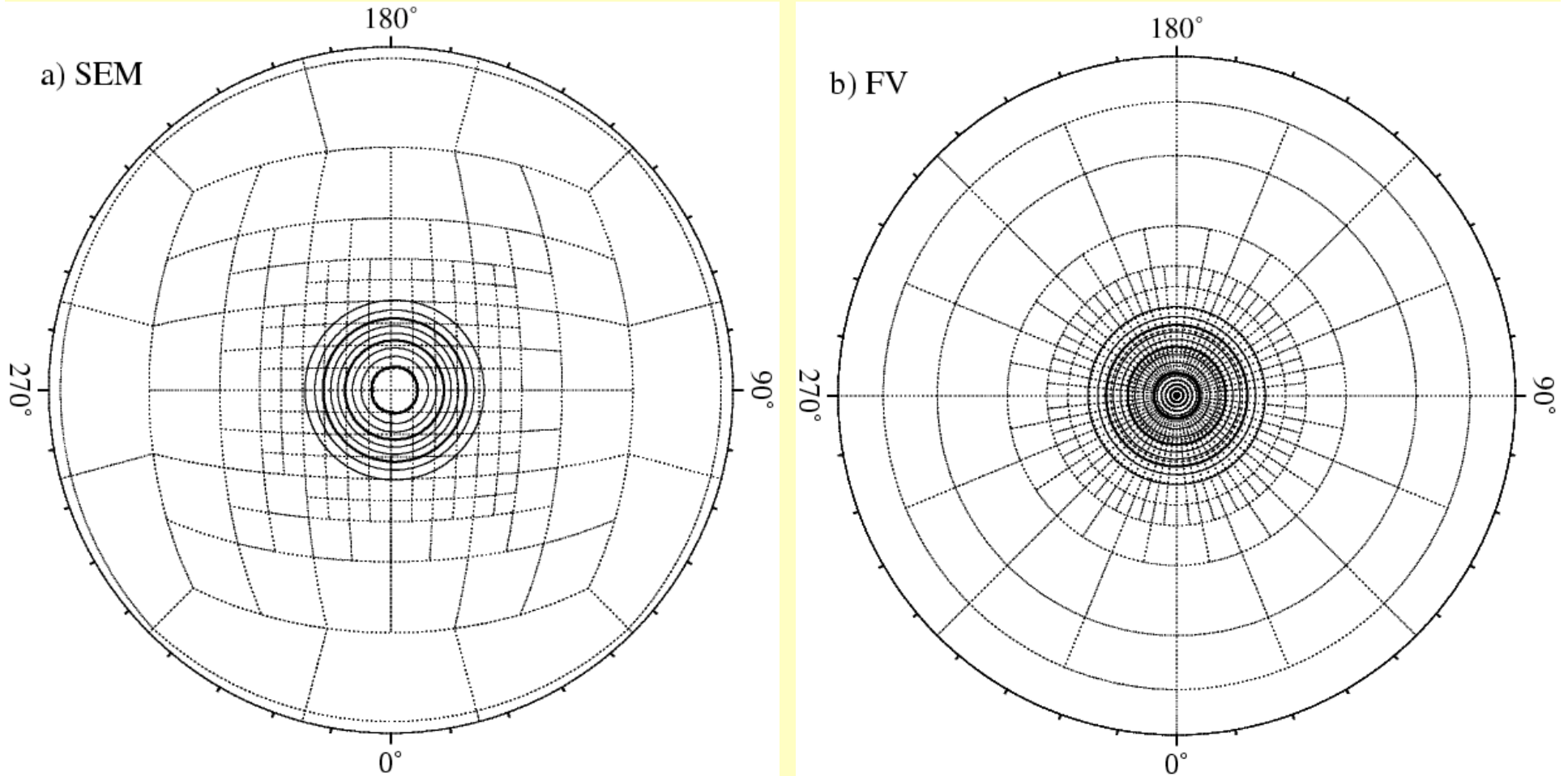
Advection of a cosine bell with 3 refinement levels,  $\alpha = 90^\circ$



Model FV

# ***Snapshots: Cosine Bell at day 3***

North-polar stereographic projection at day 3 for  $\alpha = 90$



Uniform distribution in SEM, Convergence of blocks in FV

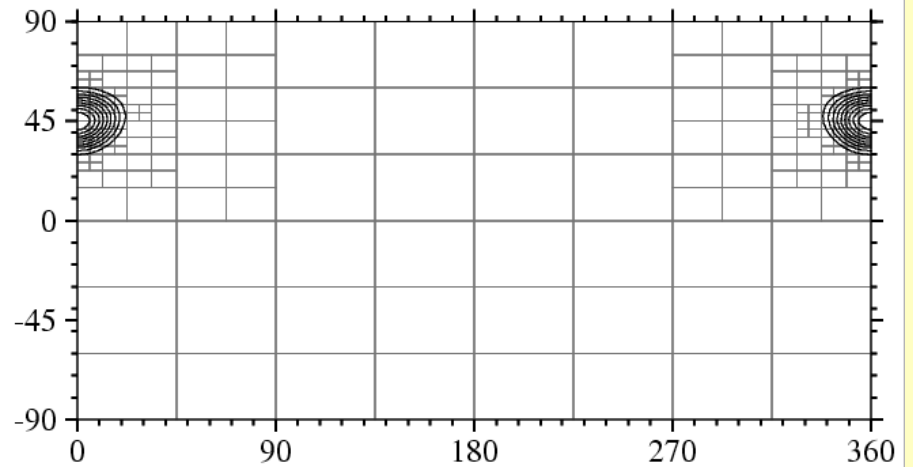
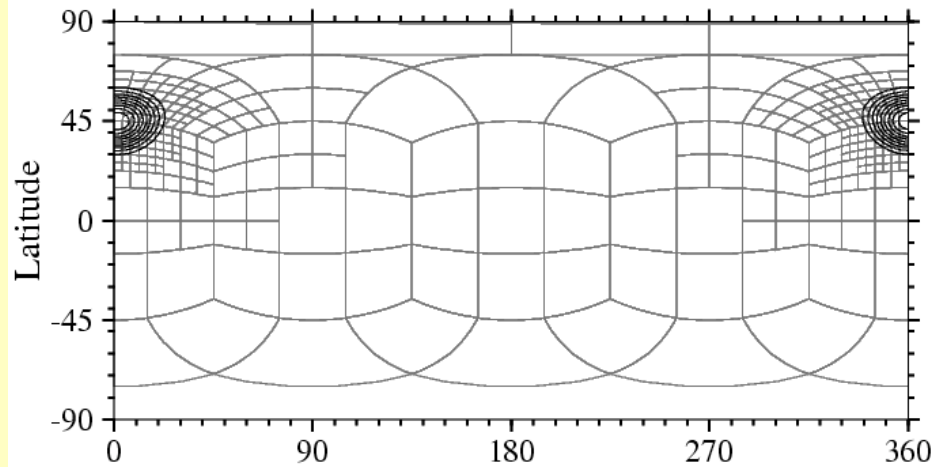
# Snapshots: Advection of a Cosine Bell

SEM

FV

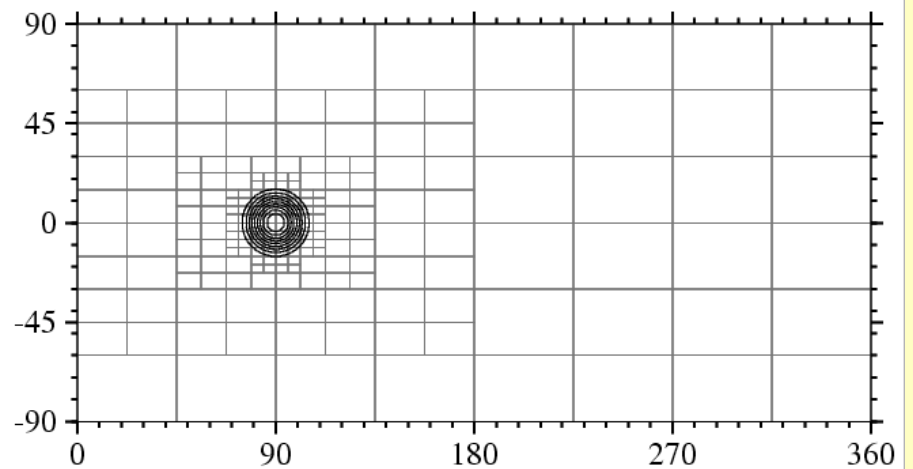
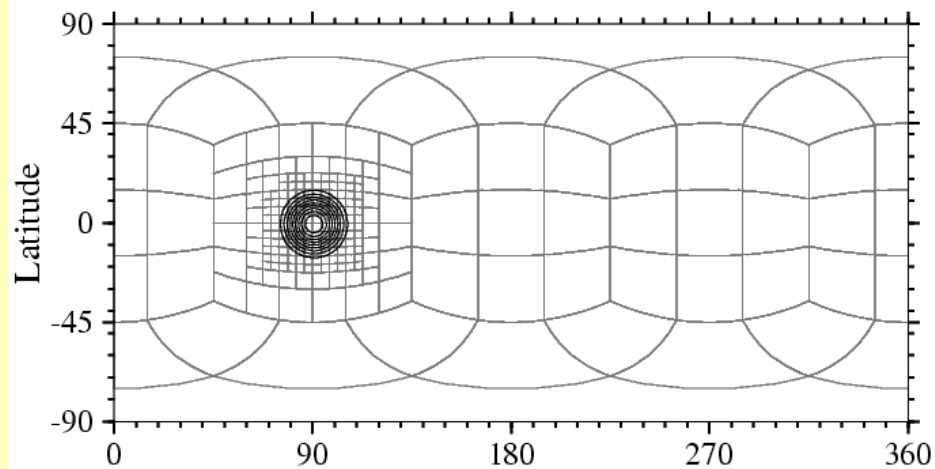
a) SEM Day 3

e) FV Day 3



b) SEM Day 6

f) FV Day 6



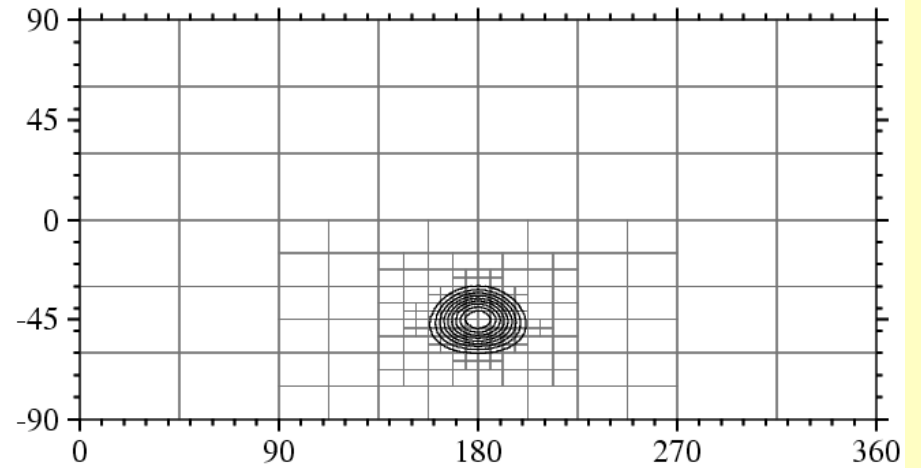
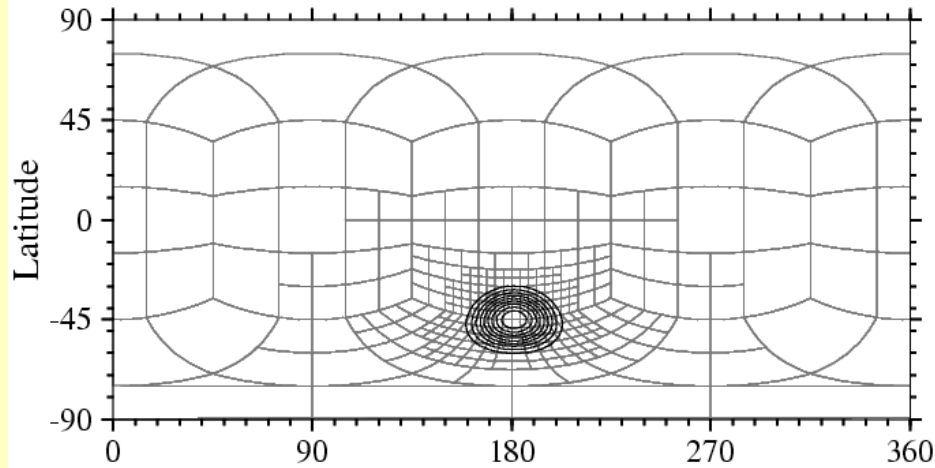
# Snapshots: Advection of a Cosine Bell

SEM

FV

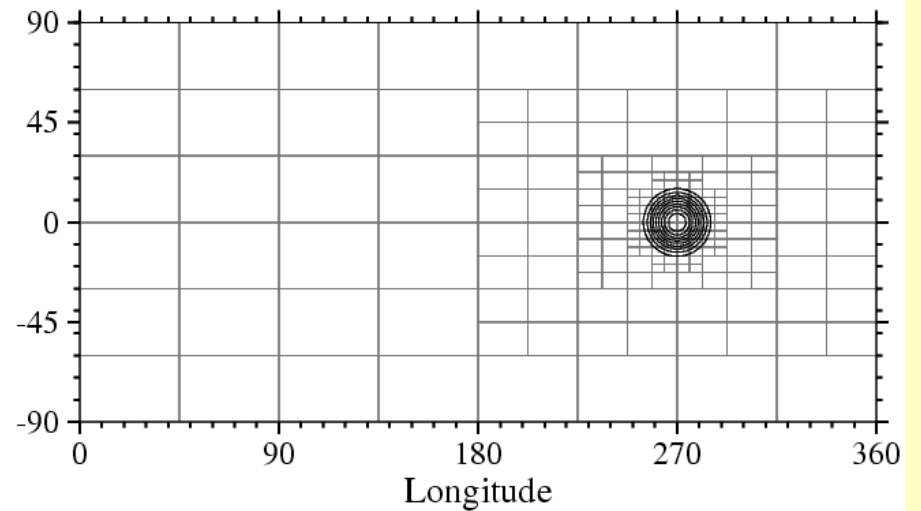
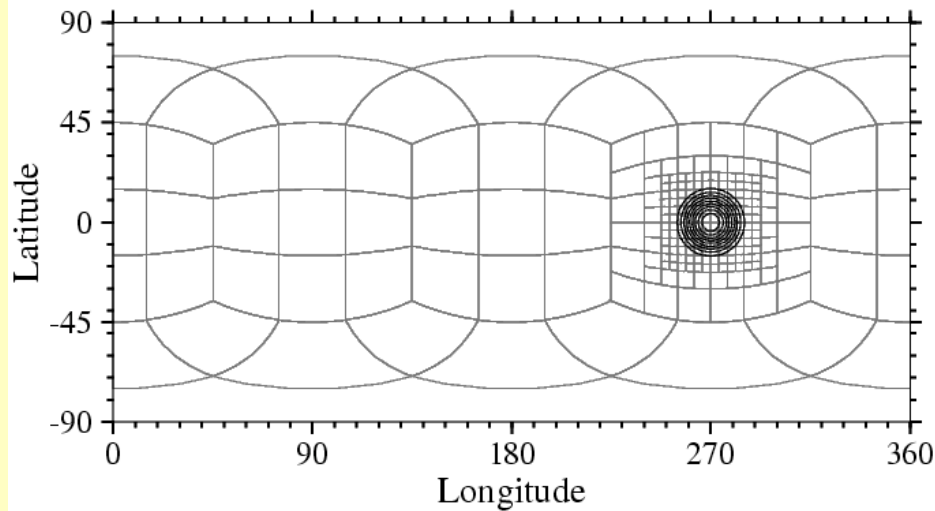
c) SEM Day 9

g) FV Day 9



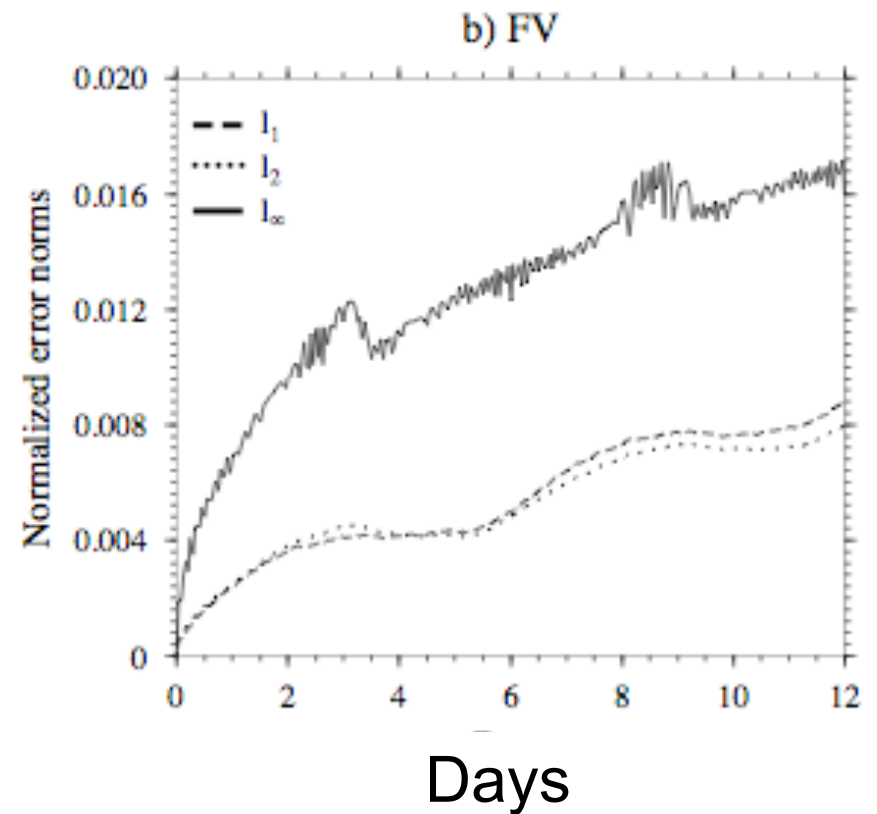
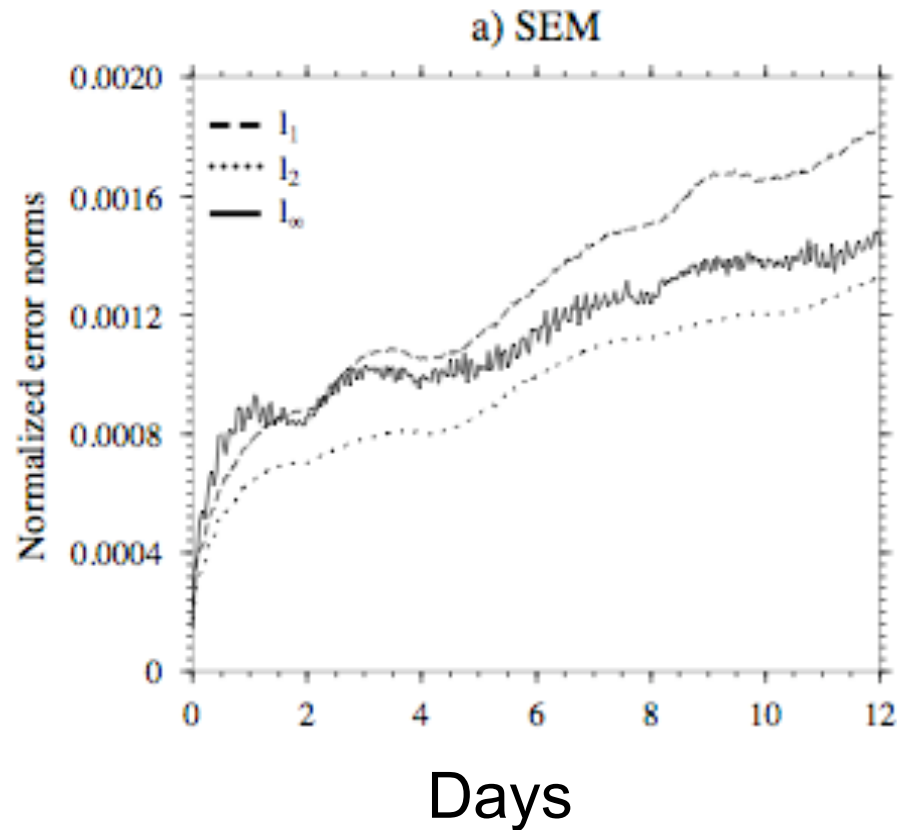
d) SEM Day 12

h) FV Day 12





# ***Error norms: Cosine Bell Advection***



Rotation angle  $\alpha = 45$ :

Errors in SEM are lower than errors in FV

## ***Error norms after 12 days***

Resolution	$l_1$	$l_2$	$l_\infty$	$h(m)$ max/min
<b>SEM</b>				
$2.5^\circ \times 2.5^\circ$	0.0503	0.0269	0.0195	991.6/-15.1
$1.25^\circ \times 1.25^\circ$	0.0085	0.0056	0.0057	997.5/-4.2
$0.625^\circ \times 0.625^\circ$	0.0019	0.0014	0.0019	999.1/-1.1
$0.3125^\circ \times 0.3125^\circ$	0.0008	0.0006	0.0015	999.7/-0.9
<b>FV</b>				
$2.5^\circ \times 2.5^\circ$	0.0341	0.0301	0.0317	949.1/0
$1.25^\circ \times 1.25^\circ$	0.0097	0.0103	0.0150	984.2/0
$0.625^\circ \times 0.625^\circ$	0.0016	0.0021	0.0044	995.0/0
$0.3125^\circ \times 0.3125^\circ$	0.0003	0.0005	0.0014	998.4/0

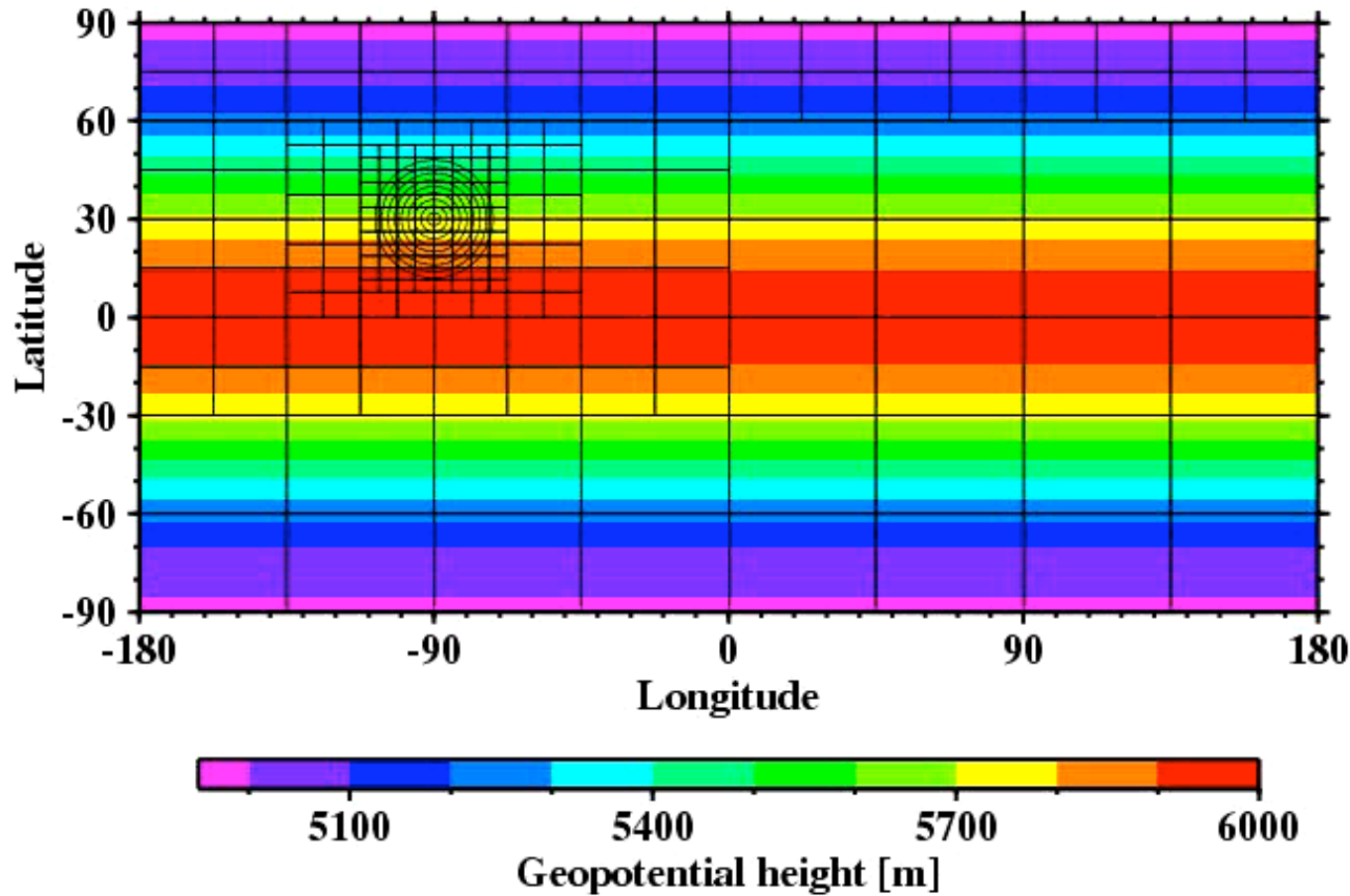
Rotation  
angle  $\alpha = 0$

SEM produces  
undershoots

Errors for  
 $\alpha = 0$  are  
comparable

# 2D Dynamic adaptations in FV

2D Flow over a mountain with 3 refinement levels



Vorticity-  
based  
adaptation  
criterion

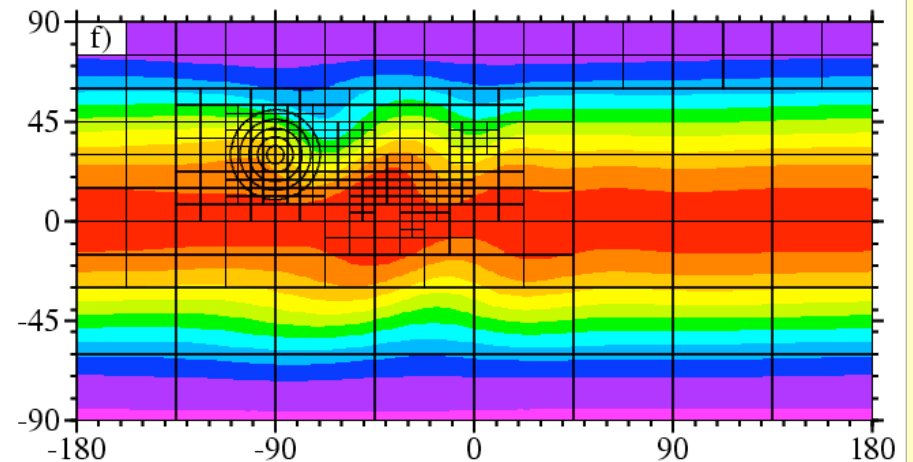
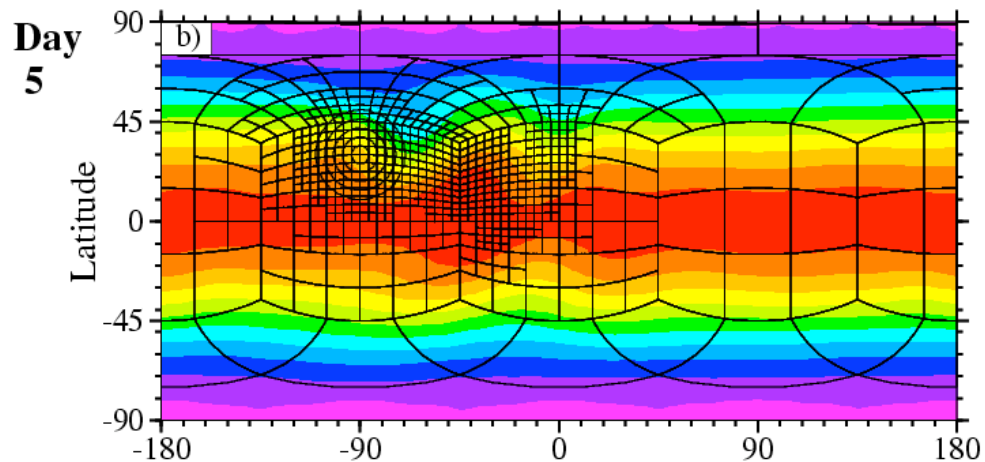
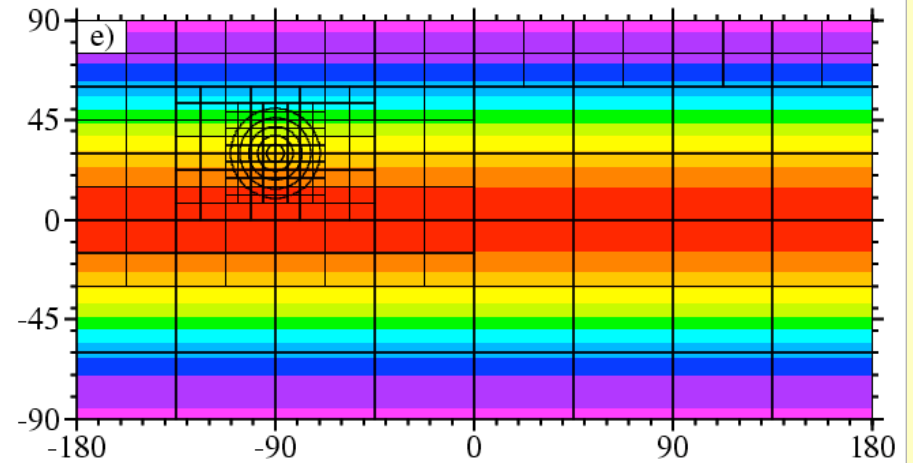
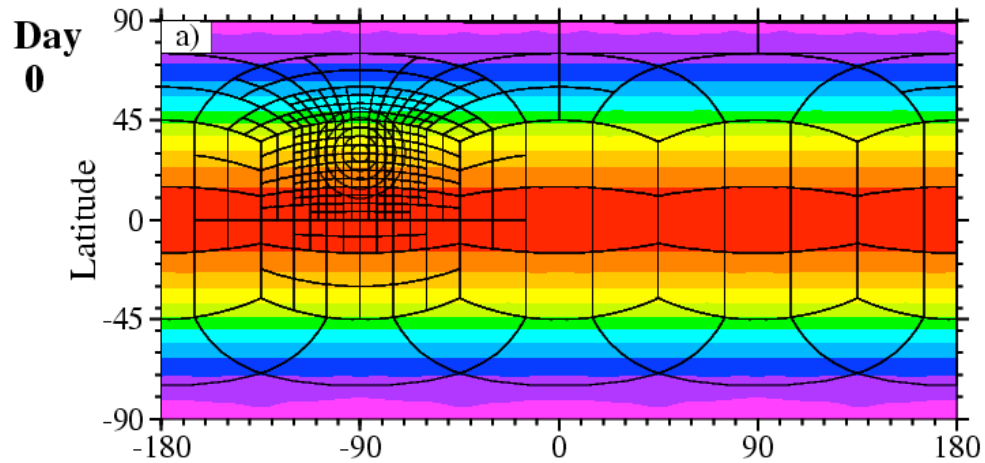
2D shallow  
water test #5:  
15-day run

# *Snapshots: Flow over a mountain*

Geopotential height field (SW test case 5)

SEM

FV



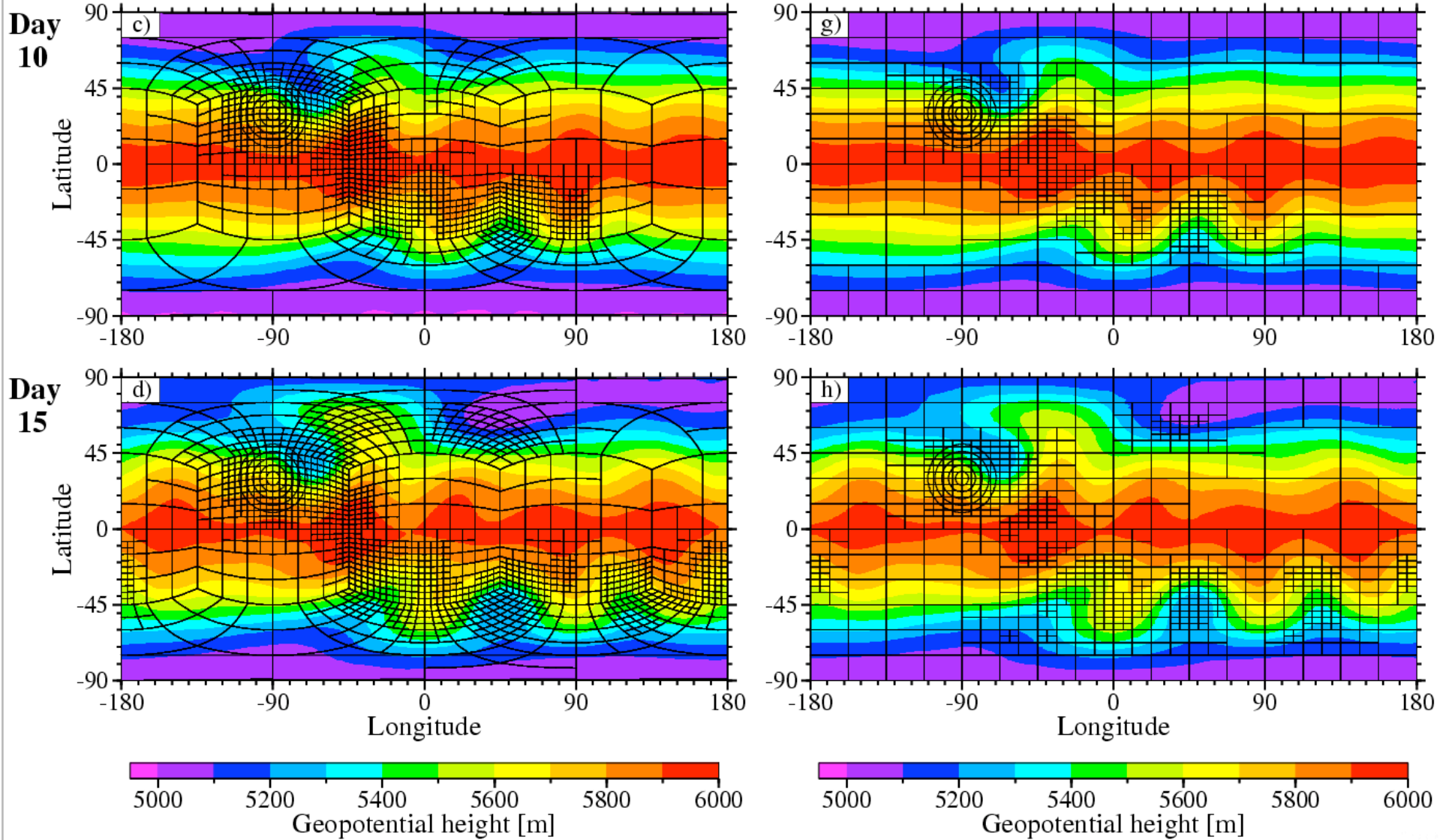
Longitude

Longitude

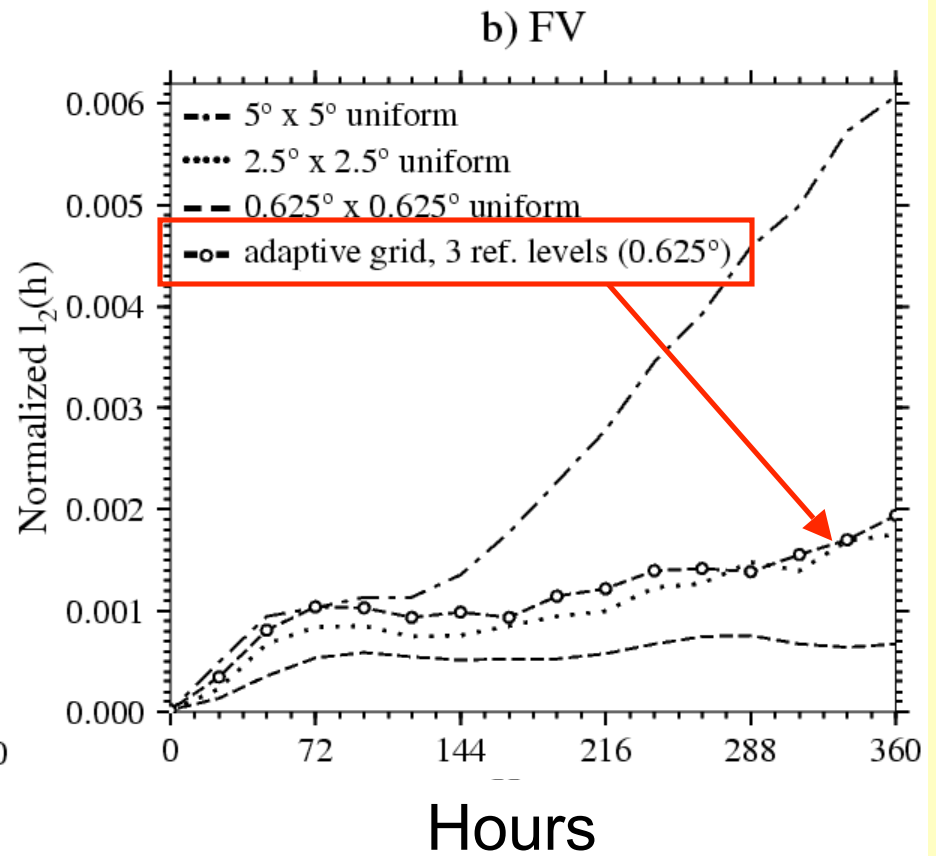
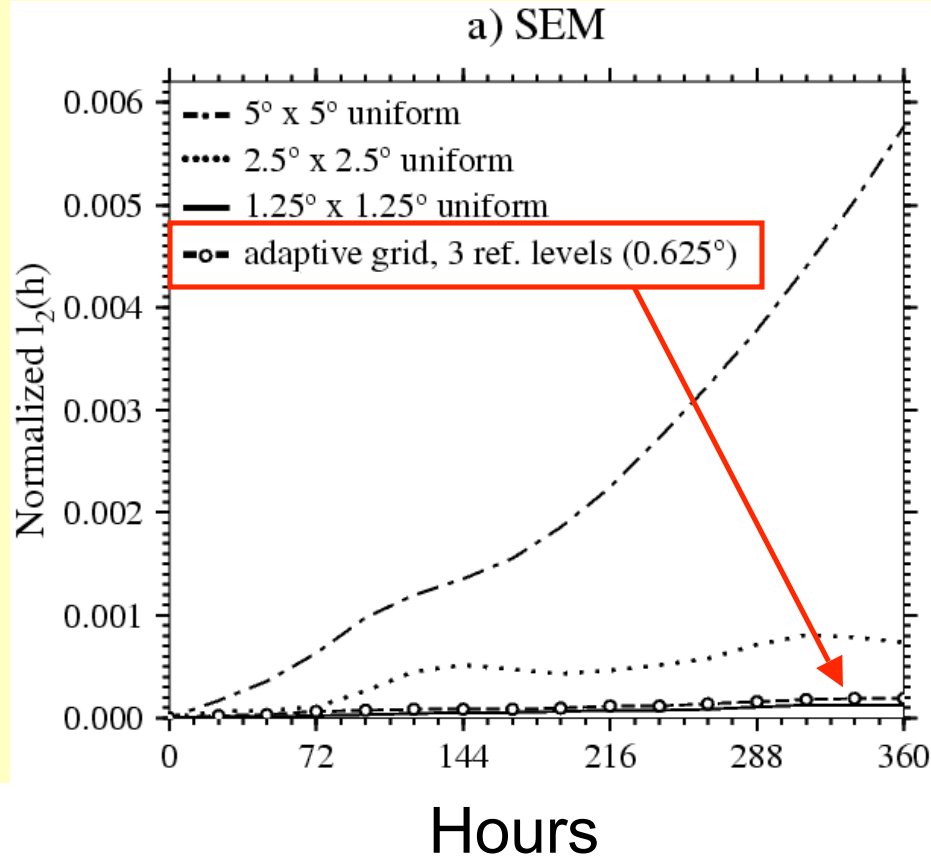


# *Snapshots: Flow over a mountain*

Geopotential height field (SW test case 5)



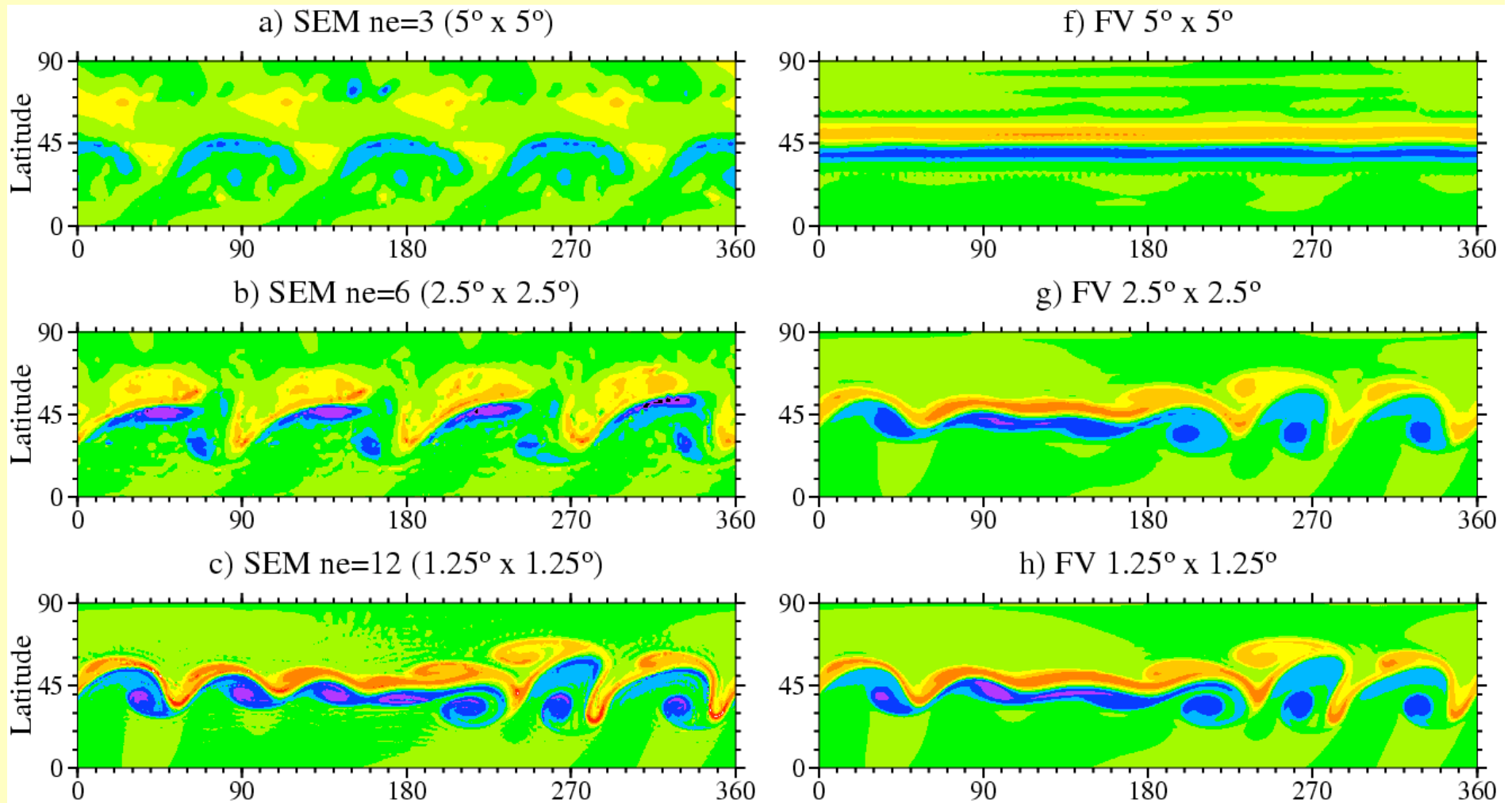
# Error norms: Test case 5



Errors in SEM converge quicker to the reference solution (T426 NCAR spectral model, provided by DWD)

# ***Barotropic instability test case***

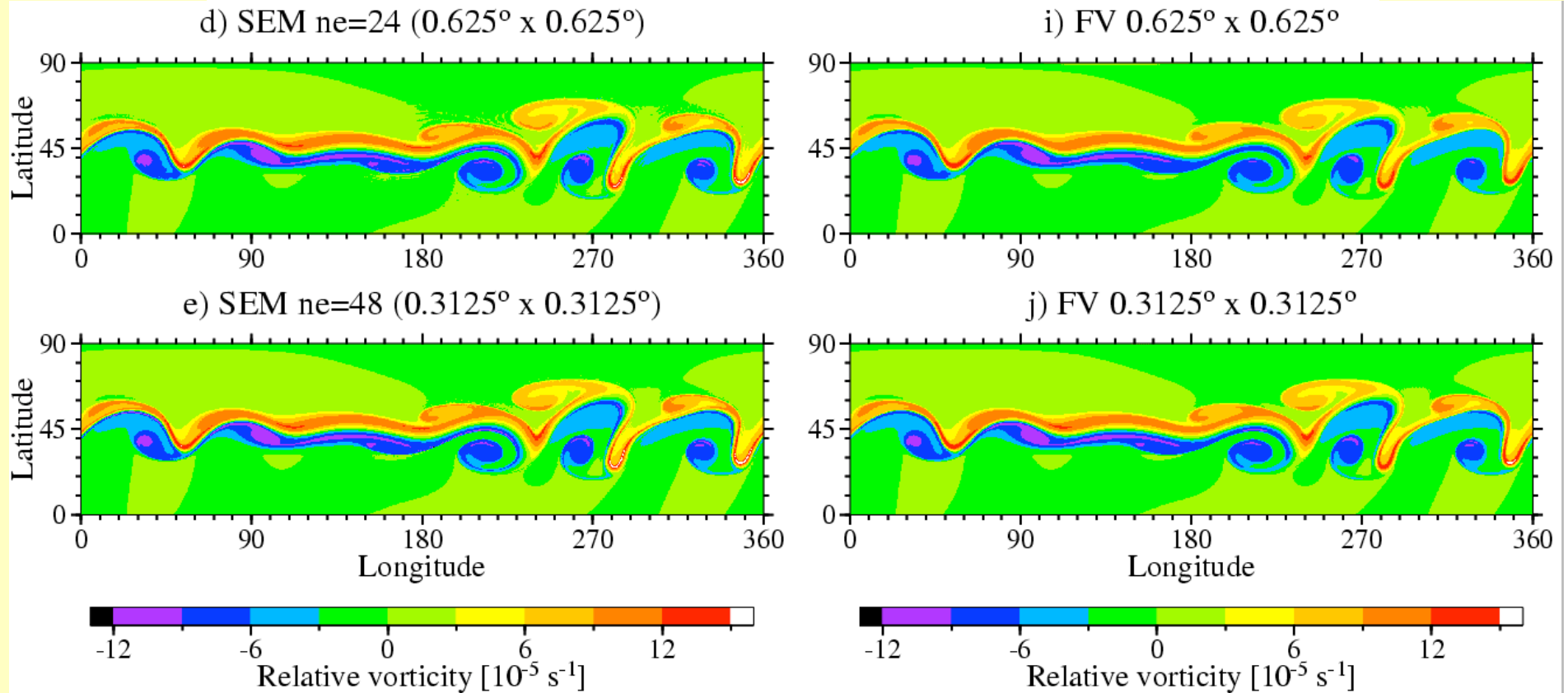
Convergence analysis: Relative vorticity at day 6



1.25° resolution is needed to get a good representation of the wave

# ***Barotropic instability test case: 'Reference Solution'***

Convergence analysis: Relative vorticity at day 6

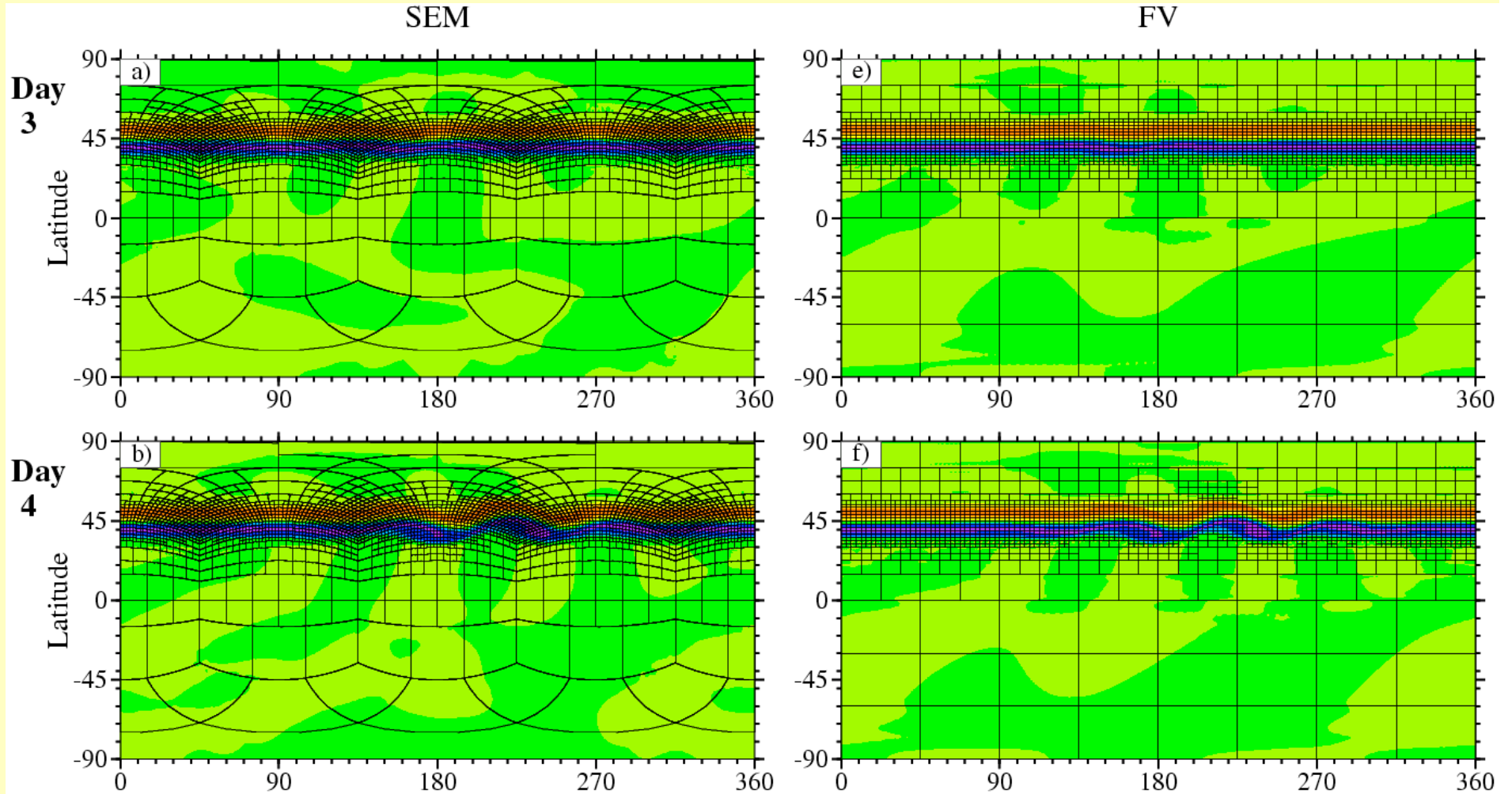


Second highest and highest resolutions are very similar to each other, SEM and FV are similar



# ***Barotropic instability test case***

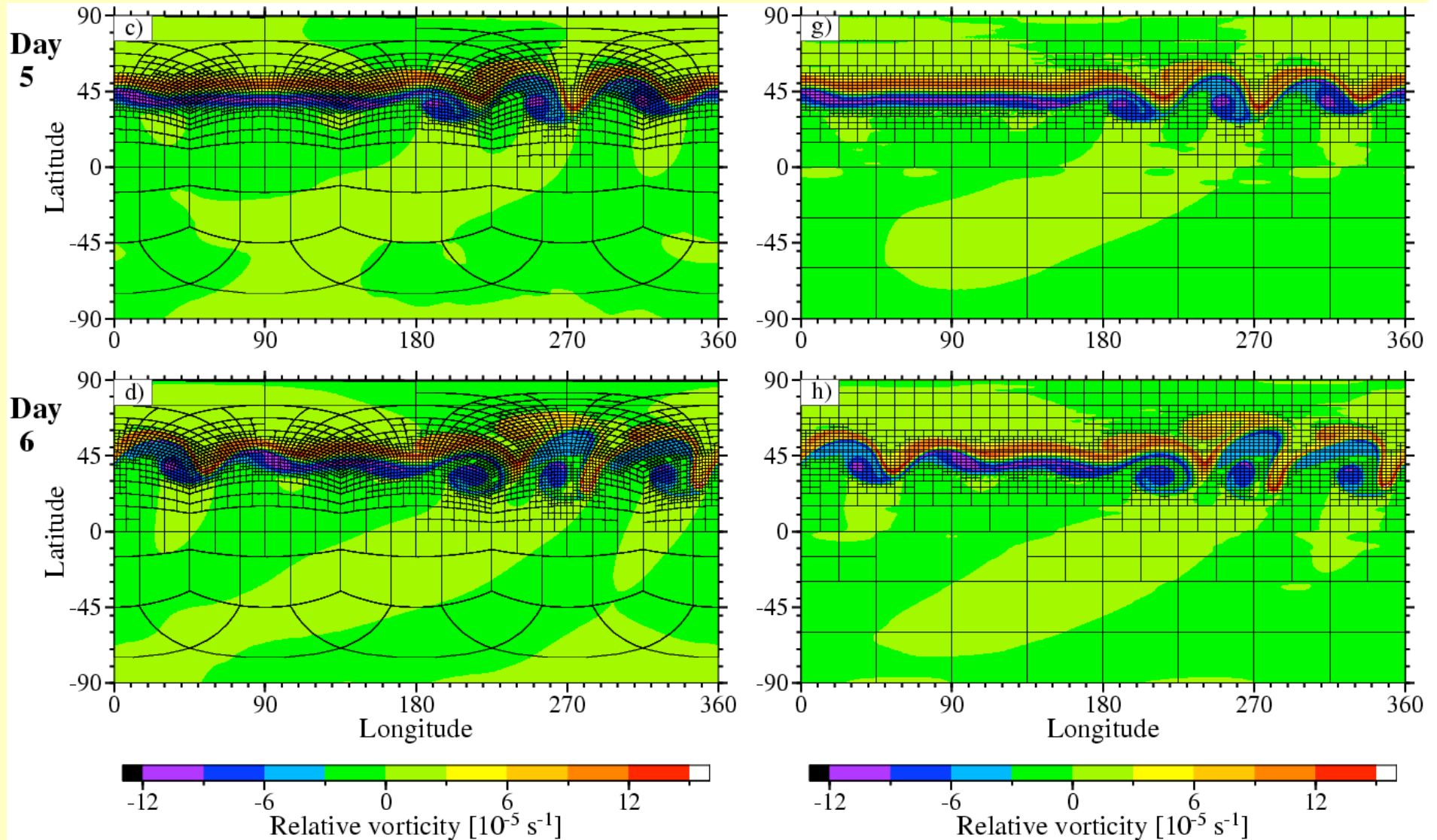
Vorticity-based adaptation criterion: Day 3 and 4



5 deg base grid, 4 refinement levels

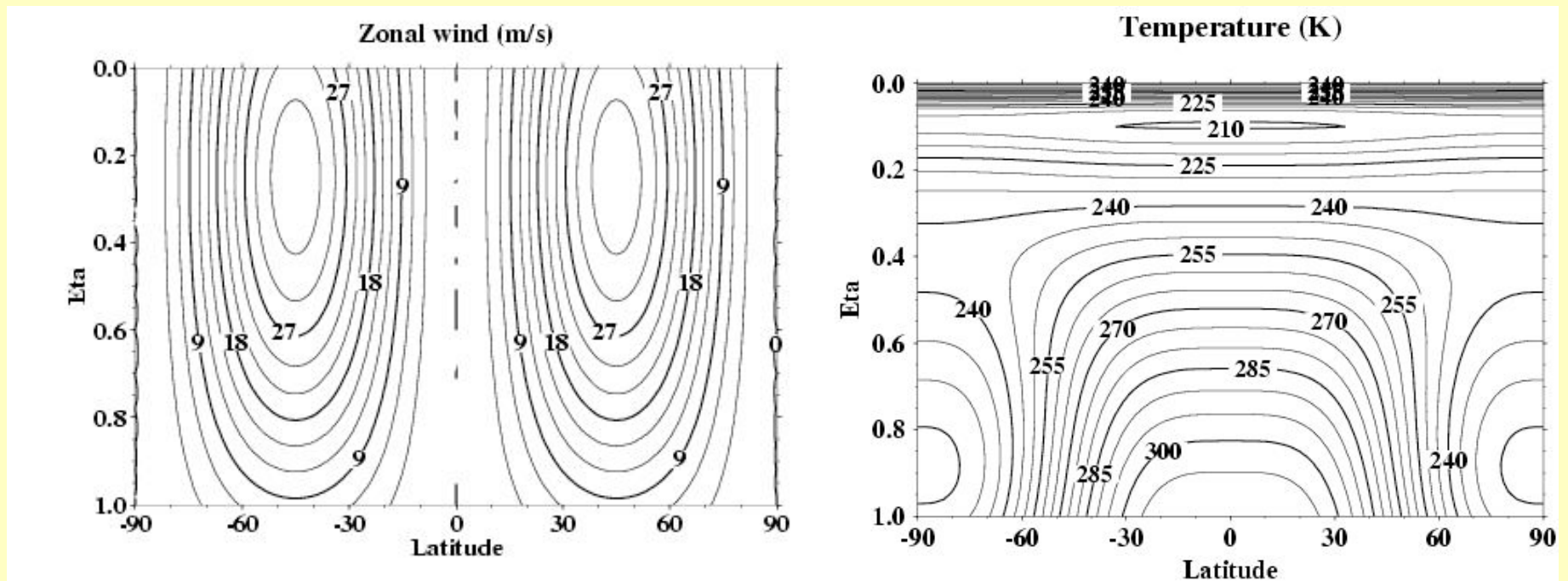
# *Barotropic instability test case*

AMR Grids and solutions in SEM and FV are very similar



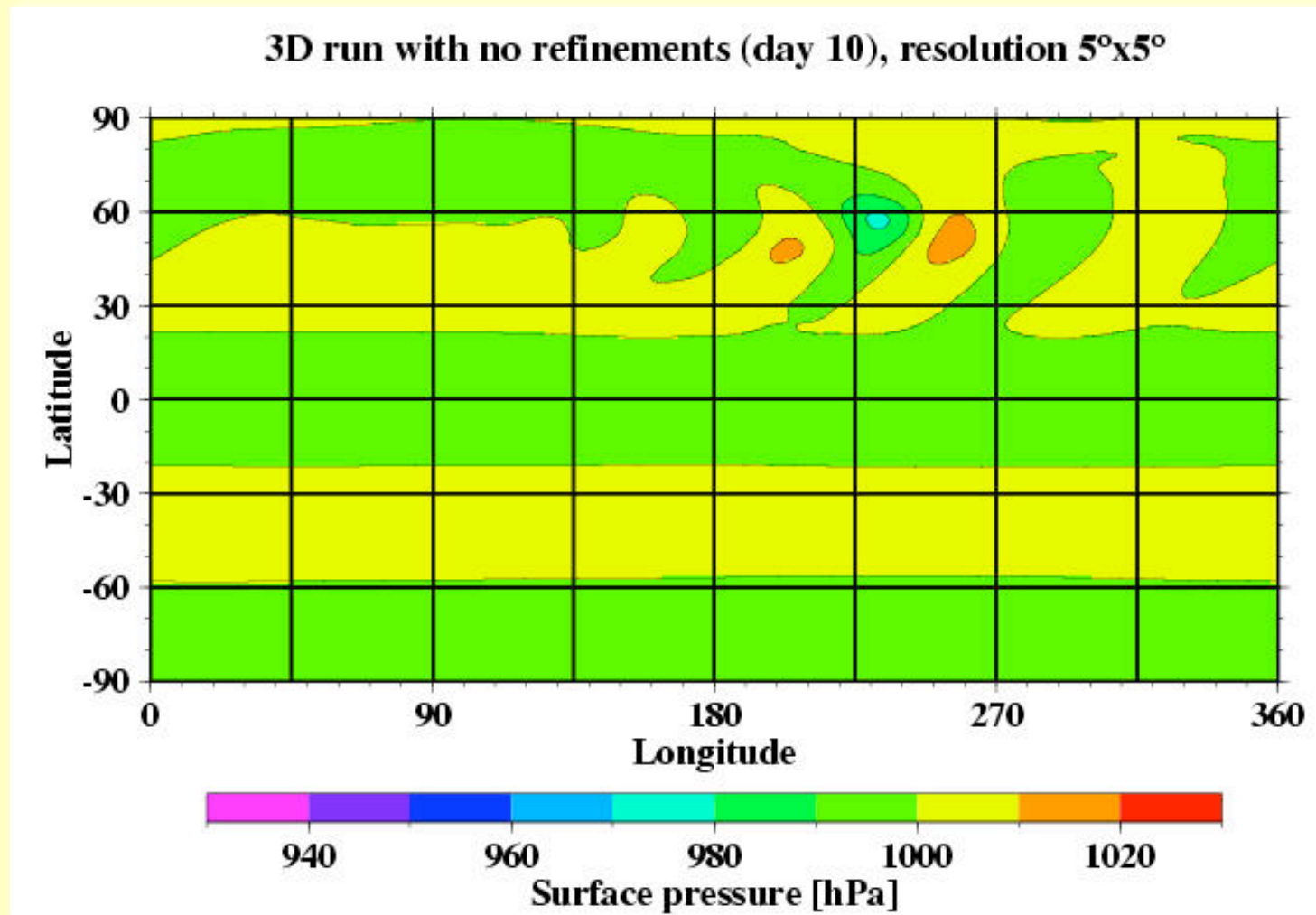
# ***3D Baroclinic wave test case***

- analytically specified balanced initial field with overlaid perturbation
- baroclinic wave develops after 5-10 days
- deterministic test that converges towards reference solution



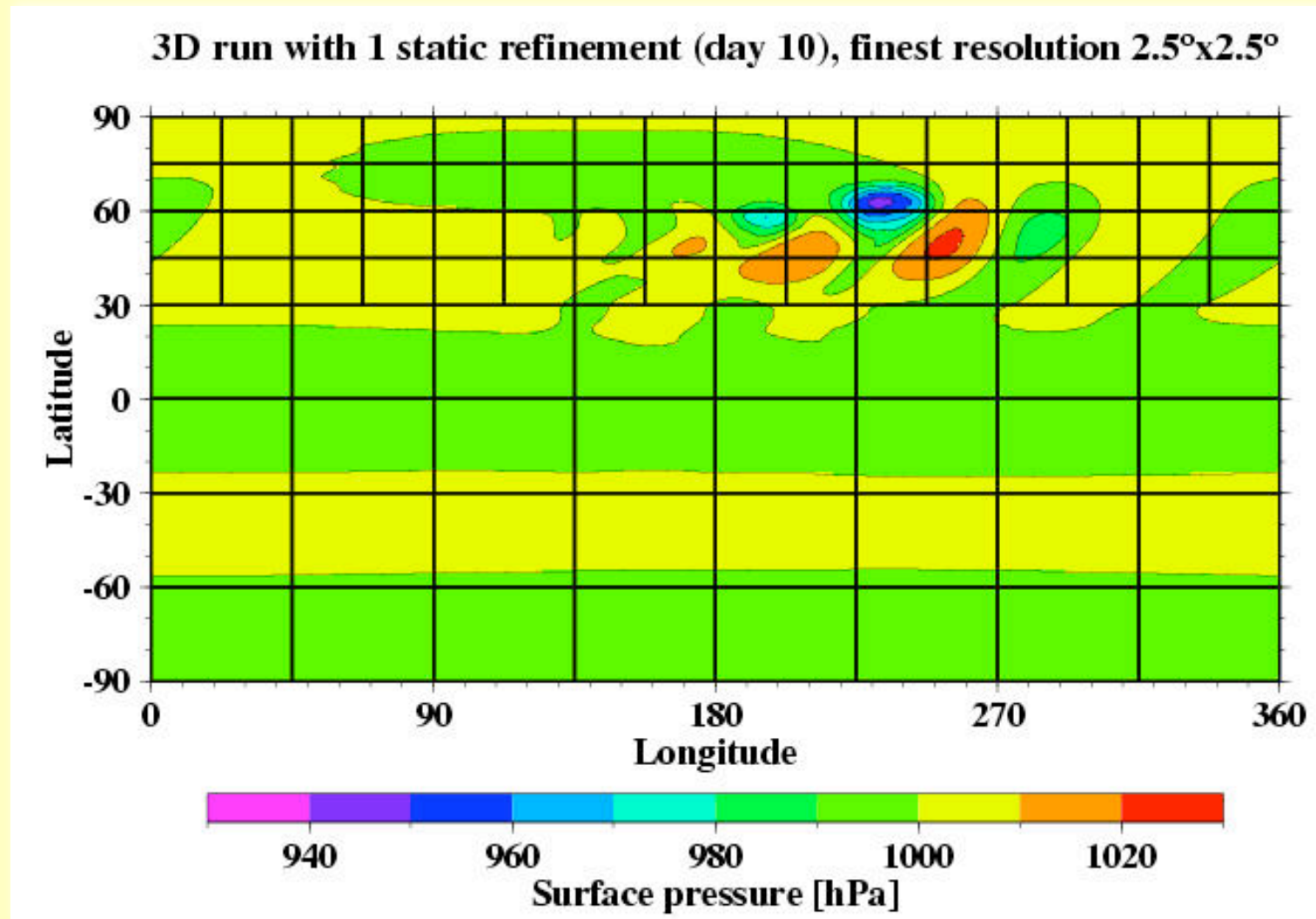
# ***Baroclinic waves: 5° x 5° resolution***

- JW baroclinic wave test case for dynamical cores (our test 2)
- Coarse resolution does not resolve the wave train



# *Static adaptations in 3D*

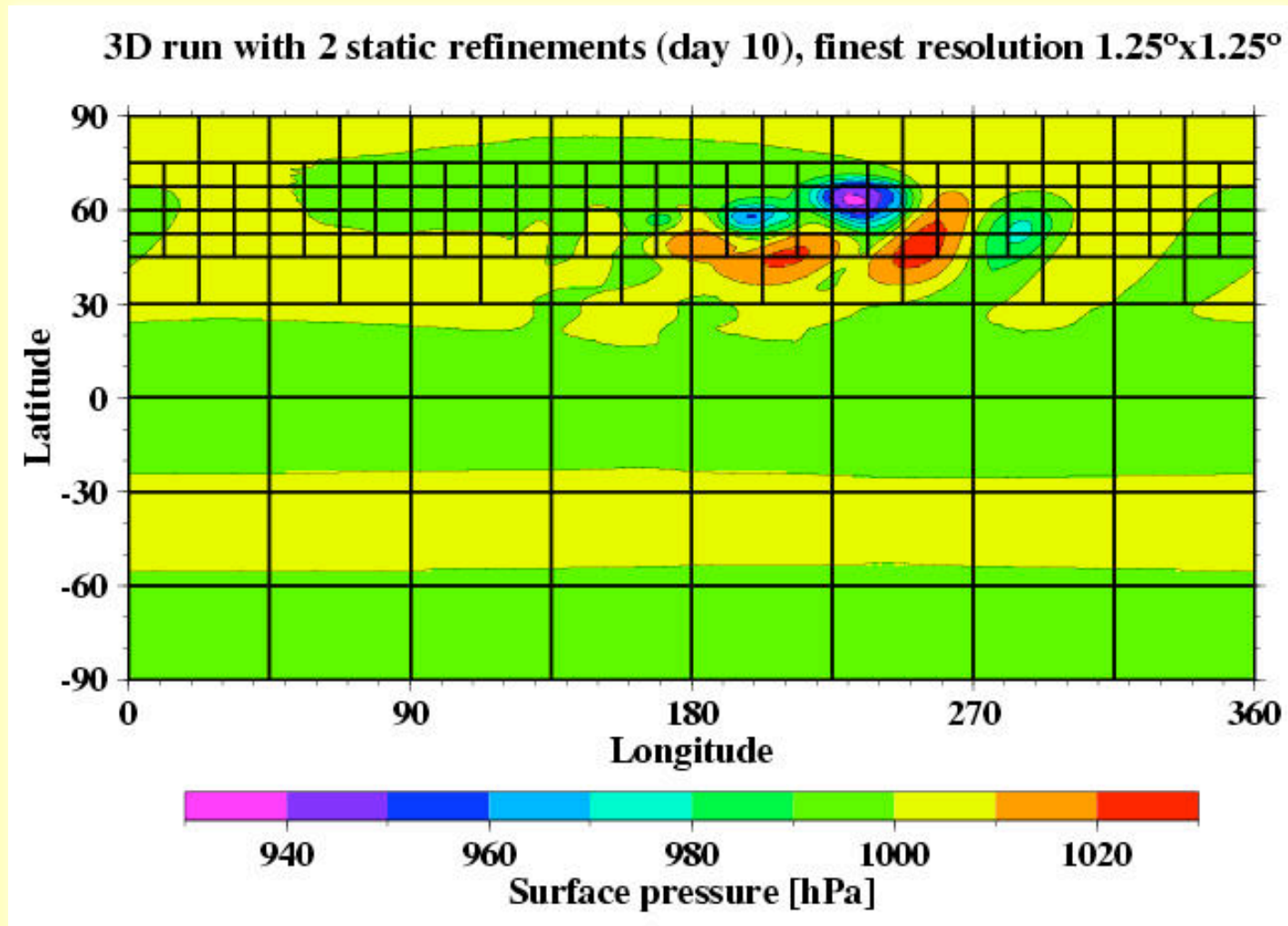
- 1 Refinement along the storm track improves the simulation





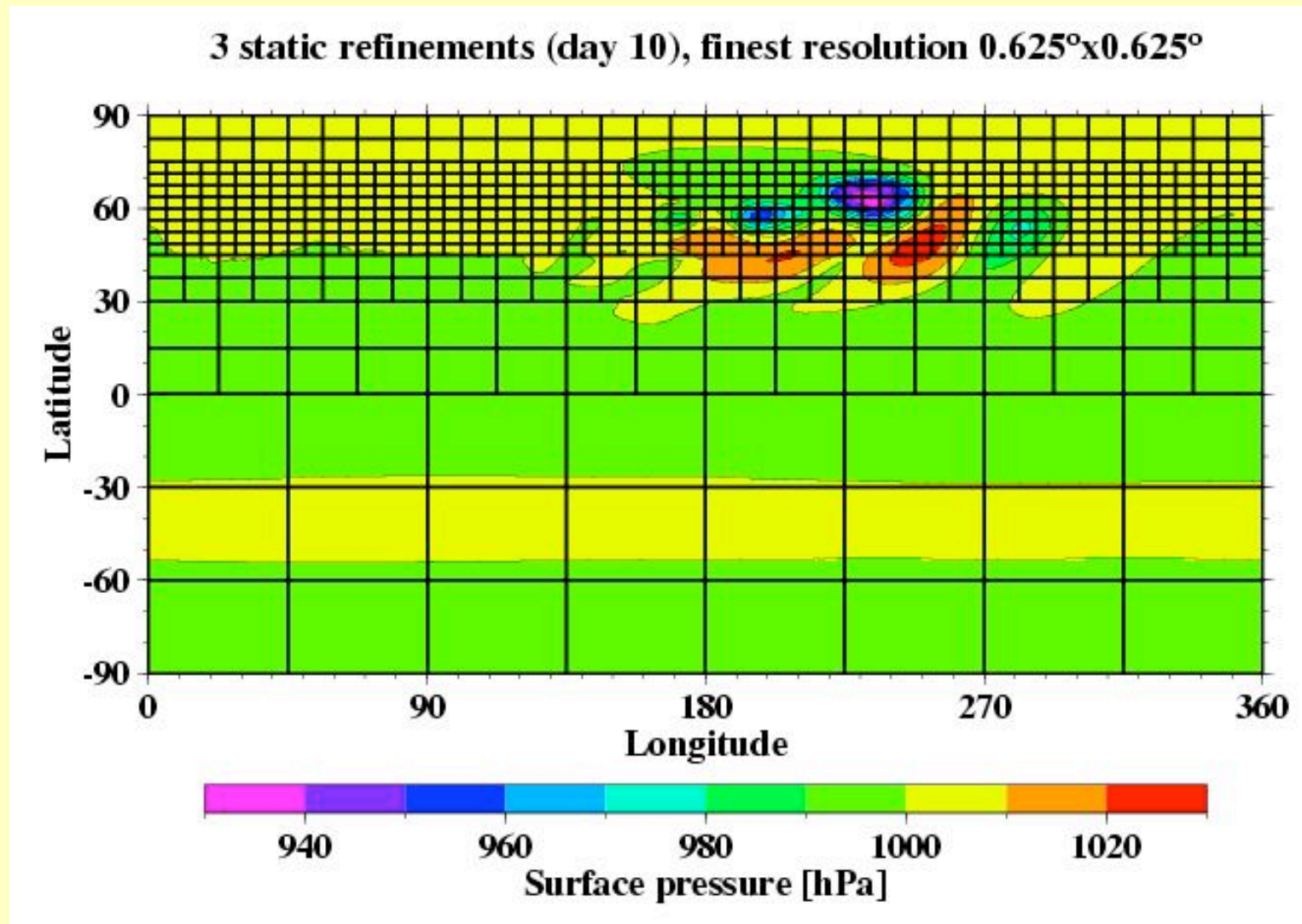
# *Static adaptations in 3D*

- 2 Refinements along the storm track capture the wave accurately



# ***Static adaptations in 3D***

- 3 Refinements along the storm track: no further intensification



# ***Conclusions***

- ✧ Static and dynamic adaptive meshes are worth exploring for weather and climate applications
- ✧ Cubed-sphere or other non-traditional meshes together with AMR have clear advantages:
  - ✧ No convergence of any grid lines, no polar filters
  - ✧ Local grid adaptations are truly local
- ✧ Moving nested grids are already used for (regional) tropical cyclone simulations today (e.g. NCAR's WRF model)
- ✧ Less clear whether dynamic adaptations can be successfully employed in climate models (remember teleconnection patterns), but static refinements are promising (e.g. near mountains, tropical channel)