

curves in \mathbf{R}^2 (tangent)

$$\mathbf{r}_{(t)} = \langle x_{(t)}, y_{(t)} \rangle$$

$$d\mathbf{r} = \langle \partial_t x, \partial_t y \rangle dt$$

$$ds = |d\mathbf{r}|$$

circulation along curve

$$\int_C \mathbf{f} \cdot d\mathbf{r}$$

length of curve

$$\int_C 1 ds$$

mass of curve with density g

$$\int_C g ds$$

curves in \mathbf{R}^2 (normal)

$$\mathbf{r}_{(t)} = \langle x_{(t)}, y_{(t)} \rangle$$

$$d\mathbf{n} = \begin{bmatrix} \mathbf{i} & \mathbf{j} \\ \partial_t x & \partial_t y \end{bmatrix} dt$$

$$ds = |d\mathbf{n}|$$

areas in \mathbf{R}^2

$$\mathbf{r}_{(u,v)} = \langle x_{(u,v)}, y_{(u,v)} \rangle$$

$$dA = \begin{bmatrix} \partial_u x & \partial_u y \\ \partial_v x & \partial_v y \end{bmatrix} du dv$$

$$dA = 1 dx dy$$

$$dA = r dr d\theta$$

scalar product

$$\mathbf{a}c = \langle a_x c, a_y c \rangle$$

gradient

$$\nabla g = \langle \partial_x g, \partial_y g \rangle$$

fundamental theorem of line integrals

$$[g]_{\text{endpts}} = \int_C (\nabla g) \cdot d\mathbf{r}$$

cross product

$$\mathbf{a} * \mathbf{b} = \begin{bmatrix} a_x & a_y \\ b_x & b_y \end{bmatrix} = a_x b_y - a_y b_x \quad \mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y$$

curl

$$\nabla * \mathbf{f} = \begin{bmatrix} \partial_x & \partial_y \\ f_x & f_y \end{bmatrix} = \partial_x f_y - \partial_y f_x \quad \nabla \cdot \mathbf{f} = \partial_x f_x + \partial_y f_y$$

curl theorem (green)

$$\int_C \mathbf{f} \cdot d\mathbf{r} = \iint_A (\nabla * \mathbf{f}) dA$$

dot product

divergence

divergence theorem (green)

$$\int_C \mathbf{f} \cdot d\mathbf{n} = \iint_A (\nabla \cdot \mathbf{f}) dA$$

curves in \mathbf{R}^3 (tangent)

$$\mathbf{r}_{(t)} = \langle x_{(t)}, y_{(t)}, z_{(t)} \rangle$$

$$d\mathbf{r} = \langle \partial_t x, \partial_t y, \partial_t z \rangle dt$$

$$ds = |d\mathbf{r}|$$

circulation along curve

$$\int_C \mathbf{f} \cdot d\mathbf{r}$$

length of curve

$$\int_C 1 ds$$

mass of curve with density g

$$\int_C g ds$$

surfaces in \mathbf{R}^3 (normal)

$$\mathbf{r}_{(u,v)} = \langle x_{(u,v)}, y_{(u,v)}, z_{(u,v)} \rangle$$

$$d\mathbf{S} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_u x & \partial_u y & \partial_u z \\ \partial_v x & \partial_v y & \partial_v z \end{bmatrix} du dv$$

$$dS = |d\mathbf{S}|$$

volumes in \mathbf{R}^3

$$\mathbf{r}_{(u,v,w)} = \langle x_{(u,v,w)}, y_{(u,v,w)}, z_{(u,v,w)} \rangle$$

$$dV = \begin{bmatrix} \partial_u x & \partial_u y & \partial_u z \\ \partial_v x & \partial_v y & \partial_v z \\ \partial_w x & \partial_w y & \partial_w z \end{bmatrix} du dv dw$$

$$dV = 1 dx dy dz$$

$$dV = r dr d\theta dz$$

$$dV = \rho^2 \sin(\phi) d\rho d\theta d\phi$$

flux through surface

$$\iint_S \mathbf{f} \cdot d\mathbf{S}$$

area of surface

$$\iint_S 1 dS$$

mass of surface with density g

$$\iint_S g dS$$

volume of volume

$$\iiint_V 1 dV$$

mass of volume with density g

$$\iiint_V g dV$$

scalar product

cross product

dot product

$$\mathbf{a}c = \langle a_x c, a_y c, a_z c \rangle$$

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = a_x b_x + a_y b_y + a_z b_z$$

gradient

curl

divergence

$$\nabla g = \langle \partial_x g, \partial_y g, \partial_z g \rangle$$

$$\nabla \times \mathbf{f} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_x & \partial_y & \partial_z \\ f_x & f_y & f_z \end{bmatrix}$$

$$\nabla \cdot \mathbf{f} = \partial_x f_x + \partial_y f_y + \partial_z f_z$$

fundamental theorem of line integrals

curl theorem (stokes)

divergence theorem (gauss)

$$[g]_{\text{endpts}} = \int_C (\nabla g) \cdot d\mathbf{r}$$

$$\int_C \mathbf{f} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{f}) \cdot d\mathbf{S}$$

$$\iint_S \mathbf{f} \cdot d\mathbf{S} = \iiint_V (\nabla \cdot \mathbf{f}) dV$$